Energy-Efficient Resource Management in OFDM-Based Cognitive Radio Networks Under Channel Uncertainty

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***Abstract*—In this paper, we investigate the energy consumption issue in cognitive radio (CR) networks. We aim to maximize the energy efficiency of the CR network while considering practical restrictions, including the power budget of the system, the interference thresholds of the primary users (PUs), the rate requirements of the secondary users, and the fairness among them. Particularly, due to the lack of explicit support from the PU system, perfect channel state information may not be acquired. Thus, the interference constraint is posed as chance-constrained form and tackled by Bernstein approximation. Then, we convert the optimization task into a quasi-convex problem via relaxing the integer variables, followed by a simple rounding technique to yield feasible subchannels assignment. We derive a fast algorithm to distribute power among subchannels by exploiting the structure of the power-allocation problem. Moreover, we give an efficient heuristic algorithm for subchannels assignment, which reduces the computation load dramatically. Simulation results show that both our proposed resource allocation schemes perform well in practical scenarios. The energy efficiency obtained by the integer subchannels assignment and the fast power distribution achieves more than 98% of the upper bound. On the other hand, the proposed heuristic subchannels assignment with optimal power allocation achieves a good tradeoff between computation complexity and energy efficiency.**

***Index Terms*—Cognitive radio, energy efficiency, proportional fairness, resource allocation.**

I. Introduction

With the ever increasing demand for mobile and wireless applications, radio spectrum becomes more and more crowded. However, investigations show that large portion of spectrum is highly underutilized due to inefficient regulatory policies [1]. As a promising technique to improve the usage efficiency of spectrum, Cognitive Radio (CR) has attracted much attention in the past decade [2]. The Secondary Users (SUs) served by the CR system sense radio spectrum environment and dynamically adjust their transmission parameters to access the licensed spectrum used by the Primary Users (PUs), as long as the interference to the PUs is kept below their tolerable thresholds. In order to meet the requirements of opportunistic access, the physical layer of the CR system should be very flexible, which necessitates multicarrier methods to operate in the CR scenario. Orthogonal Frequency Division Multiplexing (OFDM) is widely recognized as an ideal air interface for the CR system due to its flexibility in allocating radio resource among the SUs, which is the prerequisite for the CR system to acquire high throughput [3].

As an important issue in the OFDM-based systems, Resource Allocation (RA) has been studied extensively in the literature, as can be found in [4] and the references therein. The optimization objectives of the RA include maximizing the system throughput, minimizing the transmission power, or serving more users with Quality-of-Service (QoS) guarantee. For an OFDM-based CR system, there are also many fruitful results on how to improve the system throughput. RA for the single SU case is investigated in [5]–[7]. In [5], optimal and suboptimal power loading algorithms are presented. The downlink sum capacity is maximized with the constraint that the interference to the PUs is within a tolerable range. In [6], an efficient algorithm is proposed to allocate bits among all OFDM subchannels in the CR system, which can produce optimal solutions with low complexity in most cases. In [7], a fast algorithm is derived to work out optimal power allocation for the OFDM-based CR network. RA algorithms for the multiuser CR systems have been proposed in [8], [9]. In [8], both real-time and non-real-time services are considered and fast RA algorithms are developed. In [9], a general RA framework is investigated. The proposed algorithms show that RA in the OFDM-based CR network can be tackled effectively and efficiently by exploiting the structure of the problem.

Compared to the flourish on capacity enhancing, less attention has been paid to the energy efficiency of the CR system until excessive energy consumption becomes a critical issue because of consequent environment problems and operational cost [10]. With the rapid growth of high speed data services, the energy consumption of wireless systems is also growing at a staggering rate, leading to a large amount of greenhouse gas and high operation expenditure for service providers. Green communication which emphasizes on incorporating energy awareness in communication systems is becoming urgent [11]. As a result, energy-efficient RA has attracted attention in both

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industry and academia recently, especially for the OFDM-based system which is the most popular modulation technique for current wireless networks. Different from the throughput-oriented optimization targets, energy efficient RA aims at maximizing the energy efficiency of the wireless system. One of the energy-efficiency metrics is called "*bits-per-Joule*," which is defined as the system throughput to unit-energy consumption. In [12], the authors overview major international projects about energy-efficient wireless networks and discuss state-of-the-art researches on this topic. In [13], the energy efficiency of an OFDM-based system is maximized, where multiple radio access technologies are employed for parallel transmission. In [14], an energy-efficient RA scheme is developed for a downlink multiuser OFDM system with distributed antennas while considering proportional fairness among users. In [15], the RA problem for a downlink OFDM network with hybrid energy harvesting base stations is investigated.

Energy-efficient RA is also important for the CR systems because it is the prerequisite to achieve high utilization of the limited power budget, as well as to prevent unacceptable interference to the PUs. Since there are remarkable differences between energy efficiency and spectrum efficiency from the viewpoint of optimization task, many existing RA algorithms which focus on spectrum efficiency are no longer suitable for the energy efficiency scenarios. In [16] and [17], energy-efficient RA for an OFDM-based CR network is discussed, where the energy-efficiency of the system is maximized under the constraints of the transmission power budget, the interference thresholds of the PUs and the traffic demands of the SUs. However, fairness among the SUs is not involved. In fact, RA in the CR systems not only requires that the SUs can access spectrum without excessive interference to the PUs, but also demands fair resource scheduling among the SUs. Once fairness is considered, the optimization task will be more difficult to deal with.

The energy-efficient CR systems discussed above generally require that the SUs have perfect knowledge of channel gains from the transmitters of the SUs to the receivers of the PUs. However, this is possible only when the PUs are cooperative by offering a feedback mechanism for channel estimation. It is questionable because the cooperation from the PUs may not always be the case since they can use the spectrum exclusively. Therefore, it is of great importance to study the energy-efficient RA scheme under channel uncertainty. Interference constraints under channel uncertainty can be cast as chance constrained form [18], where the optimization objective is to maximize the weighted sum throughput of the system under the transmission power budget and the interference constraints raised by the PUs. Generally speaking, chance constraints are typically more difficult to handle than their deterministic counterparts as they may be nonconvex. Moreover, it is difficult to express the constraints in closed forms. As a result, the convex approximation of chance constraint is of great merit as exemplified in [19], where Bernstein approximation is employed to tackle the chance constraint.

In this paper, we study the energy efficient RA for the OFDM-based CR system. Different from the models considered in [18] and [19], our target is to maximize the overall energy efficiency of the CR system while considering proportional fairness among the SUs. Besides, the interference to the PUs is kept below their tolerable thresholds in chance constrained form. Additionally, the throughput requirement of the CR system is also required. The main contributions of this work are summarized as follows:

- We formulate a general energy-efficient RA model which covers essential constraints for the OFDM-based CR systems. Particularly, the PU interference constraint is posed as a chance constrained form. Our model can be easily extended to many practical scenarios with necessary modifications.
- We introduce Bernstein method to approximate the probabilistic constraint with a tractable convex form and propose a time-sharing method to get a tight upper bound of the objective function, based on which feasible subchannels assignment can be obtained.
- We derive a fast power allocation algorithm for given subchannels assignment by exploiting the structure of the power allocation problem, reducing computation complexity significantly.
- We develop an efficient heuristic subchannels allocation algorithm to produce (near) optimal solutions with lower complexity, which yields a good tradeoff between energy efficiency and computational complexity.

The rest of this paper is organized as follows. In Section II, we illustrate system model and formulate our optimization task. In Section III, the Bernstein method is introduced to approximate the probabilistic constraint. In Section IV, we develop a relaxation form to obtain an upper bound of the solution. In Section V, we propose a fast barrier method to solve the power distribution problem for given subchannels assignment. In Section VI, an efficient heuristic algorithm for subchannels allocation is presented, which can achieve a tradeoff between energy efficiency and complexity. Simulation results are given in Section VII, as well as discussions. Finally, we conclude the paper in Section VIII.

II. System Model and Problem Formulation

A. System Model

Consider the downlink of an OFDM-based CR system with K SUs, denoted by $\mathcal{K} = \{1, 2, \ldots, K\}$, coexisting with L active PUs served by a licensed system. The available bandwidth W is divided into N OFDM subchannels in the CR system, denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$. The interference introduced to the lth PU by the SUs' access on the nth subchannel with unit transmission power is represented as $I_{n,l}^{SP}$. On the other hand, the interference generated by the lth PU into the nth subchannel used by the kth SU is $I_{k,n,l}^{PS}$. Note that we do not assume that the PUs also adopt OFDM modulation. In practical systems, there are several mechanisms to obtain Channel State Information (CSI) between the CR transmitters and the receivers of PUs. CSI can periodically be measured by a band manager and sent back to the CR transmitter by a common control channel. Alternatively, CSI can be estimated by listening to a beacon signal and then fed-back to the CR transmitter. Besides, a centralized controller is assumed to do resource scheduling for the SUs.

Define the Signal-to-Noise Ratio (SNR) of the kth SU on the nth subchannel as

$$H_{k,n} = \frac{g_{k,n}^{SS}}{\Gamma\left(N_0 B + \sum_{l=1}^{L} I_{k,n,l}^{PS}\right)}, \tag{1}$$

where $g_{k,n}^{SS}$ is the power gain of the kth SU on the nth subchannel, N_0 is the PSD of additive white Gaussian noise, Γ is the SNR gap and can be represented as $\Gamma = -\frac{\ln(5BER)}{1.5}$ for an uncoded MQAM with a specified BER [20]. The transmission rate of the nth subchannel used by the kth SU is

$$r_{k,n} = \rho_{k,n} \log(1 + p_{k,n} H_{k,n}), \tag{2}$$

where $p_{k,n}$ is the kth SU's transmission power on the nth subchannel. $\rho_{k,n}$ can only be either 1 or 0, indicating whether the nth subchannel is used by the kth SU or not. The data rate of the kth SU, denoted by R_k, can be represented as

$$R_k = \sum_{n=1}^{N} r_{k,n}. \tag{3}$$

The energy-efficiency η_{EE} is defined as the ratio of the system throughput over the total power consumption,

$$\eta_{EE} = \frac{\sum_{k=1}^{K} R_k}{\sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} p_{k,n} + P_c}, \tag{4}$$

where P_c is the circuit power consumption which can be a constant [21]. Both the transmission power and the circuit energy consumption are considered in this work. The former is used for reliable data transmission and the latter is the average energy consumption of electronic devices.

B. Problem Formulation

Our optimization objective is to maximize the energy-efficiency of the CR system which operates in a power-limited situation while guaranteeing the proportional fairness among the SUs, controlling the interference to the PUs under their specified thresholds and satisfying the minimum throughput requirement of the CR system. Mathematically, our optimization problem can be formulated as follows,

$$\begin{aligned}
&\max_{r_{k,n},\rho_{k,n}} \ \eta_{EE} \\
\text{s.t.}\ & C1: \sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} r_{k,n} \geq R_{min} \\
& C2: \sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} p_{k,n} \leq P_t \\
& C3: P_r\left\{\sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} p_{k,n} I_{n,l}^{SP} \leq I_l^{th}\right\} \geq 1-\epsilon, \qquad \forall\, l \\
& C4: r_{k,n} \geq 0, \qquad \forall\, k, n \\
& C5: \rho_{k,n} \in \{0, 1\}, \qquad \forall\, k, n \\
& C6: \sum_{k=1}^{K} \rho_{k,n} = 1, \qquad \forall\, n \\
& C7: R_1 : R_2 : \cdots : R_K = \gamma_1 : \gamma_2 : \cdots : \gamma_K,
\end{aligned} \tag{5}$$

where P_t is the power limit of the CR system and I_l^{th} is the interference power threshold of the lth PU. C1 is the throughput requirement of the CR system. C2 is the power limitation constraint. C3 enforces that the interference power at the lth PU stays below I_l^{th} with probability no less than $1-\epsilon$, where $\epsilon \in (0, 1)$ denotes the desired upper-bound on the probability that the interference threshold is exceeded. C4 is intuitive. C5 and C6 indicate that subchannels are not shared among the SUs. That is, a subchannel can be used by only one SU. C7 is the proportional rate constraints of the SUs to achieve fairness, where γ_1 to γ_K are pre-defined constants. We introduce the proportional fairness into the system by adding a set of nonlinear constraints, based on which we can explicitly control the capacity ratios among all users and ensure that all users can obtain their target data rates if sufficient power budget is available. Fairness index $\mathcal{F}$ is defined as follows [22],

$$\mathcal{F} = \frac{\left(\sum_{k=1}^{K} \gamma_k\right)^2}{K \sum_{k=1}^{K} \gamma_k^2}, \tag{6}$$

where the maximum value of 1 indicates the greatest fairness, That is, all users achieve the same data rate.

The feasible set of C3 in (5) can be either convex or nonconvex, depending on the distribution of $I_{n,l}^{SP}$. However, even if C3 is convex, it is not straightforward to express it in closed form, rendering the optimization problem intractable. Generally, the formulated problem is nonconvex because both binary integer variables $\rho_{k,n}$'s and real variables $r_{k,n}$'s are involved. In this paper, we employ Bernstein method to obtain a convex approximation to (5). The approximation is conservative in the sense that it implies the original constraint C3 in (5) [18], [19]. Then we develop efficient subchannels assignment and power allocation algorithms to maximize the energy efficiency of the system.

III. Bernstein Approximation of Chance Constraints

Eq. (5) defines a chance-constrained optimization problem that can be tackled effectively by Bernstein approximation. Consider a chance constraint of the following form:

$$P_r\left\{f_0(\boldsymbol{p}) + \sum_{n=1}^{N} \eta_n f_n(\boldsymbol{p}) < 0\right\} \geq 1-\epsilon, \tag{7}$$

where $\boldsymbol{p}$ is a deterministic parameter and η_n is random variable with marginal distribution ξ_n. Suppose that one desires to meet this constraint for a given family of η_n distributions under the following assumptions:

1) f_n is affine in $\boldsymbol{p}$, $\forall\, n = 1, 2, \cdots, N$.
2) η_n is independent of each other.
3) ξ_n has a common bounded support of $[-1,1]$, that is $-1 \leq \eta_n \leq 1, \forall\, n = 1, 2, \cdots, N$.

Under these assumptions, the following constraint constitutes a conservative approximation,

$$\inf_{\lambda>0}\left[f_0(\boldsymbol{p}) + \lambda \sum_{n=1}^{N} \Omega_n\left(\lambda^{-1} f_n(\boldsymbol{p})\right) + \lambda \log\left(\frac{1}{\epsilon}\right)\right] \leq 0, \tag{8}$$

where $\Omega_n(y) = \max_{\xi_n} \log(\int \exp(xy)d\xi_n(x))$. Notice that (8) is convex [23], [24], such an approximation can be useful to solve (5) if $\Omega_n(y)$ could be calculated efficiently. In general, one can consider an upper bound of $\Omega_n(y)$ given by

$$\Omega_n(y) \le \max\left\{\mu_n^- y, \mu_n^+ y\right\} + \frac{\sigma_n^2}{2}y^2, \qquad n = 1, 2, \cdots, N, \quad (9)$$

where μ_n^-, μ_n^+ with $-1 \le \mu_n^- \le \mu_n^+ \le 1$ and $\sigma_n \ge 0$ are constants that depend on the given families of probability distributions. Replace $\Omega_n(\cdot)$ in (8) with this upper bound and invoke the arithmetic-geometric inequality, we have

$$f_0(\boldsymbol{p}) + \sum_{n=1}^{N} \max\left\{\mu_n^- f_n(\boldsymbol{p}), \mu_n^+ f_n(\boldsymbol{p})\right\} + \sqrt{2\log\frac{1}{\epsilon}}\left(\sum_{n=1}^{N}\sigma_n^2 f_n(\boldsymbol{p})^2\right)^{\frac{1}{2}} \le 0, \quad (10)$$

which is a convex conservative substitute for (7). Assume that the distributions of $I_{n,l}^{SP}$ have bounded supports $[a_n, b_n]$. Then we introduce constants $\alpha_n = \frac{1}{2}(b_n - a_n)$ and $\beta_n = \frac{1}{2}(b_n + a_n)$ to normalize the supports to [−1,1] as follows,

$$\xi_n = \frac{I_{n,l}^{SP} - \beta_n}{\alpha_n} \in [-1, 1]. \quad (11)$$

Let $f_0(\boldsymbol{p}) = -I_l^{th} + \beta_n \rho_{k,n} p_{k,n}$ and $f_n(\boldsymbol{p}) = \alpha_n \rho_{k,n} p_{k,n}$, then (7) is equivalent to C3 in (5). Substitute them into (10), we obtain

$$\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\gamma_n \rho_{k,n} p_{k,n} + \sqrt{2\log\frac{1}{\epsilon}}|\sigma_n \alpha_n \rho_{k,n} p_{k,n}|\right) \le I_l^{th}, \quad (12)$$

where $\gamma_n = \mu_n^+ \alpha_n + \beta_n$. Replace C3 in (5) by (12), and recall that $\sigma_n \alpha_n \rho_{k,n} p_{k,n} \ge 0$, we have

$$\begin{aligned}
&\max_{r_{k,n}, \rho_{k,n}} \ \eta_{EE} \\
\text{s.t.}\ & \text{C1}: \sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} r_{k,n} \ge R_{min} \\
& \text{C2}: \sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} p_{k,n} \le P_t \\
& \text{C3}: \sum_{n=1}^{N}\sum_{k=1}^{K}\left(\gamma_n \rho_{k,n} p_{k,n} + \sqrt{2\log\frac{1}{\epsilon}}\sigma_n \alpha_n \rho_{k,n} p_{k,n}\right) \\
&\qquad \le I_l^{th}, \qquad \forall\, l \\
& \text{C4}: p_{k,n} \ge 0, \qquad \forall\, k, n \\
& \text{C5}: \rho_{k,n} \in \{0, 1\}, \qquad \forall\, k, n \\
& \text{C6}: \sum_{k=1}^{K} \rho_{k,n} = 1, \qquad \forall\, n \\
& \text{C7}: R_1 : R_2 : \cdots : R_K = \gamma_1 : \gamma_2 : \cdots : \gamma_K.
\end{aligned} \quad (13)$$

IV. An Upper Bound of the Energy Efficiency

Since both binary integer variables $\rho_{k,n}$'s and real variables $r_{k,n}$'s are involved in (13), it defines a mixed integer programming problem. The main difficulty to solve (13) lies in the integer constraints, which generate K^N possible subchannels assignments. An intuitive strategy to tackle it is the time-sharing method which relaxes the integer variables into continuous ones. The optimal solution to the relaxed form is always an upper bound because all feasible solutions to the original problem fall into the solution space of the relaxed one. Redefine the variable $\rho_{k,n} \in [0, 1]$ as the faction of the nth subchannel allocated to the kth SU, the transmission power and the achievable rate of the nth subchannel used by the kth SU can be represented as $\rho_{k,n} p_{k,n}$ and $\rho_{k,n} \log(1 + p_{k,n} H_{k,n})$, respectively. Eq. (13) can be converted into the following form,

$$\begin{aligned}
&\max_{r_{k,n}, \rho_{k,n}} \ \eta_{EE} = \frac{\sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n}}{\sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} p_{k,n} + P_c} \\
&\text{s.t. } 0 \le \rho_{k,n} \le 1, \qquad \forall\, k, n \\
&\text{C1} \sim \text{C4, C7 in (13).}
\end{aligned} \quad (14)$$

Theorem 1: η_{EE} is strictly quasi-concave for $\mathbf{z}$, where $\mathbf{z} = (r_{1,1}, \rho_{1,1}, r_{1,2}, \cdots, \rho_{K,N})$.

Proof: Denote the superlevel sets of $\eta_{EE}(\mathbf{z})$ as

$$S_\alpha = \{\mathbf{z} \in \mathbf{dom}\ \eta_{EE} | \eta_{EE} \ge \alpha\}, \quad (15)$$

where **dom** is the domain operator. For $\alpha \in \mathcal{R}$, $\eta_{EE}(\mathbf{z})$ is strictly quasi-concave for $\mathbf{z}$ if and only if S_α is strictly convex for any real number α [25]. When $\alpha < 0$, no points exist in $\eta_{EE} < \alpha$ and $S_\alpha = \{\mathbf{z} \in \mathbf{dom}\ \eta_{EE}\}$. When $\alpha > 0$, $S_\alpha = \{\mathbf{z} \in \mathbf{dom}\ \eta_{EE} | \sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n} - \alpha(\sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} \frac{e^{\frac{r_{k,n}}{\rho_{k,n}}} - 1}{H_{k,n}} + P_c) \ge 0\}$. Obviously, S_α is strictly convex for $\mathbf{z}$. □

Furthermore, Eq. (14) is a quasi-convex optimization problem because the inequality constraint functions in (14) are all convex while the equality constraint functions are affine [25]. A general approach to the quasi-convex optimization problems relies on the representation of the sublevel sets of the quasi-convex function via a family of convex inequalities. Denote $f(\mathbf{z}) = -\eta_{EE}(\mathbf{z})$ and let $\varphi_\tau(\mathbf{z})$ be a family of convex functions that satisfy

$$f(\mathbf{z}) \le \tau \Longleftrightarrow \varphi_\tau(\mathbf{z}) \le 0, \quad (16)$$

we have

$$\varphi_\tau(\mathbf{z}) = -\sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n} - \tau\left(\sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} \frac{e^{\frac{r_{k,n}}{\rho_{k,n}}} - 1}{H_{k,n}} + P_c\right), \quad (17)$$

where $\tau \le 0$, φ_τ is convex for each τ and decreasing in τ.

Denote f^* as the optimal solution of the quasi-convex problem, if the following feasibility problem

$$\begin{aligned}
&\textit{find} \quad \mathbf{z} \\
&\text{s.t.} \quad \varphi_\tau(\mathbf{z}) \le 0 \\
&\qquad 0 \le \rho_{k,n} \le 1, \qquad \forall\, k, n \\
&\qquad \text{C1} \sim \text{C4, C7 in (13)}
\end{aligned} \quad (18)$$

has a feasible solution, we have $f^* \le \tau$; otherwise $f^* \ge \tau$. (18) can be solved by a simple bisection algorithm. The interval is divided in two parts at each iteration, and the length of the interval after k iterations is $2^{-k}(u - l)$, where $u - l$ is the length of the initial interval. Thus, it requires $\log_2((u - l)/\epsilon_b)$ iterations to converge to ϵ_b-optimality. At each iteration of the bisection method, we need to solve a feasibility problem with a given parameter τ. We introduce a crucial indicator parameter m and formulate the following minimization problem to check the feasibility of (18) [20],

$$\begin{aligned} \min_{\mathbf{x},m} \quad & m \\ \text{s.t.} \quad & \text{C1}: \varphi_\tau(\mathbf{x}) \le m \\ & \text{C2}: \sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}} \le P_t + m \\ & \text{C3}: \sum_{n=1}^{N}\sum_{k=1}^{K}\left(\gamma_n\rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}} + \sqrt{2\log\frac{1}{\epsilon}}\sigma_n\alpha_n\rho_{k,n} \times \frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}}\right) \le I_l^{th} + m, \quad \forall l \\ & \text{C4}: \sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n} r_{k,n} \ge R_{min} \\ & \text{C5}: 0 \le \rho_{k,n} \le 1, \quad \forall k, n \\ & \text{C6}: r_{k,n} \ge 0, \quad \forall k, n \\ & \text{C7}: R_1 : R_2 : \cdots : R_K = \gamma_1 : \gamma_2 : \cdots : \gamma_K. \end{aligned} \tag{19}$$

The variable m can be interpreted as an upper bound of the maximum infeasibility of the inequalities as seen from (19). If the optimal solution to (19) m^* is less than or equal zero, it means that at least one feasible solution to (18) exists. Furthermore, for a given τ that satisfies $f^* = \tau$, the solution $\{r_{k,n}, \rho_{k,n}\}$ to (19) is optimal for (14) if and only if $m^* = 0$ for (19). So the optimal resource allocation can be obtained by solving (19) in the last iteration with $m^* \le 0$.

Eq. (19) defines a convex problem whose optimal solution can be obtained by standard convex optimization technique [26]. The computation complexity is generally $O((K+N)^3)$. The solution to (19) can be taken as an upper bound of the optimal objective function value of (13).

V. Integer Subchannels Assignment and Power Distribution

A. Integer Subchannels Assignment

For practical wireless systems, each subchannel can be assigned to only one SU and the allocation indexes $\rho_{k,n}$'s are restricted to be 0's or 1's. Considering the optimal solution to the relaxation form (14), the fractional $\rho_{k,n}$ can be regarded as the metric to determine the exact assignment of subchannels. According to [27], when $K \ll N$, only few subchannels are shared among users as $\rho_{k,n}$ is mostly either 1 or 0 in the optimal solution to (14). Hence, we allocate the nth subchannel to the kth SU with the maximum $\rho_{k,n}$,

$$\rho_{k,n}^* = \begin{cases} 1, & k = \operatorname{argmax} \rho_{k,n} \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

Once subchannels assignment is fixed, the SUs can perform power allocation across the subchannels to maximize the energy efficiency of the system. Let Ω_k be the set of subchannels allocated to the kth SU, the power allocation problem is as follows,

$$\begin{aligned} \max_{r_{k,n}} \quad & \eta_{EE} \\ \text{s.t.} \quad \text{C1} \quad & \sum_{k=1}^{K}\sum_{n\in\Omega_k} r_{k,n} \ge R_{min}, \\ \text{C2} \quad & \sum_{k=1}^{K}\sum_{n\in\Omega_k} p_{k,n} \le P_t, \\ \text{C3} \quad & \sum_{k=1}^{K}\sum_{n\in\Omega_k}\left(\gamma_n p_{k,n} + \sqrt{2\log\frac{1}{\epsilon}\sigma_n\alpha_n p_{k,n}}\right) \le I_l^{th}, \quad l = 1, \ldots, L \\ \text{C4} \quad & r_{k,n} \ge 0, \quad \forall n \in \mathcal{N}, \quad \forall k, \\ \text{C5} \quad & \sum_{n\in\Omega_k} r_{k,n} = \beta_k \sum_{n\in\Omega_1} r_{1,n}, \quad k = 2, \ldots, K, \end{aligned} \tag{21}$$

where $\beta_k = \frac{\gamma_k}{\gamma_1}, k = 2, 3, \cdots K$.

B. Equivalent Transformation of (21)

Eq. (21) defines a non-linear fractional programming problem that is still difficult to solve [25]. Nevertheless, (21) has an equivalent transformation via its hypograph form [25] since the denominator of the objective function is jointly concave in $r_{k,n}$'s and the numerator is linear. The hypograph form of (21) is as follows,

$$\begin{aligned} \max_{r_{k,n},y} \quad & y \\ \text{s.t.} \quad & \eta_{EE} \ge y \\ & \text{C1} \sim \text{C5 in (21)} \\ & y \ge 0, \end{aligned} \tag{22}$$

where the domain $y \ge 0$ is determined by the inequality $\eta_{EE} \ge 0$. Such a transformation can guarantee the equivalence of (21) and (22). Collect $r_{k,n}$'s into a vector $\boldsymbol{r}$ with $\boldsymbol{r} = (r_1, r_2, \ldots, r_N)$. Eq. (22) can be analyzed geometrically as an optimization problem in the "graph space" of $(\boldsymbol{r}, y)$. That is, we can maximize y over the hypograph of η_{EE}, subject to the constraints in (21), which is equivalent to solve (21) directly. The equivalent hypograph form of (22) can be alternatively written as follows,

$$\begin{aligned} \max_{r_{k,n},y} \quad & y \\ \text{s.t.} \quad & \varphi_0(\boldsymbol{r}, y) \ge 0 \\ & \text{C1} \sim \text{C5 in (21)} \\ & y \ge 0. \end{aligned} \tag{23}$$

where $\varphi_0(\boldsymbol{r}, y) = \sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n} - y\left(\sum_{n=1}^{N}\sum_{k=1}^{K} p_{k,n} + P_c\right)$.

Now we transform (23) into a convex optimization problem in two steps.

Step 1 Transform y into $U(y)$ to make $\varphi(\boldsymbol{r}, y)$ convex. Here we take $U(y) = e^y$ and (23) is rewritten as follows,

$$\begin{aligned} \max_{r_{k,n},y} \quad & e^y \\ \text{s.t.} \quad & \varphi(\boldsymbol{r}, y) \ge 0 \\ & \text{C1} \sim \text{C5 in (21)}, \end{aligned} \tag{24}$$

where $\varphi(\boldsymbol{r}, y) = \sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n} - e^{y}\left(\sum_{n=1}^{N}\sum_{k=1}^{K} p_{k,n} + P_c\right)$.

Step 2 Transform the objective function to make the optimization problem convex. The equivalent form of (24) is as follows,

$$\begin{aligned} \min_{r_{k,n},y} \quad & e^{-y} \\ \text{s.t.} \quad & \varphi(\boldsymbol{r}, y) \geq 0 \\ & \text{C1} \sim \text{C5 in (21).} \end{aligned} \tag{25}$$

Eq. (25) defines a convex optimization problem with the same optimal solution r^* as (21). Since there are well-developed algorithms to tackle (25), the optimal solution to (21) becomes easier to obtain.

C. Optimal Power Distribution

Barrier method is a standard technique to solve convex optimization problems. Rewrite (25) as follows,

$$\begin{aligned} \min_{r_{k,n},y} \; & e^{-y} \\ \text{s.t. C1} \quad & \varphi(\boldsymbol{r}, y) \geq 0 \\ \text{C2} \quad & \sum_{k=1}^{K}\sum_{n\in\Omega_k} r_{k,n} \geq R_{min}, \\ \text{C3} \quad & \sum_{k=1}^{K}\sum_{n\in\Omega_k} \frac{e^{r_{k,n}}-1}{H_{k,n}} \leq P_t, \\ \text{C4} \quad & \sum_{k=1}^{K}\sum_{n\in\Omega_k}\left(\gamma_n \frac{e^{r_{k,n}}-1}{H_{k,n}} + \sqrt{2\log\frac{1}{\epsilon}}\sigma_n\alpha_n\frac{e^{r_{k,n}}-1}{H_{k,n}}\right) \leq I_l^{th}, \\ & \qquad l = 1, \ldots, L \\ \text{C5} \quad & r_{k,n} \geq 0, \quad \forall n \in \mathcal{N}, \quad \forall k, \\ \text{C6} \quad & \sum_{n\in\Omega_k} r_{k,n} = \beta_k \sum_{n\in\Omega_1} r_{1,n}, \qquad k = 2, \ldots, K. \end{aligned} \tag{26}$$

First, we reformulate the problem into a set of unconstrained optimization problems by making all inequality constraints implicit in an objective function. The logarithmic barrier function of (26) is

$$\begin{aligned} \phi(z) = &-\log\varphi(\boldsymbol{x}) - \log\left(\sum_{k=1}^{K}\sum_{n\in\Omega_k} r_{k,n} - R_{min}\right) \\ &- \log\left(P_t - \sum_{k=1}^{K}\sum_{n\in\Omega_k}\frac{e^{r_{k,n}}-1}{H_{k,n}}\right) \\ &- \sum_{l=1}^{L}\log\left(I_l^{th} - \sum_{k=1}^{K}\sum_{n\in\Omega_k}\left(\gamma_n\frac{e^{r_{k,n}}-1}{H_{k,n}} + \sqrt{2\log\frac{1}{\epsilon}} \right.\right. \\ &\qquad\left.\left. \times\ \sigma_n\alpha_n\frac{e^{r_{k,n}}-1}{H_{k,n}}\right)\right) \\ &- \sum_{k=1}^{K}\sum_{n\in\Omega_k}\log r_{k,n}, \end{aligned} \tag{27}$$

where all variables $\{\boldsymbol{r}, y\}$ are collected into one vector $\boldsymbol{x}$, i.e., $\boldsymbol{x} = \{r_1, r_2, \cdots, r_N, y\}$.

Thus, the optimal solution to (26) can be approximated by solving the following minimization problem with a certain parameter t,

$$\begin{aligned} \min\ & \psi_t(\boldsymbol{x}) = te^{-y} + \phi(\boldsymbol{x}) \\ \text{s.t.}\ \ & A\boldsymbol{x} = \boldsymbol{0}, \end{aligned} \tag{28}$$

where A is an $(K-1)\times N$ matrix and $\boldsymbol{0} \in \mathcal{R}^{K-1}$ with

$$A_{k,n} = \begin{cases} -\beta_{k+1}, & k = 1, 2, \cdots, K-1,\ n \in \Omega_1, \\ 1, & k = 1, 2, \cdots, K-1,\ n \in \Omega_{k+1}, \\ 0, & \text{otherwise.} \end{cases} \tag{29}$$

Eq. (28) has only equality constraints and can be solved efficiently by Newton method, where Newton step $\Delta\mathbf{x}_{nt}$ and the associated dual variables ν are given by the following Karush–Kuhn–Tucker (KKT) systems,

$$\begin{bmatrix} \nabla^2\psi_t(\boldsymbol{x}) & A^T \\ A & \mathbf{0}_n \end{bmatrix}\begin{bmatrix} \Delta\mathbf{x}_{nt} \\ \nu \end{bmatrix} = \begin{bmatrix} -\nabla\psi_t(\boldsymbol{x}) \\ \mathbf{0}_\nu \end{bmatrix}, \tag{30}$$

where $\Delta\mathbf{x}_{nt} \in \mathcal{R}^{N+1}$, $\mathbf{0}_n \in \mathcal{R}^{(K-1)\times(K-1)}$, $\mathbf{0}_\nu \in \mathcal{R}^{K-1}$.

The computation load of the barrier method mainly lies in the updating of Newton step that needs matrix inversion. Unfortunately, if we compute the inversion of the KKT matrix directly, it yields a complexity of $O((N+K)^3)$, which is too high for application because there are thousands of OFDM subchannels generally in practical wireless systems. In order to reduce the computational cost, we exploit the structure of (28) and develop a fast algorithm to calculate Newton step with low complexity.

Theorem 2: Eq. (30) can be solved with a complexity of $O(M^2K^3N)$, where $M = L + 3$.

The proof is given in detail in Appendix. It is noteworthy that the structure of (30) is totally different from that of [16] because the proportional fairness requirements in (5) introduce equality constraints, making the formulated problem more complex and difficult to address. As a result, the fast barrier method proposed in [16] cannot be used to solve (5). We develop a fast algorithm based on matrix inversion formula as shown in Appendix. In practical CR systems, $M \ll N$ and $K \ll N$ generally holds, so the complexity of the proposed algorithm is much lower than that of matrix inversion.

VI. Low Complexity Subchannels Allocation

Recall that Eq. (5) defines a mixed integer programming problem that involves both binary variables $\rho_{k,n}$'s and real variables $p_{k,n}$'s for optimization. An intuitive rounding technique can yield a feasible solution if we can get a solution of the relaxed form. However, the computational complexity of such an RA scheme is still too high even though we adopt the power distribution algorithm proposed in Section IV, since the relaxed optimization problem cannot be solved efficiently because the variables $\rho_{k,n}$'s and real variables $p_{k,n}$'s are coupled. To reduce complexity further, we propose a two-step procedure to address (5): subchannels allocation and power distribution. The latter has been discussed in Section IV. In this section, we propose an efficient heuristic subchannels allocation algorithm to figure out the binary variables $\rho_{k,n}$'s, specifying subchannels allocation assignment.

In a cognitive OFDM system, there are L additional interference constraints which should be considered. If the nth subchannel is interference limited [6], the total power allocated to it is not more than $\min_l \{I_l^{th}/I_{n,l}^{SP}\}$. Jointly consider the transmission power and interference threshold constraints, the maximum possible power allocated to the nth subchannel is

$$p_{k,n}^{\max} = \min\left(P_t, \min_l \left\{I_l^{th}/I_{n,l}^{SP}\right\}\right). \tag{31}$$

Denote $r_{k,n}^{\max}$ as the highest achievable rate of the nth subchannel used by the kth SU, we have

$$r_{k,n}^{\max} = \log\left(1 + p_{k,n}^{\max} H_{k,n}\right). \tag{32}$$

The normalized maximum rate is a practicable criterion to measure the QoS of a subchannel, giving insightful hints for subchannel allocation.

Rewrite C7 in (5) as

$$R_k = \beta_k R_1, \qquad k = 2, \ldots, K, \tag{33}$$

we have the rate requirement of the Kth SU

$$\begin{aligned} R_1 &\geq R_{1,min}, \\ R_k &\geq R_{k,min}, \end{aligned} \tag{34}$$

where

$$\begin{aligned} R_{1,min} &= \frac{R_{min}}{1 + \sum_{k=2}^{K} \beta_k}, \\ R_{k,min} &= \beta_k R_{1,min}. \end{aligned} \tag{35}$$

Let Ω_k denote the subchannel set employed by the kth SU, and $\mathcal{N}$ is the set of subchannels. The outline of our subchannels allocation scheme is described in Table I. The principle of the algorithm is that the SU whose current rate is the farthest away from his target rate has the priority to get a subchannel from the available ones. The subchannel with the highest achievable rate for this user will be chosen. This procedure continues until all SUs' rate requirements are satisfied. Then each of the remaining subchannels is allocated to the SU who has the highest achievable rate over this channel to roughly maximize the energy efficiency of the CR system. To simplify analysis and computation, the power of a subchannel is temporarily set as

$$p_{k,n} = \min\left(P_t/N, \min_l \left\{I_l^{th}/I_{n,l}^{SP}\right\}\right). \tag{36}$$

It is easy to show that the complexity of the proposed heuristic algorithm is approximately linear to the number of subchannels, that is, $O(N)$. It is much lower than the time-sharing method proposed in Section III which generally generates a complexity of at least $O((N+K)^3)$.

To conclude, the complexity of the subchannels assignment proposed in Section VI with the optimal power allocation proposed in Section V-C (SA-OP) to solve (5) is $O((L+3)^2K^3N)$. On the other hand, if we solve (5) using integer subchannels assignment proposed in Section V-A with optimal power allocation proposed Section V-C (INT-OP), the complexity will be $O((N+K)^3)$.

TABLE I
SUB-CHANNEL ALLOCATION SCHEME

1.**Initialization**:
2.$\mathcal{N}_t = \mathcal{N}$, $\Omega_k = \emptyset, \forall k$
3.Initial rate of each SU: $R_{t_i} = 0$, for $i = 1, 2, ..., K$
4.***While*** $\mathcal{N}_t \neq \emptyset$ and $R_{t_k} < R_{k,min}$ for any $1 \leq k \leq K$
5. Find k^* satisfies $R_{k^*,min} - R_{t_{k^*}} \geq R_{k,min} - R_{t_k}, 1 \leq k \leq K$
6. For k^*, find n^* satisfies $r_{k^*,n^*}^{\max} \geq r_{k^*,n}^{\max}, \forall n \in \mathcal{N}_t$
7. Update $R_{t_{k^*}} = R_{t_{k^*}} + \log_2(1 + p_{k^*,n^*} H_{k^*,n^*})$
8. Update $\Omega_{k^*} = \Omega_{k^*} \cup n^*, \mathcal{N}_t = \mathcal{N}_t \setminus n^*$
9.***Endwhile***
10.***If*** $\mathcal{N}_t \neq \emptyset$
11. ***For*** $i = 1$ to length($\mathcal{N}_t$)
12. For $n^* = \mathcal{N}_{t_i}$, find k^* satisfies $r_{k^*,n^*}^{max} \geq r_{k,n^*}^{max}$
13. Update $\Omega_{k^*} = \Omega_{k^*} \cup n^*$
14. ***Endfor***
15.***Endif***

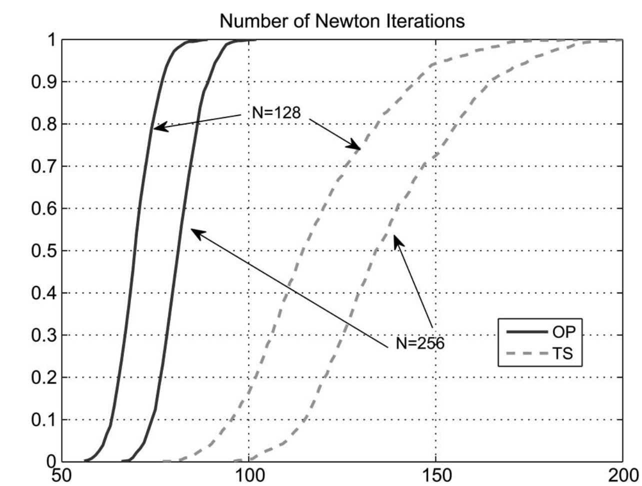

Fig. 1. CDF of the number of Newton iterations.

VII. SIMULATION RESULTS

Consider the downlink of a multiuser OFDM-based CR system, where all users (PUs and SUs) are randomly located in an area of 3×3 km. The receiver of each user is distributed in a circle within 0.5 km from its transmitter. The path loss exponent is 4, the variance of the shadowing effect is 10 dB and the multipath fading is assumed to be Rayleigh. The noise power on each subchannel is set to 10^{-13} W. The frequency bands occupied by the PUs are generated randomly with the maximum number of OFDM subchannels $2W/3L$. The parameters for the Bernstein approximations, μ_n^-, μ_n^+, and σ_n are chosen from Table I as suggested in [24] using the known first- and second-order moments of truncated channel gains.

First, we investigate the convergence of our proposed algorithms. As discussed in Section V, the computational load mainly lies in the updating of Newton step. Fig. 1 gives the Cumulative Distribution Function (CDF) of the number of Newton iterations of Time-Sharing (TS) method and Optimal Power allocation (OP) with different settings of N. Fig. 2 shows the

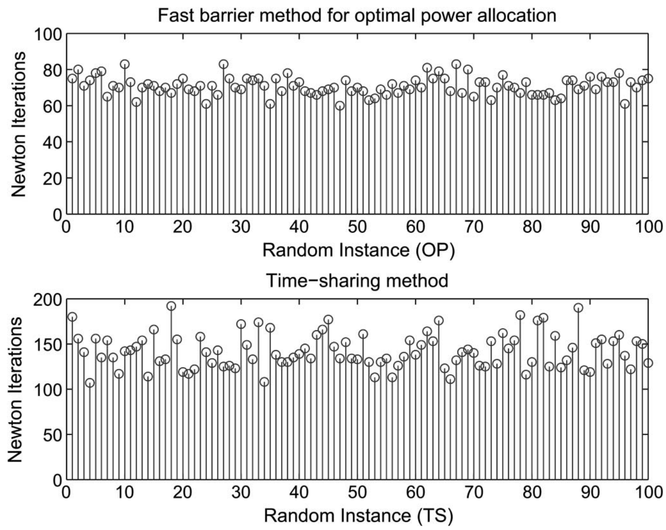

Fig. 2. Number of Newton iterations required for convergence with 100 channel realizations. $N = 256$, $K = 32$.

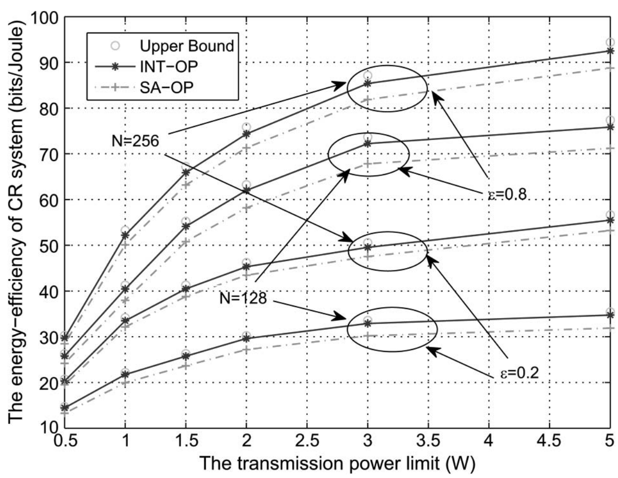

Fig. 3. The energy-efficiency of the CR system versus the transmission power limit for different ϵ.

number of Newton iterations for convergence in 100 random instances for the TS and the OP. As seen in Figs. 1 and 2, the number of Newton iterations is small and varies in a narrow range, indicating our proposed algorithm is effective and efficient.

To evaluate the energy efficiency, we compare the proposed algorithms, including efficient Sub-channels Assignment proposed in Section VI with Optimal Power allocation proposed in Section V-C (SA-OP), Integer subchannels assignment proposed in Section V-A with Optimal Power allocation proposed in Section V-C (INT-OP) and an upper bound which can be achieved by using standard optimization technique as discussed in Section IV.

Fig. 3 shows the energy-efficiency of the system as a function of the power budget. The number of SUs is 32. The static circuit power is set to 0.25 W. The minimal rate requirement of the CR system is 10 bits/symbol and the interference threshold of each PU is 5×10^{-12} W. As discussed in Section IV, the optimal solution of the relaxation form can serve as an upper bound of the energy-efficiency. As can be seen in Fig. 3, the INT-OP achieves more than 98% of the upper bound, which shows twofold meanings: First, the upper bound is tight; second, the INT-OP can almost maximize the energy efficiency of the CR system. It is worthy noticing that the gap between the SA-OP and the INT-OP is small, suggesting the SA-OP algorithm provides a good approximation to the optimal. Since the complexity of the SA-OP is much lower than that of the INT-OP, the SA-OP achieves a promising tradeoff between the energy efficiency and the complexity. We can observe that the energy-efficiency of the system grows with the increase of the transmission power budget until radio resource is sufficient to satisfy the rate requirements of the SUs. Besides, larger ϵ will enhance the energy-efficiency, which results from the more lenient chance constraint. Furthermore, the SA-OP gets closer to the INT-OP when N grows larger, which reflects that the gap between the INT-OP and the SA-OP becomes closer when $K \ll N$.

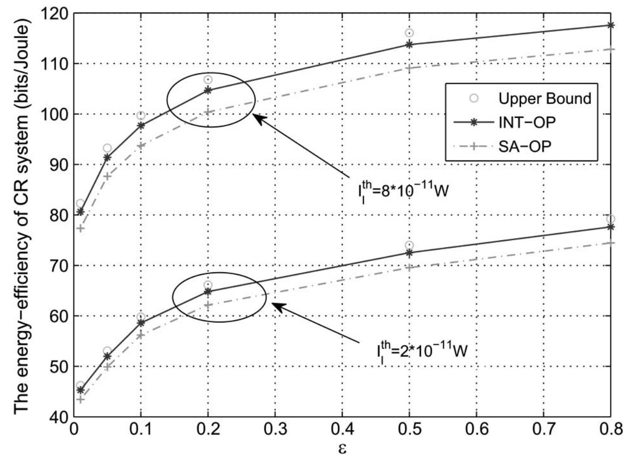

Fig. 4. The energy-efficiency of the CR system versus ϵ for different interference thresholds.

Fig. 4 shows the energy-efficiency of the CR system as a function of ϵ for different interference thresholds. There are 32 SUs, 256 subchannels. The minimal rate requirement of the CR system is 10 bits/symbol. The transmission power budget is 1 W and the static circuit power is set to 0.25 W. It can be observed that the energy efficiency of the system increases as ϵ increases since larger ϵ renders the chance constraint more lenient. Note that the SA-OP is always capable to achieve at least 90% of the INT-OP in different scenarios. And both the SA-OP and the INT-OP perform close to the upper bound. Additionally, we can observe that lower interference threshold will decrease the energy-efficiency. The reason is that more subchannels will be interference limited when the interference threshold becomes small. The system fails to maintain the rate requirements in this situation.

Fig. 5 shows the average time cost as a function of the number of subchannels over 1000 instances. The elapsed time is counted by in-built *tic-toc* function in *Matlab*. From Fig. 5 we can see the time cost of the SA-OP is much less than that of the INT-OP. Recall that the difference of the energy efficiency between the two schemes is small, we can conservatively conclude that the SA-OP achieves a good tradeoff between energy efficiency and complexity.

Fig. 6 shows the energy efficiency versus different fairness constraints defined in Table II. We consider a 6-SU case. Define

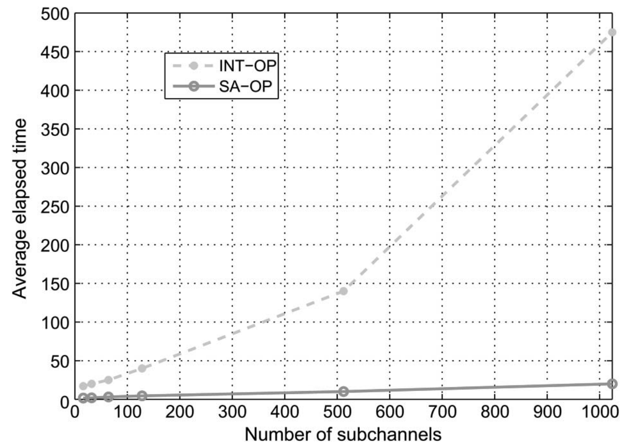

Fig. 5. Average time cost as a function of the number of subchannels.

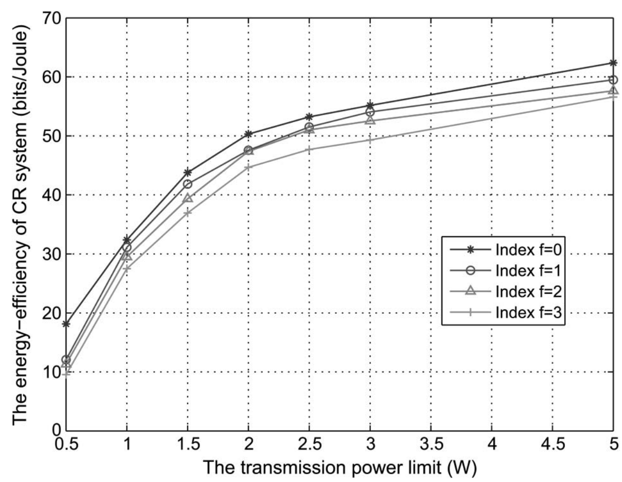

Fig. 6. Energy efficiency versus different fairness indexes (N = 256).

TABLE II
PROPORTIONAL FAIRNESS (6 SUs)

Γ-set Index f	0	1	2	3
$\gamma_1 = \gamma_2 = \gamma_3 =$	2^0	2^1	2^2	2^3
$\gamma_3 = \gamma_4 = \gamma_6 =$	1	1	1	1
Fairness Index $\mathcal{F}$	1	0.90	0.74	0.62

Γ-set index f, thus $\gamma_1 = \gamma_2 = \gamma_3 = 2^f$ and $\gamma_3 = \gamma_4 = \gamma_6 = 1$. From Fig. 6, the EE achieved by the proposed resource allocation scheme SA-OP varies with the proportional fairness constraints. The result demonstrates that the proportional fairness constraints can explicitly control the capacity ratios among the SUs and affect the energy-efficiency of the system. It can be observed that larger fairness index leads to higher energy-efficiency.

VIII. CONCLUSION

In this paper, we studied the energy-efficient resource allocation in the OFDM-based CR networks under channel uncertainty. Since the channel gains between the CR transmitters and the PU receivers often cannot be estimated accurately, the PU interference restriction is cast as chance constrained form. We introduce Bernstein approximation to remove the chance constraints. Then we relax the integer variables of the formulated mixed integer programming problem by using time-sharing method to convert it into a quasi-convex one, to which the optimal solution can be obtained by bisection-based algorithm. We also derive a fast barrier method to address the power allocation with reasonable computation load. Moreover, we develop an efficient heuristic subchannels allocation method to achieve a good tradeoff between energy efficiency and complexity. Simulation results indicate that our proposals can improve the energy efficiency of the CR system significantly and show potentials for practical applications.

APPENDIX

A. Proof of Theorem 2

Proof: Denote

$$\begin{aligned} f_0 &= P_t - \sum_{k=1}^{K}\sum_{n\in\Omega_k}\frac{e^{r_{k,n}}-1}{H_{k,n}}, \\ f_1 &= \sum_{k=1}^{K}\sum_{n\in\Omega_k} r_{k,n} - R_{min}, \\ g_l &= I_l^{th} - \sum_{k=1}^{K}\sum_{n\in\Omega_k}\left(\gamma_n\frac{e^{r_{k,n}}-1}{H_{k,n}} + \sqrt{2\log\frac{1}{\epsilon}}\sigma_n\alpha_n\frac{e^{r_{k,n}}-1}{H_{k,n}}\right) \\ &= I_l^{th} - \sum_{k=1}^{K}\sum_{n\in\Omega_k}\delta\frac{e^{r_{k,n}}-1}{H_{k,n}}, \qquad l = 1, 2, \cdots, L, \end{aligned} \tag{37}$$

where δ is a constant with $\delta = \gamma_n + \sqrt{2\log\frac{1}{\epsilon}}\sigma_n\alpha_n$.

The Hessian of $\psi_t(\boldsymbol{x})$ is

$$\nabla^2\psi_t(\boldsymbol{x}) = \begin{bmatrix} D & B^T \\ B & Y \end{bmatrix} + \sum_{i=1}^{L+3} F_i F_i^T. \tag{38}$$

where

$$D = \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \ddots & \\ & & & D_N \end{bmatrix}$$
$$B = [B_1, B_2, \cdots, B_N], \tag{39}$$

with

$$\begin{aligned} D_n &= \frac{1}{r_n^2} + \frac{1}{\varphi(\boldsymbol{x})}\frac{e^y e^{r_n}}{H_n} + \frac{1}{f_0}\frac{e^{r_n}}{H_n} + \sum_{l=1}^{L}\frac{I_{n,l}^{SP}}{g_l}\frac{e^{r_n}}{H_n}\delta \\ B_n &= -\frac{e^y}{\varphi^2(\boldsymbol{x})}\left(\sum_{k=1}^{K}\sum_{n\in\Omega_k}\frac{e^{r_n}-1}{H_n} + P_c\right)\left(1 - \frac{e^y e^{r_n}}{H_n}\right) \\ &\quad + \frac{e^y}{\varphi(\boldsymbol{x})}\left(\sum_{k=1}^{K}\sum_{n\in\Omega_k}\frac{e^{r_n}-1}{H_n} + P_c\right) \\ Y &= te^{-y} + \frac{e^y}{\varphi(\boldsymbol{x})}\left(\sum_{k=1}^{K}\sum_{n\in\Omega_k}\frac{e^{r_n}-1}{H_n} + P_c\right), \end{aligned} \tag{40}$$

and F_i with

$$F_i = \begin{cases} \frac{\nabla f_0}{f_0}, & i=1 \\ \frac{\nabla f_1}{f_1}, & i=2 \\ \frac{\nabla \varphi}{\varphi}, & i=3 \\ \frac{\nabla g_l}{g_l}, & l=1,\cdots,L, \quad i=l+3. \end{cases} \tag{41}$$

Since it is easy to prove that the matrix D is positive definite, it follows that the Hessian matrix $\nabla^2\psi_t(\mathbf{x})$ is invertible.

Rewrite the KKT system (30) as follows,

$$\Lambda_0 \boldsymbol{\mu} = G_0, \tag{42}$$

where $\boldsymbol{\mu} = \begin{bmatrix} \Delta \mathbf{x}_{nt} \\ v \end{bmatrix}$ and $G_0 = \begin{bmatrix} -\nabla\psi_t(\mathbf{x}) \\ \mathbf{0}_v \end{bmatrix}$. According to (38), Λ_0 can be written as

$$\Lambda_0 = \begin{bmatrix} H & A^T \\ A & \mathbf{0}_n \end{bmatrix} + \sum_{i=1}^{M} G_i G_i^T, \tag{43}$$

where $G_i = \begin{bmatrix} F_i \\ \mathbf{0}_v \end{bmatrix}$, $i=1,2,\cdots,M$, where $M = L+3$. Λ_0 can be decomposed into M equations,

$$\Lambda_i = \Lambda_{i+1} + G_{i+1}G_{i+1}^T, \qquad i=0,1,\cdots,M-1, \tag{44}$$

By exploiting the structure of Λ_i's, we give an M-step procedure to compute the Newton step by introducing intermediate variables.

Specifically, in Step m, we use (44) to decompose Λ_{m-1} with $\Lambda_{m-1} = \Lambda_m + G_m G_m^T$. We can update the m intermediate variables introduced in Step $m-1$ by $v_i^{m-1} = v_i^m - \frac{G_m^T v_i^m}{1+G_m v_{m+1}^m} v_{m+1}^m$, $i=1,2,\cdots,m$, where the newly introduced variables v_i^m are obtained by solving the following $m+1$ sets of linear equations, $\Lambda_m v_i^m = G_{i-1}$, $i=1,2,\cdots,m+1$.

Continue the procedure to the Mth step, and there are $M+1$ matrix systems $\Lambda_M v_i^M = G_{i-1}$, $i=1,2,\cdots,M+1$. Form the derivation process, we can find that the m variables v_i^{m-1}, $i=1,2,\cdots,m$ in the $(m-1)$th Step can be obtained by the $m+1$ variables v_i^m, $i=1,2,\cdots,m+1$ in the mth Step. Thus, if we figure out the $M+1$ variables v_i^M, $i=1,2,\cdots,M+1$, μ will be indirectly obtained.

Now we consider the matrix systems in step M in a unified form as follows,

$$\begin{bmatrix} H & A^T \\ A & \mathbf{0}_n \end{bmatrix} \begin{bmatrix} u \\ \nu \end{bmatrix} = \begin{bmatrix} h \\ \mathbf{0}_v, \end{bmatrix}. \tag{45}$$

where $u \in \mathcal{R}^{(N+1)\times 1}$ and $v \in \mathcal{R}^{(K-1)\times 1}$. The matrix inversion of (45) can be calculated with the following formula,

$$\begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix}^{-1} = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix}, \tag{46}$$

where

$$\begin{aligned} Y_1 &= X_1^{-1} + X_1^{-1}X_2\left(X_4 - X_3X_1^{-1}X_2\right)^{-1}X_3X_1^{-1}, \\ Y_2 &= -X_1^{-1}X_2\left(X_4 - X_3X_1^{-1}X_2\right)^{-1}, \\ Y_3 &= -\left(X_4 - X_3X_1^{-1}X_2\right)^{-1}X_3X_1^{-1}, \\ Y_4 &= -\left(X_4 - X_3X_1^{-1}X_2\right)^{-1}. \end{aligned} \tag{47}$$

Note that

$$H = \begin{bmatrix} D & B^T \\ B & Y \end{bmatrix} \tag{48}$$

and H^{-1} can also be calculated with (46) and (47). Since D is diagonal, the complexity of computing H^{-1} is $O(K^3)$. Obviously, a reverse derivation of the M steps is necessary after we solve the matrix system in the Mth step.

The computational complexity of our proposed algorithm can be counted as follows. Solving (26) requires M decompositions and each of them yields an additional matrix equation. First, we need to solve $M+1$ matrix systems according to (46) and (47) with a complexity $O(K^3N)$ for each system. Then, a reverse substitution with M steps is required. Thus, the complexity to work out the optimal solution to (26) is measured by $O(M^2K^3N)$. □

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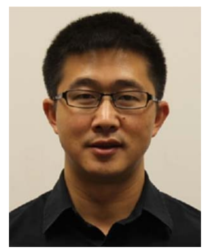

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