Energy-Efficient Power Allocation for Cooperative Relaying Cognitive Radio Networks

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***Abstract*—In this paper, we study the power allocation in Orthogonal Frequency Division Multiplexing (OFDM)-based Cognitive Radio (CR) networks with cooperative relay. Since the energy consumption is growing at a staggering rate, green radio, which puts emphasis on the energy-efficiency (EE) in wireless networks, is becoming increasingly important. Therefore, we try to maximize the EE of a relaying CR system, under the consideration of many practical limitations, such as transmission power budget, interference threshold of the primary users and the traffic demand. We first convert the formulated problem into a convex optimization one via its hypogragh form, which can be solved by the barrier method. By exploiting the special structure of the formulated convex optimization problem, we further speed up the computation of Newton step during the barrier method, significantly reducing the complexity of the algorithm. Numerical results validate that our proposal can exploit the overall EE of the CR system, while the algorithm converges efficiently and stably.**

I. INTRODUCTION

Cognitive radio (CR) arises as a promising paradigm to improve the usage efficiency of radio spectrum [1]. CR users are allowed to access the licensed spectrum, as long as the interference to the primary users (PUs) is kept below a preset threshold. However, one of the most challenging problems in CR networks is to satisfy the quality-of-service (QoS) requirements while not causing unacceptable performance degradation of PUs. In some cases, reliable transmission between a certain pair of CR nodes requires large power, leading to heavy interference to PUs. Hence, conventional end-to-end transmission schemes are not suitable for CR networks in this case. Relay technology has emerged as a key spatial diversity technique with the ability to boost the overall performance of wireless systems, especially for CR networks [2].

We find that much effort has been made to enhance the throughput of networks during past decade, such as [2–4] in relaying CR systems. However, with the ever growth of high-data-rate applications, the energy consumption is also growing at a staggering rate, which causes large amount of greenhouse gas and high operation expenditure [5]. Green radio, which emphasizes on energy-efficiency (EE) in wireless networks, is becoming increasingly important and prompting new waves of research and standard development activities [6].

Recently, a growing number of researches about the energy-efficient wireless communications have been carried out. A survey depicts the technical roadmaps of several major international projects and discusses the state-of-the-art research can be found in [7]. In [8], it aims to maximize weighted EE with a prescribed QoS requirement. [9] mainly addresses the fundamental tradeoff between EE and spectral-efficiency (SE) in downlink OFDMA networks. A low-complexity energy-efficient power allocation scheme is developed by considering time-averaged EE metrics for an uplink OFDMA system in [10]. However, it is noteworthy that there is few work on the energy-efficient resource allocation (RA) of the CR systems. Actually, dynamic energy-efficient RA is extremely important for CR networks, since it is a prerequisite to achieve the highly utilization of the limited transmission power. Moreover, many existing algorithms are no longer suitable for this case.

In this paper, we investigate the energy-efficient power allocation issue for an OFDM-based CR system, enhanced by a relay node operating with amplify-and-forward (AF) protocol. The optimization problem is formulated to maximize the overall EE of the CR relaying system under the individual transmission power budget of the CR source node (SN) and the relay node (RN), while satisfying the minimal capacity requirement. On the other hand, the interference to the PUs must be strictly kept in a tolerable range. The formulated optimization problem is a nonlinear fractional programming problem, which can be equivalently transformed into a convex optimization problem via its hypogragh form. By intensively analyzing the equivalent convex problem, an efficient barrier method is developed, which can always work out the optimal solution and converge quickly.

The rest of this paper is organized as follows. In Section II, we illustrate system model and formulate the problem as an optimization task. In Section III, an efficient barrier algorithm is proposed. Simulation results are given in Section IV, as well as discussions. Finally, we conclude the paper in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a three-node relay-enhanced CR network, which consists of an SN, an RN and a destination node (DN), coexisting with a licensed system with L PUs. The relay operates in a time-division half-duplex mode using the AF protocol. The operation of the relaying CR network is as follows: In the first time slot, the SN transmits while the RN and DN listen; then the relay amplify the signal received slot and transmits them to the DN in the second time slot.

Throughout this paper, we assume the destination receives the signals without diversity, i.e. at the end of each transmission frame, the destination processes the signals without combination. Besides, we assume the perfect channel state information (CSI) is available at both the SN and RN. We note that the results obtained by our proposed algorithms will serve as an upper bound on the achievable EE with channel estimation errors, because of the assumption of perfect CSI knowledge at the AP in the CR system.

The whole available bandwidth is divided into N subchannels, denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$. The spectrum of the nth subchannel spans from $f_0 + (n-1)B$ to $f_0 + nB$, where f_0 is the starting frequency and B is the bandwidth of each subchannel. Assume the lth PU's nominal band ranges from f_l to $f_l + B_l$, where f_l and B_l are the lth PU's starting frequency and bandwidth, respectively. Thus, the interference introduced to the lth PU in the first slot on the nth subchannel with unit transmission power is [11]

$$I_{n,l}^{SP} = \int_{f_l - f_0 - (n-1/2)B}^{f_l + B_l - f_0 - (n-1/2)B} g_{n,l}^{SP} \phi(f) df, \quad (1)$$

where $g_{n,l}^{SP}$ is the power gain from the SN's transmitter to the receiver of the lth PU on the nth subchannel. $\phi(f)$ is the baseband power spectral density (PSD) of OFDM signal with $\phi(f) = T(\frac{\sin \pi f T}{\pi f T})^2$, where T is OFDM symbol duration. Similarly, the interference to the lth PU by the CR relay with unit transmission power on the nth subchannel is

$$I_{n,l}^{RP} = \int_{f_l - f_0 - (n-1/2)B}^{f_l + B_l - f_0 - (n-1/2)B} g_{n,l}^{RP} \phi(f) df, \quad (2)$$

where $g_{n,l}^{RP}$ is the power gain from the RN to the lth PU's receiver on the nth subchannel. On the other hand, the interference cast into the RN and DN by the lth PU on the nth subchannel can be calculated by

$$\begin{aligned} I_{n,l}^{PR} &= \int_{f_0 + (n-1)B - f_l - B_l/2}^{f_0 + nB - f_l - B_l/2} g_{n,l}^{PR} \phi_l(f) df, \\ I_{n,l}^{PD} &= \int_{f_0 + (n-1)B - f_l - B_l/2}^{f_0 + nB - f_l - B_l/2} g_{n,l}^{PD} \phi_l(f) df, \end{aligned} \quad (3)$$

where $g_{n,l}^{PR}$ and $g_{n,l}^{PD}$ are the power gain from the lth PU's receiver to the RN and DN on the nth subchannel, respectively. $\phi_l(f)$ is the PSD of the lth PU's signal.

Denote $p_{s,n}$ is the transmission power on the nth subchannel in the first hop, while the relay amplifies the signal using power $p_{r,n}$ on the nth subchannel in the second hop, the amplification factor β_n on the nth subcahnnel is

$$\beta_n = \sqrt{\frac{p_{r,n}}{p_{s,n}|h_{sr,n}|^2 + N_r B + \Sigma_{l=1}^{L} I_{n,l}^{PR}}}, \quad (4)$$

where $h_{sr,n}$ is the channel gain from the SN to RN, and $N_r$is the PSD of additive white Gaussian noise (AWGN) at the CR relay. Consequently, the received signal-to-noise ratio (SNR) on the nth subchannel is

$$\begin{aligned} \gamma_n &= \frac{p_{s,n}|h_{rd,n}|^2 \beta_n^2 |h_{sr,n}|^2}{N_d B + \sum_{l=1}^{L} I_{n,l}^{PD} + (N_r B + \sum_{l=1}^{L} I_{n,l}^{PR}) \beta_n^2 |h_{rd,n}|^2} \\ &= \frac{p_{s,n} a_n p_{r,n} b_n}{1 + p_{s,n} a_n + p_{r,n} b_n}, \end{aligned} \quad (5)$$

where $h_{rd,n}$ is the channel gain of subchannel n between the CR relay and destination, N_d the PSD of AWGN at the destination. For notation brevity, let $a_n = |h_{sr,n}|^2/(N_r B + \Sigma_{l=1}^{L} I_{n,l}^{PR})$, $b_n = |h_{rd,n}|^2/(N_d B + \Sigma_{l=1}^{L} I_{n,l}^{PD})$.

Since we assume the DN receives the signals without diversity and the signal from the direct path is not taken into account, the transmission rate from source to the destination on the nth subchannel can be calculated as [12]

$$R_n = \frac{1}{2} \log(1 + \gamma_n), \quad (6)$$

where the rate is scaled by 1/2 because the transmission takes two time slots.

The EE is defined as the system throughput for unit-energy consumption in $bits/Joule$. Besides the transmission power, the energy consumption also includes circuit energy in the active mode, which is incurred by signal processing, active circuit blocks, etc. Here, the associated circuit energy consumption is generally modeled as a constant P_c [13].

In this paper, we aim to maximize the EE of the CR system with cooperative relay, under the individual transmission power budget of the SN and the RN. Besides, the total throughput should meet the minimal requirement R_{min}, while the interference to te PUs is kept below predefined thresholds. Thus, the energy-efficient optimization problem can be formulated as

$$\begin{aligned} \max_{p_{s,n}, p_{r,n}} \quad & \eta_{EE} = \frac{\Sigma_{n=1}^{N} R_n}{\Sigma_{n=1}^{N} p_{s,n} + \Sigma_{n=1}^{N} p_{r,n} + P_c} \\ s.t.\ C1 \quad & \sum_{n=1}^{N} R_n \geq R_{min} \\ C2 \quad & \sum_{n=1}^{N} p_{s,n} \leq P_S \\ C3 \quad & \sum_{n=1}^{N} p_{r,n} \leq P_R \\ C4 \quad & \sum_{n=1}^{N} p_{s,n} I_{n,l}^{SP} \leq I_l^{th}, l = 1, \ldots, L \\ C5 \quad & \sum_{n=1}^{N} p_{r,n} I_{n,l}^{RP} \leq I_l^{th}, l = 1, \ldots, L \\ C6 \quad & p_{s,n} \geq 0,\ p_{r,n} \geq 0, \forall n, \end{aligned} \quad (7)$$

where P_S and P_R are the total transmission power limit of the SN and the RN, respectively. I_l^{th} is the interference threshold of the lth PU. C1 guarantees the minimal rate requirement of the relaying CR system. C2 and C3 represent the constraints on transmission power budget, while C4 and C5 keep the interference to PUs under the tolerable range.

III. The optimal power allocation algorithm

Obviously, Eq.(7) defines a nonlinear fractional programming problem, which is generally difficult to tackle. Furthermore, the non-convexity of R_n makes the problem even tough.

To make it tractable, we adopt the following approximation

$$R_n \approx \frac{1}{2}\log\left(1+\frac{p_{s,n}a_n p_{r,n}b_n}{p_{s,n}a_n+p_{r,n}b_n}\right), \tag{8}$$

where γ_n is approximated by the harmonic mean of $p_{s,n}$ and $p_{r,n}$. Such an approximation is proved to be reasonable [14]. Note that R_n in Eq.(8) is jointly convex with $\boldsymbol{p}_s$ and $\boldsymbol{p}_r$, where we stack $\{p_{s,n}\}_{n=1}^N$ and $\{p_{r,n}\}_{n=1}^N$ into the vectors $\boldsymbol{p}_s = \{p_{s,1},\ldots,p_{s,N}\}$ and $\boldsymbol{p}_r = \{p_{r,1},\ldots,p_{r,N}\}$, respectively.

A. The equivalent hypograph problem form

Note that Eq.(7) is still a non-convex optimization problem, even with the approximation of R_n in (8). Nevertheless, an equivalent transformation of Eq.(7) can lead to a convex optimization problem via its hypograph form [15]. The hypograph form of Eq.(7) is given by

$$\begin{array}{ll} \max\limits_{\boldsymbol{p}_s,\boldsymbol{p}_r,y\geq 0} & y \\ s.t. & \eta_{EE}(\boldsymbol{p}_s,\boldsymbol{p}_r) \geq y \\ & C1 \sim C6 \text{ in Eq.(7)}. \end{array} \tag{9}$$

We can take the equivalent inequalities $\varphi(\boldsymbol{p}_s,\boldsymbol{p}_r,y) \geq 0$ to substitute for $\eta_{EE}(\boldsymbol{p}_s,\boldsymbol{p}_r) \geq y$ in Eq.(9), where $\varphi(\boldsymbol{p}_s,\boldsymbol{p}_r,y) = \sum\limits_{n=1}^{N} R_n - y(\sum\limits_{n=1}^{N} p_{s,n} + \sum\limits_{n=1}^{N} p_{r,n} + P_c)$. Thus, the equivalent hypograph problem form of Eq.(7) is established as follows,

$$\begin{array}{lll} \max\limits_{\boldsymbol{p}_s,\boldsymbol{p}_r,y} & & y \\ s.t. & C1 & \varphi(\boldsymbol{p}_s,\boldsymbol{p}_r,y) \geq 0 \\ & C2 & \sum_{n=1}^{N} R_n \geq R_{min} \\ & C3 & \sum_{n=1}^{N} p_{s,n} \leq P_S \\ & C4 & \sum_{n=1}^{N} p_{r,n} \leq P_R \\ & C5 & \sum_{n=1}^{N} p_{s,n} I_{n,l}^{SP} \leq I_l^{th}, l=1,\ldots,L \\ & C6 & \sum_{n=1}^{N} p_{r,n} I_{n,l}^{RP} \leq I_l^{th}, l=1,\ldots,L \\ & C7 & p_{s,n} \geq 0,\ p_{r,n} \geq 0, \forall n \\ & C8 & y \geq 0. \end{array} \tag{10}$$

The hypograph form problem (10) can be analyzed geometrically as an optimization problem in the 'graph space' of $(\boldsymbol{p}_s,\boldsymbol{p}_r,y)$, that is, we maximize y over the hopygraph of η_{EE}, subject to the constraints in Eq.(7), which is equivalent to solve Eq.(7) directly. It is easy to prove Eq.(10) is a convex optimization problem, since the objective function and the inequality constraints functions are all convex [15].

B. Fast barrier method

Generally, barrier method is treated as a standard technique to solve convex optimization problems, which makes all inequality constraints implicit in the optimization objective. Then the original problem is converted into a sequence of minimization problems, by introducing a logarithmic barrier function with parameter t. The solution to each minimization problem is called a central point in the central path related to the original problem. The central point will be more accurately approximated to the optimal solution as the parameter t increases. For searching the center point with a given t, Newton method is generally employed. The computational complexity of the barrier method mainly lies in the computation of Newton step that needs matrix inversion with complexity of $O(N^3)$ for our considered problem.

In order to reduce the computational cost, we propose a fast barrier algorithm by exploiting the special structure of the problem. First, a preparatory procedure is necessary for the barrier method, by transforming the objective y in Eq.(10) into a twice differentiable function $U(y)$, where U is monotone increasing. Evidently the associated problem and the original form problem (10) are equivalent; indeed, the feasible sets are identical, as well as the optimal points [15]. In this paper, we take $U(y) = \log(1+y)$ to guarantee the equivalence between the two problems.

Then, we convert all inequality constraints into a logarithmic barrier function $\phi(\boldsymbol{x})$,

$$\begin{aligned} \phi(\boldsymbol{x}) = \quad & -\log\varphi(\boldsymbol{x}) - \log y - \log(\sum_{n=1}^{N} R_n - R_{min}) \\ & -\log(P_S - \sum_{n=1}^{N} p_{s,n}) - \log(P_R - \sum_{n=1}^{N} p_{r,n}) \\ & -\sum_{l=1}^{L}\log(I_l^{th} - \sum_{n=1}^{N} p_{s,n} I_{n,l}^{SP}) - \sum_{n=1}^{N}\log p_{s,n} \\ & -\sum_{l=1}^{L}\log(I_l^{th} - \sum_{n=1}^{N} p_{r,n} I_{n,l}^{RP}) - \sum_{n=1}^{N}\log p_{r,n}, \end{aligned} \tag{11}$$

where all variables $\{\boldsymbol{p}_s,\boldsymbol{p}_r,y\}$ are collected into one vector $\boldsymbol{x}$, i.e.$\boldsymbol{x} = \{p_{s,1},p_{r,1},\ldots,p_{s,N},p_{r,N},y\}$. Thus, the optimal solution to the (10) can be approximated by solving the following minimization problem

$$\min \quad \psi_t(\boldsymbol{x}) = -t\log(1+y) + \phi(\boldsymbol{x}). \tag{12}$$

The optimal solution to (12) is an approximation of the original problem (10). As t increases, the approximation becomes more and more close to the optimal solution.

During the centering step of barrier method, Newton method is executed to compute the central point. With a given parameter t, Newton step $\Delta\boldsymbol{x}$ can be given by the following Karush-Kuhn-Tucker (KKT) system,

$$\bigtriangledown^2\psi_t(\boldsymbol{x})\Delta\boldsymbol{x}_{nt} = -\bigtriangledown\psi_t(\boldsymbol{x}), \tag{13}$$

where $\Delta\boldsymbol{x}_{nt} \in \Re^{2N+1}$. $\bigtriangledown^2\psi_t(\boldsymbol{x})$ and $\bigtriangledown\psi_t(\boldsymbol{x})$ are the Hessian and the gradient of $\psi_t(\boldsymbol{x})$, respectively.

The outline of the barrier method is summarized in Table II. ϵ and ϵ_n are the tolerances of the barrier method and the Newton step, respectively. α and β are two constants utilized in backtracking line search with $\alpha \in (0, 0.5)$ and $\beta \in (0, 1)$. The step size of the backtracking line search is s with $s > 0$. t and μ is a parameter that is associated with a tradeoff between outer iterations and inner iterations.

If calculating the Newton step in (13) by matrix inversion directly, it will generate a complexity of $O(N^3)$. We analyze

TABLE I
THE BARRIER METHOD

Algorithm 2
1.**Initialization:** A feasible point $\boldsymbol{x}$, $t > 0$, tolerance $\epsilon > 0, \mu > 1$
2.**Outer Loop for Barrier method**
3. Centering step: Compute $\boldsymbol{x}^*(t)$ derived by problem (12)
4. **Initialization for Newton method**
5. Starting point $\boldsymbol{x}$, tolerance $\epsilon_n > 0, \alpha \in (0, 1/2), \beta \in (0, 1)$
6. **Inner Loop for Newton method**
7. Compute $\Delta \boldsymbol{x}_{nt}$ and $\lambda = - \bigtriangledown \psi_t(\boldsymbol{x}) \Delta \boldsymbol{x}_{nt}$;
8. Quit if $\lambda^2/2 \le \epsilon_n$
9. Backtracking line search on $\psi_t(\boldsymbol{x})$, $s := 1$
10. ***while*** $\psi_t(\boldsymbol{x} + s\Delta \boldsymbol{x}) > \psi_t(\boldsymbol{x}) - \alpha s \lambda^2$
11. $s := \beta s$
12. ***endwhile***
13. Update:$\boldsymbol{x} = \boldsymbol{x} + s\Delta \boldsymbol{x}$,
14. Update:$\boldsymbol{x}^*(t) = \boldsymbol{x}$.
15. Stopping criterion:$(2N + 2L + 4)/t < \epsilon$.
16. Increase:$t := \mu t$.

the problem (12) and develop a fast computation of the Newton step by exploiting its special structure. For simplicity, denote

$$\begin{aligned} f_s &= P_s - \sum_{n=1}^{N} p_{s,n}, & f_{s,l} &= I_l^{th} - \sum_{n=1}^{N} p_{s,n} I_{n,l}^{SP}, \\ f_r &= P_r - \sum_{n=1}^{N} p_{r,n}, & f_{r,l} &= I_l^{th} - \sum_{n=1}^{N} p_{r,n} I_{n,l}^{RP}, \\ g_r &= \sum_{n=1}^{N} R_n - R_{min}. \end{aligned} \tag{14}$$

The Hessian of $\psi_t(\boldsymbol{x})$ is given by

$$\begin{aligned} \bigtriangledown^2 \psi_t(\boldsymbol{x}) &= D + \frac{\bigtriangledown f_s \bigtriangledown f_s^T}{f_s^2} + \sum_{l=1}^{L} \frac{\bigtriangledown f_{s,l} \bigtriangledown f_{s,l}^T}{f_{s,l}^2} \\ &\quad + \frac{\bigtriangledown f_r \bigtriangledown f_r^T}{f_r^2} + \sum_{l=1}^{L} \frac{\bigtriangledown f_{r,l} \bigtriangledown f_{r,l}^T}{f_{r,l}^2} \\ &\quad + \frac{\bigtriangledown \varphi \bigtriangledown \varphi^T}{\varphi^2} + \frac{\bigtriangledown g_r \bigtriangledown g_r^T}{g_r^2} \\ &= D + \sum_{i=1}^{2L+4} G_i G_i^T, \end{aligned} \tag{15}$$

where $D = \text{diag}(D_1, \dots, D_N, Y) \in \Re^{2N+1}$ with

$$\begin{aligned} D_n &= \begin{bmatrix} \frac{1}{p_{s,n}^2} & 0 \\ 0 & \frac{1}{p_{r,n}^2} \end{bmatrix} - \left(\frac{1}{g_0} + \frac{1}{g_r} \right) \\ &\quad \begin{bmatrix} \frac{\partial^2 R_n}{\partial p_{s,n}^2} & \frac{\partial^2 R_n}{\partial p_{s,n} \partial p_{r,n}} \\ \frac{\partial^2 R_n}{\partial p_{r,n} \partial p_{s,n}} & \frac{\partial^2 R_n}{\partial p_{r,n}^2} \end{bmatrix}, \\ Y &= t/(1+y)^2 + 1/y^2. \end{aligned} \tag{16}$$

The Hessian is positive definite because the diagonal matrix D positive definite matrixes, and all $G_i G_i^T \ge 0$. Denote $G_0 = - \bigtriangledown \psi_t(\boldsymbol{x})$, $H_0 = \bigtriangledown^2 \psi_t(\boldsymbol{x})$ and $M = 2L + 4$. Instead of computing $\Delta \boldsymbol{x}_{nt}$ via matrix inversion, we propose a fast algorithm by exploiting the structure of the problem as analyzed above, to speedup the Newton step with an M-step procedure as follows [16],

Step 1 Let $H_0 = H_1 + G_1 G_1^T$.
Then we have $\Delta \boldsymbol{x}_{nt} = \upsilon_1^1 - \frac{G_1 \upsilon_1^1}{1 + G_1 \upsilon_2^1} \upsilon_2^1$,
Where $H_1 \upsilon_1^1 = G_0$ and $H_1 \upsilon_2^1 = G_1$.

Step 2 We further decompose Λ_1 with $H_1 = H_2 + G_2 G_2^T$, Similarly, the two variables in Step 1 can be updated by $\upsilon_i^1 = \upsilon_i^2 - \frac{G_2 \upsilon_i^2}{1 + G_2 \upsilon_3^2} \upsilon_3^2, i = 1, 2$,
where $H_2 \upsilon_i^2 = G_{i-1}, i = 1, 2, 3$.

...

Step m Let $H_{m-1} = H_m + G_m G_m^T$.
We can update the m variables in Step $m - 1$ by $\upsilon_i^{m-1} = \upsilon_i^m - \frac{G_m^T \upsilon_i^m}{1 + G_m \upsilon_{m+1}^m} \upsilon_{m+1}^m, i = 1, \dots, m$,
where $H_m \upsilon_i^m = G_{i-1}, i = 1, \dots, m+1$ with $H_i = D + \sum_{j=i+1}^{M} G_j G_j^T$.

Through M step as discussed above, there produces $M + 1$ matrix systems $\Lambda_M \upsilon_i^M = G_{i-1}, i = 1, \dots, M + 1$. During the derivation, we can find that the m variables $\upsilon_i^{m-1}, i = 1, \dots, m$ in Step $m - 1$ can be obtained by the $m + 1$ variables $\upsilon_i^m, i = 1, \dots, m + 1$ in Step m. Hence, if we can figure out the $M + 1$ variables $\upsilon_i^M, i = 1, \dots, M + 1$, the Newton step and the associated dual variable in (13) will be indirectly obtained. Obviously, a reverse derivation of the M steps' decomposition discussed above is necessary to be executed, after we solve the $M + 1$ matrix system in the Step M.

Then we demonstrate the exhaustive process to solve the matrix system $H_M \upsilon_i^M = G_{i-1}$ as follows. Without loss of generality, we convert the equations into a unified form $Du = G$ since we have $\Lambda_M = D$, where $u, G \in \Re^{2N+1}$. It follows

$$\begin{aligned} \begin{bmatrix} u_{2n-1} \\ u_{2n} \end{bmatrix} &= D_n^{-1} \begin{bmatrix} G_{2n-1} \\ G_{2n} \end{bmatrix}, n = 1, \dots, N, \\ u_{2N+1} &= G_{2N+1}/Y. \end{aligned} \tag{17}$$

Thus, the $M + 1$ matrix systems in Step M can be solved in the same way.

C. Warm start procedure for barrier method

In the initialization of the barrier method in Section III-B, a strictly feasible starting point is required. Thus, a preparatory procedure is necessary to obtain a feasible points or prove its inexistence. We execute the warm start procedure in two step. First, we try to find a feasible point $(\boldsymbol{p}_s^0, \boldsymbol{p}_r^0)$, satisfying the constraints $C3 \sim C7$ in Eq.(10). Then we can take any value in the interval $(0, \eta_{EE}(\boldsymbol{p}_s^0, \boldsymbol{p}_r^0))$ as a feasible y, denoted as y^0.

During the first step, finding a feasible solution is equivalent to solve a minimization problem by introducing a crucial indicator parameter z as discussed in [15]. The optimization problem for the warm start procedure can be formulated as

$$\begin{aligned} \min_{p_{s,n}, p_{r,n}, z} \quad & z \\ s.t.\ C1 \quad & \textstyle\sum_{n=1}^{N} R_n \ge R_{min} - z \\ & C3 \sim C7 \ \text{ in } \ \text{Eq.}(10), \end{aligned} \tag{18}$$

where z can be interpreted as a bound on the maximum infeasibility of the inequality C1 and our goal is to drive it below zero. Since it is easy to find $\boldsymbol{p}_s, \boldsymbol{p}_r$ to satisfy C3∼C7 in Eq.(10), we can choose a feasible z to satisfy C1. So the feasible solution to Eq.(18) always exists.

Note that Eq.(18) also defines a convex problem whose structure is similar to Eq.(10). Due to its special structure, we can also apply the fast barrier method developed in section III-B to solve the problem (18). By solving Eq.(18), a strictly feasible point $(\boldsymbol{p}_s^0, \boldsymbol{p}_r^0, y^0)$ may be computed, or it proves no feasible point exists. If the optimal solution to Eq.(18) satisfies $z \leq 0$, the associated solution of $\boldsymbol{p}_s$ and $\boldsymbol{p}_r$ can be used as the starting point of the barrier method to solve Eq.(10). Otherwise, no feasible point exists for the Eq.(10) and we regard such a case as system outage.

D. On the complexity

The computational complexity can be counted roughly as follows. The fast barrier algorithm of solving Eq.(10) consumes M decomposition, while each decomposition yields an additional equation. First, we need to solve $M+1$ matrix system according to (17) with the computational complexity $O(N)$ for each one. Then, a reverse substitution procedure is carried out and the total computational cost for the fast barrier method is $O(M^2N)$, after M reverse substitution steps. Thus, we can conclude the complexity for solving the optimal solution to Eq.(10) is measured by $O(M^2N)$. If we take the standard barrier method and compute the Newton step directly by matrix inversion, the total complexity is $O(N^3)$.

Since we can also apply the proposed fast algorithm to solve the warm start problem, the complexity is roughly equal to that of solving Eq.(10), because of the similar structure. Therefor, we conclude the complexity of the optimal power allocation is $O(M^2N)$. Notice that the number of PUs is always much smaller than that of the subchannels in wireless systems, that is, $M \leq N$, the complexity is reduced dramatically.

IV. Simulation results

Consider a CR system where the SN and PUs are uniformly distributed within a 1km circle area. The frequency-selective fading is assumed and the path loss exponent is set to 4. The variance of logarithmic normal shadow fading is 10 dB and the amplitude of multipath fading is Rayleigh. We assume that each PU's bandwidth is randomly generated by uniform distribution and the maximum value is $2W/3L$. The noise power is 10^{-13}W. To emphasize the advantages of energy-efficient power loading scheme for green communication, we compare the EE of our proposed optimal energy-efficient power allocation algorithm (EE-PA), with that of rate adaptive power allocation algorithm (RM-PA) which always maximizes the throughput of the relaying CR system.

First, we illustrate the EE of CR system versus the transmission power limit at the SN and RN in Fig.1 and Fig.2, respectively. Two cases of different numbers of subchannels, that are $N=32$ and $N=64$, are considered. There are 2 PUs and the interference threshold of each PU is uniformly set to 5×10^{-12}W. The static circuit power is fixed to 0.5W, with $P_r = 1$W in Fig.1 and $P_s = 1$W in Fig.2. Besides, the minimal rate requirement is 20bits/symbol. For both cases of N=32 and N=64, the EE of EE-PA grows with the increase of P_s (P_r), until the power is sufficient to satisfy the rate requirements. Because more power budger will lower the probability of system outage and enhance the EE of the CR system.

For the RM-PA, the curve of EE first increases with the growth of P_s (P_r), and the loss of EE occurs at a cut-off of the transmission power limit, where a decrease of EE can be found when P_S (P_r) becomes larger. This phenomenon can be explained intuitively. When the power limit is relatively small, P_c is the main part of the total power consumption and larger P_s or P_r can achieves more capacity. However, when P_s or P_r gets larger enough to ignore the static circuit power, the EE will decrease since the logarithmic growth of the capacity exhausted the total power budget.

Fig.3 shows the EE of the CR system as a function of the minimal rate requirements for different numbers of PUs. There are 64 subchannels with the transmission power budgets $P_s = 10$W and $P_r = 1$W, while the static circuit power $P_c = 0.5$W.

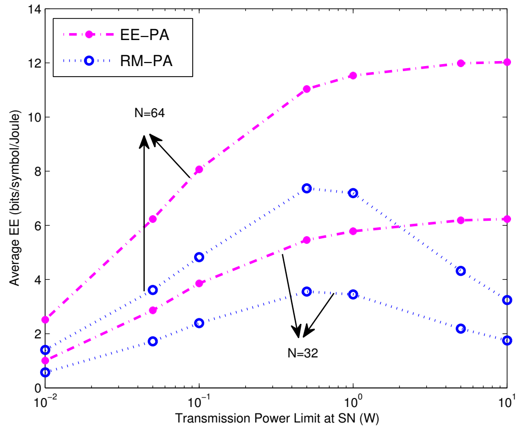

Fig. 1. The EE of CR system as a function of transmission power limit at SN. $L=2$, $P_r = 1$W, $R_{min} = 20$bits/symbol

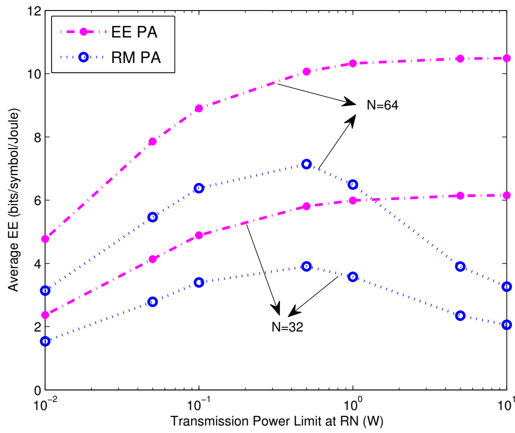

Fig. 2. The EE of CR system as a function of transmission power limit at RN. $L=2$, $P_s = 1$W, $R_{min} = 20$bits/symbol

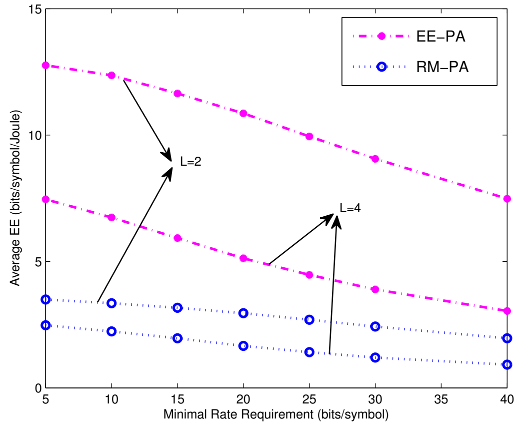

Fig. 3. The EE of CR system as a function of minimal rate requirement. $L = 2$, $P_s = 10$W, $R_r = 1$W

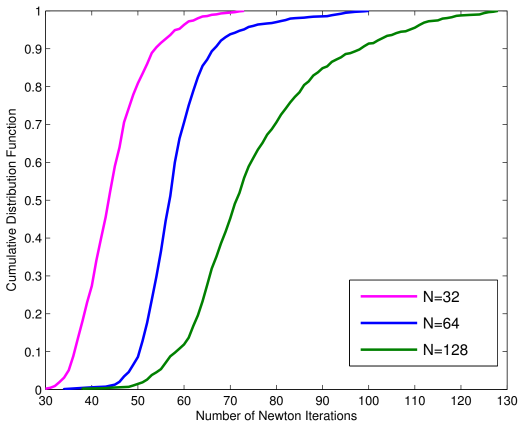

Fig. 4. CDF of the number of Newton iterations required for convergence for 1000 channel realizations.

The interference threshold of each PU is 5×10^{-12}W. We can observe that the EE of CR system decreases with the growth of the rate requirements for both $L = 2$ and $L = 4$. Because the growth of rate requirements will not only result in exponentially increase of power consumption, but also more frequently system outage. Comparing the curves of the two cases, we find more PUs will lower the total EE, since more subchannels will be interference limited and fail to maintain the rate requirements.

Finally, we investigate the convergence of our proposed algorithm in Fig.4. As discussed in Section III-B, the computational load of the algorithm mainly lies in the computation of Newton step. If the number of Newton iterations is large or varies in a wide range, the algorithm would be difficult to be applied in practical wireless systems. Fig.4 demonstrates that it is not the case for our proposed algorithm. Fig.4 gives the cumulative distribution function (CDF) of the number of Newton iterations for our proposal with different settings of N. It shows that the number of Newton iterations varies in a narrow range with a given N, which validates that our proposed algorithm is effective and efficient.

V. Conclusion

This paper investigated the energy-efficient power allocation in the relaying CR networks. Since the formulated problem is non-convex and difficult to tackle, it is first converted into a convex optimization problem in its hypogragh form. We analyzed the equivalent problem and developed a fast barrier algorithm to work out the optimal solution quickly, which always updates the Newton step in an ingenious way by exploiting its special structure. Numerical simulations verified the effectiveness and efficiency of our proposed method.

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References

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. on Sel. Areas in Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[2] Q. Zhang, J. Jia, and J. Zhang, "Cooperative relay to improve diversity in cognitive radio networks," *IEEE Commun. Mag.*, vol. 47, no. 2, pp. 111–117, Feb. 2009.

[3] L. Li, X. Zhou, H. Xu, G. Li, D. Wang, and A. Soong, "Simplified relay selection and power allocation in cooperative cognitive radio systems," *IEEE Trans. on Wire. Commun.*, vol. 10, no. 1, pp. 33–36, Jan. 2011.

[4] D. Bharadia, G. Bansal, P. Kaligineedi, and V. Bhargava, "Relay and power allocation schemes for OFDM-based cognitive radio systems," *IEEE Trans. on Wire. Comm.*, vol. 10, no. 9, pp. 2812–2817, Sep. 2011.

[5] C. Han, T. Harrold, S. Armour, I. Krikidis, S. Videv, P. Grant, H. Haas, J. Thompson, I. Ku, C.-X. Wang, T. A. Le, M. Nakhai, J. Zhang, and L. Hanzo, "Green radio: radio techniques to enable energy-efficient wireless networks," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 46–54, June 2011.

[6] Y. Chen, S. Zhang, S. Xu, and G. Li, "Fundamental trade-offs on green wireless networks," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 30–37, Jun. 2011.

[7] D. Feng, C. Jiang, G. Lim, L. Cimini, Jr., G. Feng, and G. Li, "A survey of energy-efficient wireless communications," *IEEE Commun. Surv. & Tutor.*, vol. PP, no. 99, pp. 1–12, 2012.

[8] C. Xiong, G. Li, S. Zhang, Y. Chen, and S. Xu, "Energy-efficient resource allocation in OFDMA networks," in *Proc. of the IEEE GLOBECOM 2011*, Dec. 2011, pp. 1–5.

[9] ——, "Energy- and spectral-efficiency tradeoff in downlink OFDMA networks," *IEEE Trans. on Wire. Commun.*, vol. 10, no. 11, pp. 3874–3886, Nov. 2011.

[10] G. Miao, N. Himayat, G. Li, and S. Talwar, "Low-complexity energy-efficient scheduling for uplink OFDMA," *IEEE Trans. on Commun.*, vol. 60, no. 1, pp. 112–120, Jan. 2012.

[11] G. Bansal, M. Hossain, and V. Bhargava, "Adaptive power loading for OFDM-based cognitive radio systems," in *Proc. of the IEEE ICC 2007*, June 2007, pp. 5137–5142.

[12] Y. Li, W. Wang, J. Kong, W. Hong, X. Zhang, and M. Peng, "Power allocation and subcarrier pairing in ofdm-based relaying networks," in *Proc. of the IEEE ICC 2008*, May 2008, pp. 2602–2606.

[13] S. Cui, A. Goldsmith, and A. Bahai, "Energy-constrained modulation optimization," *IEEE Trans. on Wire. Commun.*, vol. 4, no. 5, pp. 2349–2360, Sep. 2005.

[14] M. Hasna and M.-S. Alouini, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. on Commun.*, vol. 52, no. 1, pp. 130–135, Jan. 2004.

[15] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[16] C. D. Meyer, *Matrix Analysis & Applied Linear Algebra*. SIAM Press, 2000.