Joint User Association and Base Station Switching on/off for Green Heterogeneous Cellular Networks

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Abstract—Heterogeneous cellular network (HCN), which generally consists of small cell base stations (SBSs) and macro base stations (MBSs), is proposed as a promising scheme to improve the capacity of the cellular network. However, huge energy consumption by the densely deployed SBSs arises as a challenging problem that should be addressed properly to fulfill the potential of HCNs. In this paper, we aim to minimize the total power consumption of the HCNs by jointly designing energy-efficient user association and SBSs switching schemes. We develop an approximation algorithm to solve the intractable user association problem efficiently, based on which an efficient local search procedure is introduced to minimize the total power consumption of the HCN by controlling active/inactive state of each SBS dynamically. Numerical results demonstrate that our proposed method reduces the energy consumption of the HCN significantly as compared to other representative methods.

Index Terms—Base station sleeping, energy efficiency, heterogeneous cellular networks, user association.

I. INTRODUCTION

With the evolution of information and communication technology (ICT), mobile data traffic is forecasted to grow rapidly [1], raising rigorous requirements on network capacity. Densely deployed small cell base stations (SBSs) are deemed as a prominent architecture to tackle such a serious challenge by offloading data traffic from conventional macro base stations (MBSs) in the cellular systems, which is referred to as heterogeneous cellular network (HCN) [2]. Although the densely deployed SBSs can provide high network throughput, they consume a huge amount of energy. It is reported that the amount of energy consumption caused by ICT is growing at 15%–20% per year, doubling every five years [3]. As a result, it is necessary to shift attention from improving spectral efficiency to energy efficiency so that the sustainable development of the ICT can be achieved. Meanwhile, the excessive energy consumption is contradictory to green resource management and has become a matter of great concern for the coming fifth generation (5G) networks [4].

Recent studies show that base stations (BSs) accounts for about 80% of energy consumption of the whole network [5]. Therefore, the core idea of green cellular network is reducing the energy consumption of BSs, which motivates us to focus on the energy consumption problem of densely deployed SBSs in this work. Cellular network is generally designed to satisfy peak traffic demand in its coverage range rather ignoring the change of traffic demand [6]. Due to the fluctuation of temporal traffic demand, some active BSs with light traffic load still consume a huge amount of energy, causing unnecessary energy consumption. Thus, a reasonable approach to save energy for the cellular network is to turn off part of BSs with no/low load. In [7], the authors optimize the number of active SBSs by analyzing the traffic load model in the network. Numerical results indicate that about 50% SBSs can be turned off without deteriorating the system performance. A fixed time sleeping scheme is presented in [8], which focuses on the energy saving of femtocells. In [9], a dynamic programming algorithm is proposed to minimize the system power consumption by adaptively adjusting the on/off states of the BS. In [10], the power consumption in C-RAN is minimized, where the power consumption tradeoff between the backhauls and the transmission power of access points is investigated.

The switching on/off operation of BSs inevitably involves user association. More specifically, users served by the BS that is turned off should be associated with other active BS. The conventional user association strategies, e.g., maximum received signal power (MRSP) or signal-to-interference-plus-noise ratio (SINR), is unsuitable for the HCN scenarios due to the transmit power disparity between MBS and SBS, resulting that most of the users would be associated with the MBSs in the cellular system. An alternative scheme, called as cell range expansion (CRE), has been proposed by 3GPP in Release 10. CRE scheme increase the number of users served by SBSs by adding a positive bias to the user’s received power from the SBS [11]. However, the value of bias should be adjusted dynamically to accommodate the fluctuations of traffic load.

In this paper, we aim to minimize the total power consumption of the HCNs while satisfying the rate requirements of all users. The main contributions of this work are summarized as follows:

- We present a general power consumption model for the HCNs. Based on the power consumption model, we formulate our optimization problem, where both user association and SBSs switching operation are jointly considered.
- We develop an efficient approximation algorithm for the user association problem in the HCN, which is different from traditional MRSP and SINR schemes. Simulation results demonstrate that our proposal brings more power saving, especially when the SBSs have limited radio
resources.
- We introduce three local search operations that improves the local optimal solutions and obtains the (near) optimal solutions for the SBS switch on/off problem, which minimizes the number of active SBSs while satisfying the rate requirements of users. Numerical results show that our proposed algorithm performs quite well for all considered scenarios.

The rest of this paper is organized as follows. Section II presents system model and formulate optimization task. In Section III, our proposed algorithms are proposed in detail. Section IV shows numerical results, as well as discussions. Conclusions are drawn in Section V.

II. NETWORK MODEL AND PROBLEM FORMULATION

A. Network Model

Consider a typical scenario of HCN, where all SBSs are deployed within the coverage of MBS. We consider that there only exists one MBS for simplifying analysis. The MBS is always active for basic coverage, while the SBSs are located in the coverage of MBS to provide high traffic demands. In contrary to the MBS, the SBSs can be turned off for energy saving or turned on for traffic offloading according to the variation of traffic.

The set of all BSs is denoted as $\mathcal{N} = \{0,1,2,\ldots,N\}$. For each BS $n \in \mathcal{N}$, $p_n^{\text{max}}$ is the maximum transmission power.

In general, the transmission power budget of a MBS (e.g., 46 dBm) is much higher than that of a SBS (e.g., 30 dBm). Note that $n = 0$ denotes the BS is a MBS. As for the spectrum resource, we assume that MBS and SBSs use orthogonal frequency bandwidths to avoid cross-tier interference among them. Denote $h_n^{\text{max}}$ as the available bandwidths of BS $n$.

The spatial distribution of users follows nonhomogeneous Poisson Point Process (PPP), where the density of user within the SBSs and outside the SBSs is different. The MBS can serve users within or outside the SBSs while SBSs can only serve the user within its coverage. The set of users is denoted as $\mathcal{U}_n \subset \mathcal{N}$, respectively.

The transmission rate between BS $n$ and user $k$ can be calculated as

$$r_{k,n} = h_{k,n} \log_2 \left[1 + \frac{b_{k,n} p_{k,n}}{b_{k,n} (N_0 + I_{k,n})}\right],$$

where $N_0$ is the power spectral density (PSD) of additive white Gaussian noise (AWGN), $I_{k,n}$ is the interference introduced by neighbour BSs with unit bandwidth. In this paper, we assume that the co-tier interference and cross-tier interference can be eliminated by introducing coordinated multi-point (CoMP) transmission/reception and inter-cell interference coordination (ICIC) [12]. Thus, $I_{k,n}$ is approximately zero.

B. Power Consumption Model

Based on the power consumption model presented in EARTH project [13], a BS in active mode consume a fixed power and a radio frequency (RF) related power:

$$P_{BS} = P_{fiz} + P_{RF},$$

where $P_{fiz}$ denotes the power consumption for baseband processor, cooling equipment and so on, coefficient $\eta$ is the power amplifier efficiency factor, and $P_{RF}$ is the RF power. Note that the RF power is proportional to the utilized bandwidth $\omega$, i.e.,

$$P_{RF} = \frac{\omega}{\omega_{\text{max}}} P_T, \quad 0 < \omega \leq \omega_{\text{max}},$$

where $P_T$ is the transmit power. In this paper, we assume that users served by BS $n$ consume all the available bandwidth to reduce transmission power consumption, that is, $\omega = B$ and the RF power is only related to the transmission power consumption of BSs. Substituting $P_{RF}$ in (2) with (3), we have

$$P_{BS} = P_{fiz} + \frac{p_n}{\eta},$$

where $p_n$ is the transmission power of BS $n$.

Note that BS still consumes a small amount of power to ensure that the BS can be reactivated from its sleep mode. Compared with $P_{fiz}$, however, the power consumption in inactive state is negligible. Therefore, we assume that the power consumption in sleep mode is zero.

C. Problem Formulation

A binary variable $z_n$ is introduced to indicate whether BS $n$ is active or not,

$$z_n = \begin{cases} 1 & \text{BS } n \text{ is turned on}, \\ 0 & \text{BS } n \text{ is turned off}. \end{cases} \quad \forall n \in \mathcal{N}. \quad (5)$$

For each user $k \in \mathcal{K}$, we introduce $\rho_{k,n}$ to denote the index that indicates whether user $k$ is associated with BS $n$ or not,

$$\rho_{k,n} = \begin{cases} 1 & \text{user } k \text{ is served by BS } n, \\ 0 & \text{otherwise}. \end{cases} \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N}. \quad (6)$$

Our optimization objective is to select a subset of $\mathcal{N}$ to minimize total power consumption while satisfying practical network constraints. The problem can be mathematically formulated as follows:

minimize $z_n, \rho_{k,n}, b_{k,n}, p_{k,n}$

$$\sum_{n \in \mathcal{N}} P_{fiz} z_n + \frac{1}{\eta} \sum_{k \in \mathcal{K}} \rho_{k,n}$$

s.t. $C_1: \sum_{k \in \mathcal{K}} b_{k,n} \leq z_n b_{\text{max}}^{\text{max}}, \forall n \in \mathcal{N},$

$C_2: \sum_{k \in \mathcal{K}} \rho_{k,n} \leq z_n b_{\text{max}}^{\text{max}}, \forall n \in \mathcal{N},$

$C_3: \sum_{n \in \mathcal{N}} \rho_{k,n} \geq R_k^{\text{min}}, \forall k \in \mathcal{K},$

$C_4: \rho_{k,n} \leq z_n, \forall k \in \mathcal{K}, \forall n \in \mathcal{N},$

$C_5: b_{k,n} \geq 0, \rho_{k,n} \geq 0, \forall k \in \mathcal{K}, \forall n \in \mathcal{N},$

$C_6: z_n, \rho_{k,n} \in \{0,1\}, \forall k \in \mathcal{K}, n \in \mathcal{N}.$

$C_1$ and $C_2$ are the bandwidth and power budgets of BS $n$, respectively. $C_3$ means that the rate requirement of user $k$ should be satisfied. $C_4$ indicates that user $k$ should be served by an active BS.
The optimization problem in (7) is very challenging for solving since highly complex coupling of BSs’ on/off decision and user association. Though [14] proposed a generalized algorithm to deal with the cell planning problem, the bandwidth and power requirements of users are not taken into consideration. Before introducing our algorithms, we need to solve the bandwidth and power allocation problem when the set of users is given, where the rate requirements of all users should be satisfied.

A. Bandwidth and Power Allocation Algorithm

The bandwidth and power allocation problem for a given BS and its serving users is illustrated as follows: Given a set of users $K_n$ served by BS $n$, we try to obtain a feasible bandwidth and power assignment that satisfies the rate requirements of all users. To make this problem straightforward, we follow the assumption that users in $K_n$ occupy all available bandwidth $b_{k,n}^{max}$. Then, our optimization objective is to calculate the required minimum power consumption of BS $n$ when the rate requirements of all users can be satisfied. Mathematically, the optimization problem is denoted as follows:

$$\min_{b_{k,n}, p_{k,n}} \sum_{k \in K_n} p_{k,n}$$

s.t. $C_1: \sum_{k \in K_n} b_{k,n} = b_{n}^{max}$, $R_{k,n} = R_{k,min}, \forall k \in K_n$, $b_{k,n} \geq 0, p_{k,n} \geq 0, \forall k \in K_n$.

Substituting (1) into $C_2$ in (8), we have

$$p_{k,n} = \frac{b_{k,n}}{H_{k,n}} \left( 2^{\frac{R_{k,min}}{b_{k,n}}} - 1 \right),$$

where $H_{k,n} = b_{k,n}/N_0$. Thus, the optimization problem in (8) can be converted into

$$\min_{b_{k,n}} \sum_{k \in K_n} b_{k,n} \left( 2^{\frac{R_{k,min}}{b_{k,n}}} - 1 \right)$$

s.t. $C_1: \sum_{k \in K_n} b_{k,n} = b_{n}^{max}$, $C_2: b_{k,n} \geq 0, \forall k \in K_n$.

Since the objective function is convex and all the constraints are affine [15], (10) defines a convex problem that can be solved by standard convex optimization techniques. The Lagrangian of (10) is

$$L = \sum_{k \in K_n} b_{k,n} \left( 2^{\frac{R_{k,min}}{b_{k,n}}} - 1 \right) + \lambda \left( \sum_{k \in K_n} b_{k,n} - b_{n}^{max} \right) - \mu_{k,n} b_{k,n},$$

where $\lambda$ and $\mu_{k,n}$ are the Lagrange multipliers. Let $b^{*}_{k,n}$ and $\lambda^*$, $\mu_{k,n}$ be the primal and dual optimal points with zero duality gap [15]. By introducing KKT conditions [15], we are able to obtain the following equations:

$$\lambda^* = \frac{1}{H_{k,n}} \left[ \left( 1 - \frac{R_{k,min} \ln 2}{b^{*}_{k,n}} \right) 2^{\frac{R_{k,min}}{b^{*}_{k,n}}} - 1 \right],$$

$$\sum_{k \in K_n} b^{*}_{k,n} = b_{n}^{max},$$

$$\mu_{k,n} = 0, b^{*}_{k,n} > 0.$$  (13)

Eq. (13) indicates that BS cannot satisfy the rate requirement of user $k$ if there is no power margin. By introducing bisection method, we can obtain the value of $b^{*}_{k,n}$ and $\lambda^*$. The bandwidth and power allocation algorithm is shown in Table I, where $\epsilon$ is a tolerance and $\Gamma$ is a appropriate positive integer. Denote $P_T(K_n) = \sum_{k \in K_n} b^{*}_{k,n}$ as the obtained optimal value of (10) by Algorithm 1. If $P_T(K_n)$ satisfies the budget power, we claim that BS $n$ can satisfy the rate requirements of all users in $K_n$, in other words, BS $n$ can cover $K_n$. Otherwise, BS $n$ cannot serve all users in $K_n$.

B. Energy-Efficient User Association

As the coupling of binary variables $z_n$ and $\rho_{k,n}$, the optimization problem defined by (7) is hard to solve. Generally speaking, the required time of user association is much less than that of BS turning on/off. This assumption follows the intuition that dynamic BS operation can be executed every a couple of hours, whereas user association is determined over a much finer time granularity. Thus, we first focus on the user association problem without considering the BS turning on/off. The optimization objective of user association is to determine which BS that each user should be associated with.

We can achieve high energy-efficiency by associating user with the BS that consumes the minimum power to serve it under the limitation of the minimum rate requirement. In other words, we need to serve all users with the minimum energy consumption by choosing an appropriate BS set. Define
TABLE II

1\(^{2}\) - APPROXIMATION ALGORITHM FOR USER ASSOCIATION

<table>
<thead>
<tr>
<th>Algorithm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Initialization: ( K_n = \emptyset, \forall n \in \mathcal{N} ); ( \mathcal{K}_r = \mathcal{K}; \mathcal{N}_c = \mathcal{N} );</td>
</tr>
<tr>
<td>2. Calculate the required minimum power ( p_n(k) ), ( k \in \mathcal{K}_r, n \in \mathcal{N}_c );</td>
</tr>
<tr>
<td>3. Repeat</td>
</tr>
<tr>
<td>4. ((k', n') = \arg \min_{(k, n) \in \mathcal{K} \times \mathcal{N}_c} p_n((k)));</td>
</tr>
<tr>
<td>5. If ( p_n(K_{n'} \cup {k'}) \leq p_{n_{\text{max}}} ), ( \mathcal{K}<em>r = \mathcal{K}</em>{n'} \cup {k'} );</td>
</tr>
<tr>
<td>6. ( \mathcal{K}_r = {} );</td>
</tr>
<tr>
<td>7. ( \mathcal{K}_r = {} );</td>
</tr>
<tr>
<td>8. Else</td>
</tr>
<tr>
<td>9. ( \mathcal{K}_r = \mathcal{K} \setminus {n'} );</td>
</tr>
<tr>
<td>10. End if</td>
</tr>
<tr>
<td>11. Until ( \mathcal{K}_r = \emptyset ) or ( \mathcal{N}_c = \emptyset );</td>
</tr>
<tr>
<td>12. Return ( K_n )</td>
</tr>
</tbody>
</table>

Theorem 1. Algorithm 2 is a \( \frac{1}{2} \)-approximation algorithm for user association.

Theorem 2. The total power consumption by \( \mathcal{K}_n \) is \( \frac{1}{2} \) of the optimal solution.

Proof: Let \( \mathcal{K}_n \) be the set of totally satisfied users in the optimal solution. For each BS \( n \in \mathcal{N}_s \), the set of users served by it in the optimal solution is denoted as \( \mathcal{K}_n \). Let \( \mathcal{K}' \) and \( \mathcal{K}'_n \) be the set of users selected by Algorithm 2 and the set of users associated with BS \( n \in \mathcal{N}_s \), respectively.

Based on Algorithm 2, users served by BS \( n \) consume less power compared to the users in \( \mathcal{K}_n \). Therefore, we have

\[
p_n(\{k_2\}) \geq p_n(\{k_1\}), \quad \forall k_1 \in \mathcal{K}_{n'_1}, k_2 \in \mathcal{K}_n \setminus \mathcal{K}'.
\]

Moreover, we can obtain

\[
p_n(\mathcal{K}'_n \cup \{k_2\}) \geq p_{n_{\text{max}}},
\]

for each user \( k_2 \in \mathcal{K}_n \setminus \mathcal{K}' \) since user \( k_2 \) is not assigned to BS \( n \). As the users in \( \mathcal{K}_n \) are served by BS \( n \), we also have

\[
p_n(\mathcal{K}_n) \leq p_{n_{\text{max}}},
\]

Combining (16) and (17), we can obtain

\[
p_n(\mathcal{K}_n) > p_{n_{\text{max}}},
\]

which can be expressed as

\[
|\mathcal{K}_n'| \geq |\mathcal{K}_n| + 1.
\]

Note that \( \mathcal{K}_n \cap K' \) denotes the selected users in \( \mathcal{K}_n \). Thus \( (\mathcal{K}_n \cap K') \cup (\mathcal{K}_n \cap K') = \mathcal{K}_n \) and finally we get

\[
|\mathcal{K}_n'| \geq \frac{1}{2}|\mathcal{K}_n|.
\]

C. Energy-Efficient SBS Turning on/off

We introduce an algorithm to reduce the total power consumption by turning off some lightly-load SBSs. When \( z_n \) is considered as a variable, the problem (7) falls into a combinatorial problem with \( O(2^V) \) possible cases. The complexity of finding an optimal solution by exhaustive search is too high even for medium scale case. Here, we introduce three efficient local improvement operations to minimize the total power consumption.

Starting with a feasible solution, e.g., \( \mathcal{N}_f = \mathcal{N}_s \), the total power consumption is denoted as \( P(\mathcal{N}_f) \). Then, we can perform three local improvement operations to minimize the total power consumption:

- 'open' operation: Turn on an SBS in sleep mode and perform Algorithm 2. If \( P(\mathcal{N}_f \cup \{n\}) < P(\mathcal{N}_f) \), add SBS \( n \) to \( \mathcal{N}_s \); \( \mathcal{N}_f = \mathcal{N}_f \cup \{n\} \).

- 'close' operation: Turn off an SBS in active state \( n \in \mathcal{N}_f \) and perform Algorithm 2. If \( P(\mathcal{N}_f \setminus \{n\}) < P(\mathcal{N}_f) \), remove SBS \( n \) from \( \mathcal{N}_f \); \( \mathcal{N}_f = \mathcal{N}_f \setminus \{n\} \).
`exchange` operation: Turn on an SBS \(n' \in \mathcal{N}\backslash\mathcal{N}_f\) in sleep mode and turn off an active SBS \(n \in \mathcal{N}_f\). If \(P(\mathcal{N}_f \backslash \{n\} \cup \{n'\}) < P(\mathcal{N}_f),\) set \(\mathcal{N}_f \leftarrow \mathcal{N}_f \backslash \{n\} \cup \{n'\}\).

Obviously, \(\mathcal{N}_f\) is locally optimal solution when the power consumption can not be reduced. The SBS turning on/off procedure terminates then.

IV. NUMERICAL RESULTS AND DISCUSSIONS

We give a series of numerical results to evaluate the performance of our proposed algorithm. We consider a two-tier HCN with a MBS and a given set of SBSs. The simulation parameters of the HCNs, such as system bandwidth, maximum transmission power, path-loss model, etc., are given in [16]. All results are averaged over 500 Monte Carlo simulations.

The region served by the HCN has an area of \(2 \times 2 \text{ km}^2\). The bandwidth of each SBS is chosen from \([20 40 60 80 100]\) MHz randomly, while the bandwidth of MBS is 100 MHz. The maximum transmission power of MBS is 40 W, while the maximum transmission powers of SBSs are 1 W. As for path loss model, path loss (in dB) from MBS to user is \(128.1 + 37.6 \log_{10}(D)\) (in dB), while path loss from SBS to user is calculated as \(140.7 + 36.7 \log_{10}(D)\), where \(D\) (in km) is the distance between BS and user. The standard deviation of lognormal shadowing is 10 dB and the noise power spectral density is \(-184\) dBm/Hz. Besides, the required minimum rate of each user is chosen randomly from \([0.1 1 10]\) Mbps. For BS power consumption model, we do not consider the fixed power consumption of the MBS since it is always active for basic coverage. The fixed power consumption for each active SBS is 3.1 W. In general, the coefficient \(\eta\) of RF power amplifier is 25%.

First, we evaluate the influence of the number of users on the transmission power consumption under the case that \(N = 20\). As can be seen in Fig. 1, our proposed user association algorithm has advantages over other user association schemes. Intuitively, SINR-based scheme has poor performance since the majority of users are served by the MBS, resulting in high amount of power consumption. Under the CRE strategy, the decrease of power consumption benefits from the offloading traffic from MBS to SBSs with low transmission power. The power consumption gap has an increasing trend with the increase of number of users. We can draw a conclusion that our proposed user association scheme performs well even if radio resources of BSs are strained.

Then, we investigate the convergence of the proposed SBS turning on/off algorithm. Fig. 2 illustrates the total power consumption of the HCNs during each iteration, where \(K = 60, N = 20\) and \(K = 100, N = 20\). As shown in Fig. 2, we can claim that the proposed SBS turning on/off algorithm converges rapidly. The required number of iterations for convergence is inversely proportional to the number of users. 20 iterations are needed to converge for the case \(K = 60, N = 20\), while only 9 iterations are required for the case \(K = 100, N = 20\). It makes sense that more SBSs could be closed for the fewer number of users. Also, Fig. 2 demonstrates that our proposal can reduce the power consumption effectively compared to no SBS turning on/off algorithm, about 56.8% and 30.2% total power consumption can be reduced with our proposal for the case that \(K = 60, N = 20\) and \(K = 100, N = 20\), respectively.

Finally, we evaluate the performance of our proposal. We compare the proposed SBS turning on/off algorithm with the following ones: no SBS turning on/off and greedy-off (GOFF) algorithm. All SBSs are active for the former, whereas the SBS that achieves the largest power saving with sleep mode is turned off for the latter. For the sake of comparison, we employ user association algorithm shown in Algorithm 2. Fig. 3 shows that the total power consumption of the HCNs as a function of \(\eta\). The number of users and SBSs are 80 and 20, respectively. We can observe from Fig. 3 that our proposal outperforms the GOFF algorithm. Specifically, up to 38%-43% energy reduction can be achieved by applying our proposal. We can make a conclusion that our proposal can reduce the total power consumption of the HCNs effectively. An interesting observation can be obtained from Fig. 3 is that...
the total power consumption has an increasing trend with the decrease of $\eta$ when $\eta$ is small enough. This is because the benefit that is achieved by turning off SBS will be eliminated when transmission power consumption takes a large part of total power consumption.

V. CONCLUSION

In this paper, we studied how to minimize the total power consumption by dynamically adjusting the on/off states of the SBSs in the heterogeneous cellular networks. First, we develop an approximation algorithm with approximation ratio of $\frac{1}{2}$ to deal with the user association problem for a given set of BSs. Then, we introduce an efficient local search algorithm to minimize the number of active SBSs so that the total energy consumption can be reduced. Numerical results demonstrate that our proposed scheme can dramatically reduce the energy consumption of the HCNs while satisfying the rate requirements of all users.

APPENDIX A

PROOF OF LEMMA 1

**Fact 1.** Given positive numbers $A, a_1, a_2$ such that $a_1 - 1 \geq A \cdot (a_2 - 1)$, where $a_1 > 1, a_2 > 1$ and $A > 0$, then

$$a_1^n - 1 \geq A \cdot (a_2^n - 1), \forall n \geq 1, n \in \mathbb{R}.$$  

Let $p_{k_1,n}^*$ and $b_{k_1,n}^*$ be the optimal power and bandwidth allocation for $p_n(K_1)$, respectively. According to (14) and $p_n(K_2) = p_n(K_1)$, $\forall k_1 \in K_1, k_2 \in K_2$, we have

$$\frac{1}{H_{k_1,n}} \left(2^{R_{min}^{k_1}} b_{k_1,n}^{max} - 1 \right) \geq \frac{1}{H_{k_2,n}} \left(2^{R_{min}^{k_2}} b_{k_2,n}^{max} - 1 \right).$$  

For each $k_1 \in K_1, k_2 \in K_2$, we have

$$p_{k_1,n}^* = \frac{b_{k_1,n}^*}{H_{k_1,n}} \left(2^{R_{min}^{k_1}} b_{k_1,n}^{max} - 1 \right) \geq \frac{b_{k_1,n}^*}{H_{k_2,n}} \left(2^{R_{min}^{k_2}} b_{k_2,n}^{max} - 1 \right)$$  

$$= \frac{b_{k_1,n}^*}{H_{k_2,n}} \left(2^{R_{min}^{k_2}} b_{k_2,n}^{max} - 1 \right),$$  

where the inequality follows Fact 1 since $b_{k_1,n}^* \leq b_{k_2,n}^{max}$ is always holds. For users in $K_2$, we allocate as same bandwidth as users in $K_1$, e.g. $b_{k_1,n}^*$. Therefore, we have

$$p_n(K_1) = \sum_{k_1 \in K_1} p_{k_1,n}^* \geq \sum_{k_2 \in K_2} \frac{b_{k_2,n}^*}{H_{k_2,n}} \left(2^{R_{min}^{k_2}} b_{k_2,n}^{max} - 1 \right) \geq p_n(K_2).$$

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