Energy- and Spectrum-Efficiency Tradeoff in OFDM-based Cognitive Radio Systems

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***Abstract*—In this paper, we investigate the Energy Efficiency (EE)- Spectrum Efficiency (SE) tradeoff issue in an OFDM-based cognitive radio (CR) network. A multi-objective resource allocation problem is formulated, where we try to maximize the EE and the SE simultaneously. The Pareto optimal set of the formulated problem is characterized by analyzing the relationship between the EE and the SE. To find a unique globally optimal solution, we proposed a unified EE-SE tradeoff metric, based on which the original optimization task is transformed into a single-objective problem that has a D.C. (Difference of two Convex functions/sets) structure. Then an efficient barrier method is developed, where we speeds up the time-consuming computation of Newton step by exploiting the structure of the D.C. programming problem. Simulation results validate the effectiveness and efficiency of the proposed algorithm. Our general problem formulation sheds some insights on how to design an energy- and spectrum-efficient CR system.**

I. Introduction

Spectrum scarcity crisis has been existing for many years in wireless applications, especially in the band below 6GHZ. On the other hand, investigations show that large portions of spectrum are highly underutilized due to the inefficient conventional regulatory policies [1]. Cognitive Radio (CR) is deemed as a highly promising technology to improve the spectrum usage efficiency and has gained more and more attentions. It has been proposed as a solution to the underutilization problem by allowing Secondary Users (SUs) to sense radio spectrum environment and opportunistically access licensed frequency, as long as the interference to the Primary Users (PUs) can be kept under their tolerances, such as interference temperature. In order to meet the requirements of opportunistic access, the physical layer of a CR system should be very flexible, which necessitates multicarrier methods to operate in CR networks. Orthogonal Frequency Division Multiplexing (OFDM) has been widely recognized as a fascinating air interface for CR systems due to its flexibility in adapting spectral environments and allocating radio spectrum among SUs, which is the prerequisite for the CR system to acquire high performance [2].

Resource Allocation (RA) is an important issue in OFDM-based wireless networks and has been studied extensively for more than a decade. Spectral efficiency (SE), defined as the system throughput per unit of bandwidth, is a widely accepted criterion for wireless network optimization. For an OFDM-based CR network, there are many research results on how to improve the SE or equivalently, minimize the power consumption with given throughput constraints [3]. On the other hand, energy efficiency (EE) is the main problem in wireless networks, and attracted attentions in both industry and academia nowadays [4, 5]. It is noteworthy that there is few works on the joint optimization of the EE and the SE of a CR network. Ideally, it is desirable to maximize the EE and the SE simultaneously. However, the EE and the SE do not always coincide and may even conflict in many cases [6]. Thus, it is important to understand the tradeoff between the EE and the SE, which offers a balanced view to the nature of a communication system and can provide guidelines for wireless system design and optimization.

In [7], the EE-SE tradeoff in the downlink of an Orthogonal Frequency Division Multiple access (OFDMA) network is formulated to maximize EE with the minimal SE requirement. In [8], a multi-criteria optimization method is proposed to investigate the relationship between the EE and the SE in distributed antenna systems. In [9], the EE and the SE is jointly optimized, where a unified metric for the EE-SE tradeoff is developed. However, the EE-SE relationship in an OFDM-based CR network is more complicated because the interference to the PUs should be considered carefully. To allocate radio resource to tradeoff the EE and the SE in a CR system efficiently is a nontrivial question.

In this paper, we address the EE-SE tradeoff issue in an OFDM-based CR network. We formulate a multi-objective optimization problem to optimize the EE and the SE simultaneously. First, for a general system model, we prove that the EE is quasiconcave in the SE. The Pareto optimal set of the formulated multi-objective optimization problem is characterized. To find a unique globally optimal solution, we proposed a unified EE-SE tradeoff metric to transform the multi-objective optimization problem into a single-objective one, which has a D.C. (difference of two convex functions/sets) structure and can be solved by Frank-and-Wolfe (FW) procedure. In the FW procedure, a convex optimization problem needs to be solved, for which we derive a fast algorithm by exploiting the structure of the problem.

The rest of this paper is organized as follows. In Section II, we illustrate the system model and formulate our optimization task. In Section III, we analyze the EE-SE relationships and propose the power allocation scheme. Simulation results are

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given in Section VI, as well as discussions. In Section V, we draw our conclusions.

II. System Model and Problem Formulation

A. System Model

Consider the downlink of a CR system with OFDM modulation coexisting with a licensed system. The available bandwidth W is divided into N OFDM subchannels in the CR system, denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$. The bandwidth of the nth subchannel spans from $f_0 + (n-1)W/N$ to $f_0 + nW/N$, where f_0 is the starting frequency. The licensed system serves L PUs. The lth PU's nominal band ranges from f_l to f_l+B_l, where f_l and B_l are the starting frequency and the lth PU's occupied bandwidth, respectively.

Perfect channel-state information is assumed to be available at the transceivers of the SUs and the PUs. The interference introduced to the lth PU by SUs' access on the nth subchannel with unit transmission power can be represented as follows,

$$I_{n,l}^{SP} = \int_{f_l - f_0 - (n-1/2)W/N}^{f_l + B_l - f_0 - (n-1/2)W/N} g_{n,l}^{SP} \phi_n^{SU}(f) df, \quad (1)$$

where $g_{n,l}^{SP}$ is the power gain from the SU's transmitter to the lth PU's receiver on the nth subchannel. $\phi_n^{SU}(f)$ is the Power Spectrum Density (PSD) of the OFDM subchannel used by an SU, which can be expressed as $\phi_n^{SU}(f) = T_s(\frac{\sin \pi f T_s}{\pi f T_s})^2$, where T_s is OFDM symbol duration. On the other hand, the interference generated by the lth PU into the nth subchannel used by the SU is

$$I_{n,l}^{PS} = \int_{f_0 + (n-1)W/N - f_l - B_l/2}^{f_0 + nW/N - f_l - B_l/2} g_{n,l}^{PS} \phi_l^{PU}(f) df, \quad (2)$$

where $g_{n,l}^{PS}$ is the power gain from the lth PU's transmitter to the SU's receiver on the nth subchannel and $\phi_l^{PU}(f)$ is the PSD of the lth PU's signal. Note that we do not assume that the PUs also adopt OFDM modulation.

Define the Signal-to-Noise Ratio (SNR) of the nth OFDM subchannel with unit power as

$$H_n = \frac{g_n^{SS}}{\Gamma(N_0 B + \sum_{l=1}^{L} I_{n,l}^{PS})}, \quad (3)$$

where $B = \frac{W}{N}$, g_n^{SS} is the power gain of the SU on the nth subchannel, N_0 is the power spectrum density of additive white Gaussian noise, Γ is the SNR gap and can be represented as $\Gamma = -\frac{\ln(5BER)}{1.5}$ for an uncoded MQAM with a specified BER [10].

The η_{SE} of the CR system is defined as the throughput per unit of bandwidth

$$\eta_{SE} = \sum_{n=1}^{N} \log(1 + p_n H_n), \quad (4)$$

where p_n is the power allocated to the channel n. Energy-efficiency η_{EE} is defined as the ratio of the system throughput per unit of bandwidth over the total power consumption

$$\eta_{EE} = \frac{\sum_{n=1}^{N} \log(1 + p_n H_n)}{\sum_{n=1}^{N} p_n + P_c}, \quad (5)$$

where P_c is the circuit power consumption. It can be a constant or a function of throughput [11]. To design an energy-efficient wireless system, both the transmission power and the circuit energy consumption should be taken into consideration, where the former is used for reliable data transmission and the latter represents average energy consumption of device electronics.

B. Problem Formulation

Our goal is to simultaneously maximize the EE and the SE of the CR system which operate in a power-limited situation, while keeping the interference to the PUs not exceeding their specified thresholds. Thus, the optimization problem can be formulated as follows,

$$\begin{aligned} & \max_{p_n} \quad \{\eta_{SE}, \eta_{EE}\} \\ s.t. \quad & \text{C1}: p_n \geq 0, \forall n \\ & \text{C2}: \sum_{n=1}^{N} p_n \leq P_t, \\ & \text{C3}: \sum_{n=1}^{N} p_n I_{n,l}^{SP} \leq I_l^{th}, l = 1, \ldots, L \end{aligned} \quad (6)$$

where P_t is the power limit of the CR system and I_l^{th} is the interference power threshold of the lth PU. C1 is intuitive. C2 and C3 are the power limitation and the interference constraints, respectively.

III. EE-SE tradeoff power allocation

A. Relationship between EE and SE

We first study the EE-SE relationship and demonstrate the quasiconcavity of the EE in the SE. As the CR system is power-limited and interference-limited [12], there is a maximum value of η_{SE}. Assume the maximal value of η_{SE} is η_{SE}^{max} and the SE region is $[0, \eta_{SE}^{max}]$.

Theorem 1: For any given SE, achieved with power allocation matrix P that satisfys all constraints in (6), the maximum EE, $\eta_{EE}^{m}(\eta_{SE}) = \max_{P} \eta_{EE}(\eta_{SE})$, is strictly quasiconcave in η_{SE}. Moreover, in the SE region $[0, \eta_{SE}^{max}]$, the EE $\eta_{EE}^{m}(\eta_{SE})$
(i) strictly increases with η_{SE} and is maximized at $\eta_{SE} = \eta_{SE}^{max}$ if $\frac{d\eta_{EE}^{m}(\eta_{SE})}{d\eta_{SE}}|_{\eta_{SE}=\eta_{SE}^{max}} \geq 0$,
(ii) first strictly increases and then strictly decreases with η_{SE} and is maximized at $\eta_{SE} = \eta_{SE}^{*}$ if $\frac{d\eta_{EE}^{m}(\eta_{SE})}{d\eta_{SE}}|_{\eta_{SE}=\eta_{SE}^{max}} < 0$, where $\eta_{SE}^{*} = \arg\max \eta_{EE}^{m}(\eta_{SE})$.

The proof of *Theorem 1* is in Appendix A. From *Theorem 1*, we can see that the EE is quasiconcave in the SE.

Pareto optimal solution is a basic concept in multi-object optimization field. A solution P_1 is called to dominate feasible solution P_2, if $\eta_{SE}(P_1) \geq \eta_{SE}(P_2)$ and $\eta_{EE}(P_1) \geq \eta_{EE}(P_2)$. A feasible solution P_0 is called Pareto optimal if there is no other feasible solution dominating it. There may be multiple Pareto optimal solutions for a multi-object optimization problem. Pareto optimal set is the set of all Pareto optimal points.

Theorem 2: The Pareto optimal set of the problem (6) is

$$P^{POS} = \begin{cases} \{P|\eta_{SE}^* \leq \eta_{SE} \leq \eta_{SE}^{max}\} & \text{if } \eta_{SE}^* < \eta_{SE}^{max} \\ \{P|\eta_{SE} = \eta_{SE}^{max}\} & \text{if } \eta_{SE}^* \geq \eta_{SE}^{max} \end{cases} \quad (7)$$

Theorem 2 is proved in Appendix B. According to (7), in the case of $\eta_{SE}^* \geq \eta_{SE}^{max}$, P^{POS} contains a single point, which means that the globally optimal solution for (6) is unique. So we only need to analyze the case of $\eta_{SE}^* < \eta_{SE}^{max}$.

B. EE and SE Tradeoff Metric

To facilitate system design, we should try to find a unique global solution from the Pareto optimal set P^{POS}. Scalarization method is efficient to distinguish a unique point in the Pareto optimal set. We can transform the multi-object optimization problem (6) into a single-object optimization one by the scalarization method.

Define an EE and SE tradeoff metric [9] as

$$U(P) = [\eta_{SE}(P)]^{\omega} \times [\eta_{EE}(P)]^{1-\omega}, \quad (8)$$

where $\omega \in [0, 1]$. $(\omega, 1-\omega)$ is a given preference configuration for SE and EE. $U(P)$ is referred as the utility function.

Then (6) can be transformed into the following single-object optimization problem,

$$\begin{aligned} &\max_{p_n} \quad U(P) \\ s.t. \quad &\text{C1}: p_n \geq 0, \forall n \\ &\text{C2}: \sum_{n=1}^{N} p_n \leq P_t, \\ &\text{C3}: \sum_{n=1}^{N} p_n I_{n,l}^{SP} \leq I_l^{th}, l = 1, \dots, L \end{aligned} \quad (9)$$

C. D.C.Programming

Consider the following utility transformation,

$$\begin{aligned} V(P) &= \log U(P) \\ &= \omega \log \eta_{SE}(P) + (1-\omega)\log \eta_{EE}(P) \\ &= \log \eta_{SE}(P) - (1-\omega)\log(P_{sum} + P_c), \end{aligned} \quad (10)$$

where $P_{sum} = \sum_{n=1}^{N} p_n$, (9) is equivalent to the following form,

$$\begin{aligned} &\max_{p_n} \quad f(P) - g(P) \\ s.t. \quad &\text{C1}: p_n \geq 0, \forall n \\ &\text{C2}: \sum_{n=1}^{N} p_n \leq P_t, \\ &\text{C3}: \sum_{n=1}^{N} p_n I_{n,l}^{SP} \leq I_l^{th}, l = 1, \dots, L, \end{aligned} \quad (11)$$

where $f(P) = \log \eta_{SE}(P)$ and $g(P) = (1-\omega)\log(P_{sum} + P_c)$. The term $(f(P) - g(P))$ is a D.C. function (Difference of two Convex functions) [13] as both $f(P)$ and $g(P)$ are concave. The gradient of $g(P)$ is $\nabla g(P) = (\frac{\partial g}{\partial p_1}, \frac{\partial g}{\partial p_2}, \cdots, \frac{\partial g}{\partial p_n})$, where

$$\frac{\partial g}{\partial p_n} = \frac{1-\omega}{\sum_{n=1}^{N} p_n + P_c}. \quad (12)$$

We propose an FW procedure which generates a sequence of improved feasible solutions [14, 15]. Initialized from a feasible solution $P^{(0)}$, and $P^{(t+1)}$ at the tth iteration is generated as the optimal solution of the following convex optimization problem,

$$\begin{aligned} &\max_{p_n} \quad f(P) - g(P^{(t)}) - < \nabla g(P^{(t)}), P - P^{(t)} > \\ s.t. \quad &\text{C1}: p_n \geq 0, \forall n \\ &\text{C2}: \sum_{n=1}^{N} p_n \leq P_t, \\ &\text{C3}: \sum_{n=1}^{N} p_n I_{n,l}^{SP} \leq I_l^{th}, l = 1, \dots, L, \end{aligned} \quad (13)$$

where $< x, y >= x^T y$.

Function $g(P)$ is not sensitive to the change of P, so $g(P)$ is well approximated by its first order approximation $g(P^{(t)}) + < \nabla g(P^{(t)}), P - P^{(t)} >$ at a fairly large neighborhood of $P^{(t)}$. Thus, (11) can be well approximated by (13).

As function $g(P)$ is concave, its gradient $g(P^{(t)})$ is also the super-gradient, so

$$g(P) \leq g(P^{(t)}) + < \nabla g(P^{(t)}), P - P^{(t)} > . \quad (14)$$

The convex optimization problem (13) provides a well approximated lower bound maximization of the nonconvex optimization problem (11). Besides, as

$$\begin{aligned} &f(P^{(t+1)}) - g(P^{(t+1)}) \geq f(P^{(t)}) - [g(P^{(t)}) + \\ &< \nabla g(P^{(t)}), P^{(t+1)} - P^{(t)} >] \geq f(P^{(t)}) - g(P^{(t)}), \end{aligned} \quad (15)$$

$P^{(t+1)}$ is always better than the previous $P^{(t)}$.

Since the constraint set is compact, the sequence of improved solutions $\{P^{(t)}\}$ always converges based on By Cauchy theorem. The procedure terminates after finite iterations under the condition of either $|P^{(t)} - P^{(t-1)} \leq \epsilon|$ or $|U(P^{(t)}) - U(P^{(t-1)})| \leq \epsilon$ where ϵ is a threshold for convergence.

The FW procedure to solve (11) is summarized as follows. Initialization: Set $t = 0$, choose $P^{(0)}$ and calculate $U(P^{(0)})$; at the tth iteration: Solve the convex optimization problem (13) to obtain the solution P^*, set $t = t + 1$, $P^{(t)} = P^*$ and calculate $U(P^{(t)})$; stop: when $|V(P^{(t)}) - V(P^{(t-1)})| \leq \epsilon$.

D. Fast Algorithm for Solving (13)

(13) defines a convex optimization problem and many standard techniques can be employed to solve it. However, resource allocation should be tackled in an online way, so fast algorithms are always preferred. We develop a fast barrier method [13, 16] to solve it. First, reformulate the problem (13)

TABLE I
THE BARRIER METHOD

Barrier method
Initialization for the Barrier method
Find feasible point x, $t := t^{(0)} > 0$, tolerance $\epsilon > 0$, $\mu > 1$
Outer Loop for Barrier method
Centering step: Compute $x^*(t)$ derived by problem (13)
Initialization for Newton method
Starting point x,
tolerance $\epsilon_n > 0$, $\alpha \in (0, 1/2)$, $\beta \in (0, 1)$
Inner Loop for Newton method
Compute Δx_{nt} and $\lambda := -\nabla\psi_t(x)\Delta x_{nt}$;
Quit if $\lambda^2/2 \leq \epsilon_n$
Backtracking line search on $\psi_t(x)$, $s := 1$
while $\psi_t(x + s\Delta x_{nt}) > \psi_t(x) - \alpha s \lambda^2$
$s := \beta s$
endwhile
Update: $x = x + s\Delta x_{nt}$
Update: $x^*(t) = x$.
Stopping criterion: $(N + L + 1)/t < \epsilon$
Increase: $t := \mu t$.

into a set of unconstrained optimization problems by making all inequality constraints implicit in a logarithmic function,

$$\begin{aligned}\phi(x) = & -\log(P_t - \sum_{n=1}^{N} p_n) - \sum_{l=1}^{L} \log(I_l^{th} - \sum_{n=1}^{N} p_n I_{n,l}^{SP}) \\ & - \sum_{n=1}^{N} \log(p_n),\end{aligned} \tag{16}$$

where $x = (p_1, p_2, \cdots, p_N)$. Denote $Q(x) = f(P) - \sum_{n=1}^{N} \nabla g(P^{(t)})(p_n - p_n^{(t)}) - g(P^{(t)})$. Thus, the optimal solution to (13) can be approximated by solving the following unconstrained minimization problem with a parameter t

$$\min \psi_t(x) = -tQ(x) + \phi(x). \tag{17}$$

(17) can be solved efficiently by Newton method. Newton step at x, denoted by Δx_{nt} is given by the following Karush-Kuhn-Tucker (KKT) systems,

$$\nabla^2\psi_t(x)\Delta x_{nt} = -\nabla\psi_t(x), \tag{18}$$

where $\nabla^2\psi_t(x)$ and $\nabla\psi_t(x)$ are the Hessian and the gradient of $\psi_t(x)$, respectively.

The outline of the barrier method is summarized in Table I. The computational complexity of the barrier method mainly lies in the computation of Newton step that needs matrix inversion. In order to reduce the computational cost, we exploit the structure of (17) and develop a fast algorithm to calculate the Newton step with lower complexity. Denote

$$\begin{aligned} f_0 &= P_t - \sum_{n=1}^{N} p_n \\ g_l &= I_l^{th} - \sum_{n=1}^{N} p_n I_{n,l}^{SP}, l = 1, 2, \cdots, L. \end{aligned} \tag{19}$$

The Hessian of $\psi_t(x)$ is

$$\begin{aligned}\nabla^2\psi_t(x) &= \begin{bmatrix} D1 & & & \\ & D2 & & \\ & & \ddots & \\ & & & D_N \end{bmatrix} \\ & + \frac{\nabla f_0 \nabla f_0^T}{f_0^2} + \sum_{l=1}^{L} \frac{\nabla g_l \nabla g_l^T}{g_l^2} \\ &= D + \sum_{i=1}^{M} F_i F_i^T. \end{aligned} \tag{20}$$

where $M = L+1$ and $D = diag(D_1, D_2, \cdots, D_N) \in \mathcal{R}^{N\times N}$ with

$$D_n = \frac{tH_n^2(1 + \sum_{n=1}^{N} \log(1 + p_n H_n))}{(\sum_{n=1}^{N} \log(1 + p_n H_n))^2 (1 + p_n H_n)^2} + \frac{1}{p_n^2}. \tag{21}$$

$F_i^{'} s$ are all vectors with N elements,

$$F_i = \begin{cases} \frac{\nabla f_0}{f_0}, & i = 1 \\ \frac{\nabla g_l}{g_l}, & l = 1, \cdots, L, i = l + 1. \end{cases} \tag{22}$$

Since it is easy to prove that the matrix D is positive definite, it follows that the Hessian matrix $\nabla^2\psi_t(x)$ is invertible. However, if we compute the inversion of the KKT matrix directly, it yields a complexity of $O(N^3)$, which is too high for application because there are thousands of OFDM subchannels in practical wireless systems.

Rewrite the KKT system (18) as follows,

$$\Lambda_0 \Delta x_{nt} = F_0, \tag{23}$$

where $\Lambda_0 = \nabla^2\psi_t(x)$ and $F_0 = -\nabla\psi_t(x)$. According to (20), Λ_0 can be written as

$$\Lambda_0 = D + \sum_{i=1}^{M} F_i F_i^T, \tag{24}$$

which can be decomposed into M equations,

$$\Lambda_i = \Lambda_{i+1} + F_{i+1} F_{i+1}^T, i = 0, 1, \cdots, M - 1, \tag{25}$$

By exploiting the structure of Λ_i's, we give an M-step procedure to compute the Newton step.

- **Step 1:** Use (25) to decompose Λ_0, $\Lambda_0 = \Lambda_1 + F_1 F_1^T$. Denote two intermediate variables as the solutions of the following two sets of linear equations, $\Lambda_1 v_1^1 = F_0$ and $\Lambda_1 v_2^1 = F_1$. Then Δx_{nt} can be obtained by $\Delta x_{nt} = v_1^1 - \frac{F_1 v_1^1}{1+F_1 v_2^1} v_2^1$. So we can figure out μ if obtaining the two new variables v_1^1 and v_2^1.
- **Step 2:** decompose Λ_1 with $\Lambda_1 = \Lambda_2 + F_2 F_2^T$. Then the two variables introduced in step 1 can be updated by $v_i^1 = v_i^2 - \frac{F_2 v_i^2}{1+F_2 v_3^2} v_3^2, i = 1, 2,$ where $\Lambda_2 v_i^2 = F_{i-1}, i = 1, 2, 3$.
- **Step m:** decompose Λ_{m-1} with $\Lambda_m = \Lambda_m + F_m F_m^T$. We can update the m variables introduced in Step $m-1$ by $v_i^{m-1} = v_i^m - \frac{F_m v_i^m}{1+F_m v_{m+1}^m} v_{m+1}^m, i = 1, 2, \cdots, m$, which

is obtained by solving the following $m+1$ sets of linear equations, $\Lambda_m v_i^m = F_{i-1}, i = 1, 2, \cdots, m+1$.

Continue the procedure to the Mth step, and there are $M+1$ matrix systems $\Lambda_M v_i^M = F_{i-1}, i = 1, 2, \cdots, M+1$. Form the derivation process, we can find that the m variables $v_i^{m-1}, i = 1, 2, \cdots, m$, at the $(m-1)$th Step can be obtained by the $m+1$ variables $v_i^m, i = 1, 2, \cdots, m+1$ in the mth Step. Thus, if we figure out the $M+1$ variables $v_i^M, i = 1, 2, \cdots, M+1$, μ will be indirectly obtained.

Consider the matrix systems at step M. As $\Lambda_M = D$, these equations can be unified into

$$Dv = h, \tag{26}$$

where $v, h \in \mathcal{R}^{N \times 1}$. Since D is a diagonal matrix, we can easily obtained

$$v_i = D_i^{-1} h_i, i = 1, 2, \cdots, N. \tag{27}$$

The complexity of computing v is $O(N)$. Thus it costs $O(NM)$ to work out the M variables at the Mth Step. A reverse derivation of the M steps is necessary to figure out Δx_{nt} and the total computational complexity can be measured by $O(NM^2)$.

In practical CR systems, $M \ll N$ generally holds, so the complexity of the proposed algorithm is much lower than that of matrix inversion.

IV. Simulation Results

Experiments are conducted to evaluate the performance of our proposed scheme. Consider a multiuser OFDM-based CR system, where all users are randomly located in an area of 3×3km, and the SU's receiver is distributed in a circle within 0.5km from its transmitter. The path loss exponent is 4, the variance of the shadowing effect is 10dB, and the multipath fading is assumed to be Rayleigh. The noise power on a subchannel is set to 10^{-13} W. The frequency bands occupied by PUs are generated randomly with the maximum number of OFDM subchannels $2W/3L$. The interference threshold of each PU is 5×10^{-12}W.

First, we investigate the convergence of the proposed algorithms. As discussed in Section III, the computational load mainly lies in the computation of Newton step. Fig.1 gives the Cumulative Distribution Function (CDF) of Newton iterations for solving (13) with different settings of N. As seen in Fig.1, the number of Newton iterations is small and varies in a narrow range, indicating our proposed algorithm is effective and efficient.

Fig.2 shows the normalized SE, EE and the unified efficiency defined in (8) (Unified) with optimal transmit powers. It can be observed that the normalized SE (SE-norm) will be non-decreasing while the normalized EE (EE-norm) will be non-increasing with ω, and the intersection of the three lines is shown in Fig.2. Furthermore, the normalized SE and EE are still constants when ω is close to 1 because of the power constraints and interference constraints.

Fig.3 illustrates the EE-SE relationship. It can be observed that maximal EE is achieved with $\omega = 0$ while maximal SE

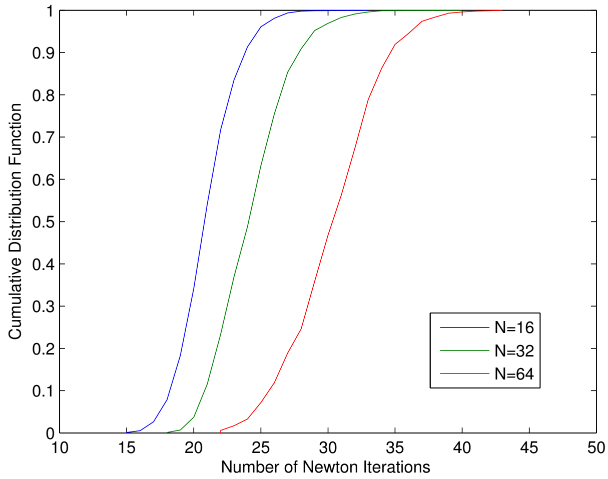

Fig. 1. CDF of the number of Newton iterations

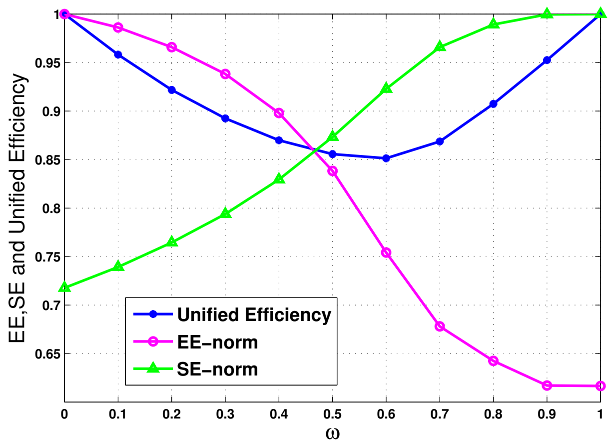

Fig. 2. SE-norm, EE-norm and Unified utility with optimal transmit powers ($N = 32$)

is achieved with $\omega = 1$. So we can make a tradeoff between the EE and the SE for different preferences based on the EE-SE tradeoff metric. A certain ω can be chosen for a practical system to work out the unique globally optimal solution.

V. Conclusion

In this paper, we studied the EE-SE tradeoff issue in an OFDM-based CR system, where our optimization objective is to jointly optimize the spectrum-and energy efficiency of the CR system. Our general problem formulation yields a multi-objective resource allocation problem. We characterized the Pareto optimal set of the problem and proposed a unified EE-SE tradeoff metric to find a unique globally optimal solution. A fast algorithm is also developed to speed up the time-consuming computation of Newton step by exploiting

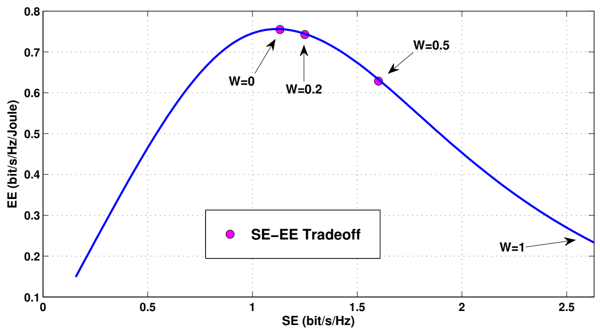

Fig. 3. EE-SE relationship ($N = 32$)

the structure of the problem. Simulation results validate the effectiveness of our proposed algorithms.

APPENDIX

A. Proof of Theorem 1

Proof: Denote $\mathbf{R_1^*}$, $\mathbf{R_2^*}$ and $\mathbf{R_3^*}$ as the the optimal rate vectors corresponding to the overall throughput R_1, R_2 and R_3, respectively, and they satisfy all constraints in (6). Without loss of generality, assume that $R_1 < R_2 < R_3$. Rewrite $\mathbf{R_2}$ as follows,

$$\begin{aligned} \mathbf{R_2} &= \tfrac{R_3-R_2}{R_3-R_1}\mathbf{R_1^*} + \tfrac{R_2-R_1}{R_3-R_1}\mathbf{R_3^*} \\ &= \gamma\mathbf{R_1^*} + \gamma\mathbf{R_3^*}, \end{aligned} \tag{28}$$

where $\gamma = \frac{R_3-R_2}{R_3-R_1}$ and $0 < \gamma < 1$. It is obvious that $\mathbf{R_2}$ is in the feasible region of (6) and its sum rate is R_2. According to [17–19], $P^*(\mathbf{R})$ and $I(\mathbf{R})$ is strictly convex in $\mathbf{R}$, where $P^*(\mathbf{R})$ is the optimal power corresponding to the optimal rate vector $\mathbf{R}$ and $I(\mathbf{R}) = \sum_{n=1}^{N} \frac{e^{r_n}-1}{H_n} I_{n,l}^{SP} - I_l^{th}$. Thus, $P^*(\mathbf{R}_2) < \gamma P^*(\mathbf{R}_1^*) + (1-\gamma)P^*(\mathbf{R}_3^*)$. Since $\mathbf{R_2}^*$ is the optimal rate vector corresponding to the overall throughput R_2, we have $P^*(\mathbf{R}_2^*) \le P^*(\mathbf{R}_2)$, and $P^*(\mathbf{R}_2^*) < \gamma P^*(\mathbf{R}_1^*) + (1-\gamma)P^*(\mathbf{R}_3^*)$. Thus, for any given $R(= B\eta_{SE})$, the minimum transmit power $P^*(R) = P^*(\mathbf{R}^*)$ is strictly convex in R (and η_{SE}).

Denote the superlevel set of $\eta_{EE}^m(\eta_{SE})$ as $\mathbf{S}_\beta = \{R|\eta_{EE}^m(\eta_{SE}) \ge \beta, \beta \le \mathcal{R}\}$. $\mathbf{S}_\beta$ is equivalent to $\{R|\beta P^*(\eta_{SE}) + \beta P_c - \eta_{SE} \le 0\}$. As a result of the convexity of $P^*(\eta_{SE})$ proved above, $\mathbf{S}_\beta$ is strictly convex in η_{SE}. Thus, $\eta_{EE}^m(\eta_{SE})$ is strictly quasiconcave and has a unique global maximum. We can see that

$$\begin{aligned} \lim_{\eta_{SE}\to\infty} \eta_{EE}^m(\eta_{SE}) &= \lim_{\eta_{SE}\to\infty} \max_{\eta_{SE}} \tfrac{\eta_{SE}}{P^*(\eta_{SE})+P_c} \\ &= \lim_{P^*(\eta_{SE})\to\infty} \tfrac{o(P^*(\eta_{SE}))}{P^*(\eta_{SE})} \\ &= 0. \end{aligned} \tag{29}$$

Thus, starting from $\eta_{SE} = 0$, $\eta_{EE}^m(\eta_{SE})$ either strictly increases with η_{SE} if $\frac{d\eta_{EE}^m(\eta_{SE})}{d\eta_{SE}}|_{\eta_{SE}=\eta_{SE}^{max}} \ge 0$ or first strictly increases and then strictly decreases with η_{SE} if $\frac{d\eta_{EE}^m(\eta_{SE})}{d\eta_{SE}}|_{\eta_{SE}=\eta_{SE}^{max}} < 0$. The maximum EE in the SE region $[0, \eta_{SE}^{max}]$ is straightforward as indicated in Theorem 1. ■

B. Proof of Theorem 2

Proof: If $\eta_{SE}^* \ge \eta_{SE}^{max}$, $\eta_{EE}^m(\eta_{SE})$ is increasing at $[0, \eta_{SE}^{max}]$ according to Theorem 1. Thus $\forall \eta_{SE} \in [0, \eta_{SE}^{max})$, we have $\eta_{EE}^m(\eta_{SE}^{max}) > \eta_{EE}^m(\eta_{SE})$ and $\eta_{SE}^{max} > \eta_{SE}$, which results in $P^{POS} = \{P|\eta_{SE} = \eta_{SE}^{max}\}$. If $\eta_{SE}^* < \eta_{SE}^{max}$, $\eta_{EE}^m(\eta_{SE})$ is increasing at $[0, \eta_{SE}^*)$ while decreasing at $[\eta_{SE}^*, \eta_{SE}^{max}]$ due to Theorem 1. $\forall \eta_{SE} \in [0, \eta_{SE}^*)$, we have $\eta_{EE}^m(\eta_{SE}^*) > \eta_{EE}^m(\eta_{SE})$ and $\eta_{SE}^* > \eta_{SE}$, which means $\{P|0 \le \eta_{SE} < \eta_{SE}^*\} \cap P^{POS} = \emptyset$. However, $\forall P \in \{P|\eta_{SE} \in [\eta_{SE}^*, \eta_{SE}^{max}]\}$, there does not exist any other point P' such that $\eta_{EE}^m(\eta_{SE}(P')) > \eta_{EE}^m(\eta_{SE}(P))$ and $\eta_{SE}(P') > \eta_{SE}(P)$. Thus, $P^{POS} = \{P|\eta_{SE}^* \le \eta_{SE} \le \eta_{SE}^{max}\}$. ■

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