Low Complexity Power Allocation for Device-to-Device Communication Underlaying Cellular Networks

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***Abstract*—Device-to-device (D2D) communication, which underlays a cellular network to improve the reuse of spectrum resources, is a promising technique that can increase system capacity, enhance cell coverage and extend battery lifetime of user equipments. In this paper, we study the power allocation for a D2D underlaying cellular network. Our optimization task is to maximize the sum rate of the cellular network subject to the maximum power budgets of both the cellular link and the D2D link. The formulated problem is nonconvex which is generally difficult to obtain the global optimum. We propose a low complexity iterative algorithm to work out promising solutions with reasonable complexity. We develop a linear complexity barrier method to get a concave lower bound of the sum rate, and prove that an improved solution can be obtained after each iteration until the proposed algorithm converges, when the lower bound is tight. Simulation results validate the effectiveness and the efficiency of our proposed algorithm.**

I. Introduction

Device-to-device (D2D) communication as an underlay to a cellular network has received significant attention due to the potential of increasing spectrum efficiency, offloading cellular traffic, and decreasing power consumption of user equipments. As the proximity of users allows for high transmission rate, low delays and low power consumption, D2D users can transmit data to each other directly under the control of base stations (BSs) in a cellular network [1, 2].

D2D links may share the spectrum with cellular links in either orthogonal or nonorthogonal way [3–5]. The former is that cellular links use a part of spectrum resources and leave the remaining resources exclusively to D2D links, where intracell interference can be completely eliminated. The latter is that D2D links share the same spectrum resources with cellular links, where the spectrum efficiency can be significantly improved. However, to prevent the harmful interference and enhance cellular capacity, intracell interference management becomes one of most important issues for such a D2D mode [6–12].

A series of resource sharing modes for the network scenario of one cellular user (CU) and one D2D pair sharing a subchannel are introduced in [6–8]. In [6], the transmission power of D2D link is adjusted by the power of cellular link to limit the intracell interference. In [7], the D2D pair can share the resource with the CU in orthogonal or nonorthogonal way. Additional, the BS can act as a relay to help the D2D pair achieve a better capacity gain. In [8], a D2D user can act as a relay node to assist the communication between the BS and the CU when they are far apart.

To achieve high capacity gain, multiuser diversity gain can be achieved by properly pairing multiple CUs and D2D pairs [9–12]. The case of one D2D pair and multiple CUs is considered in [9, 10]. In [9], the authors propose a interference management strategy, where the D2D pair and the CUs are forbidden to coexist in a predefine area to protect the D2D link. In [10], each CU occupies a subchannel and the D2D pair can reuse all subchannels. The objective is to maximize the transmission rate of the D2D pair while guaranteeing each CU's required rate. The optimal power allocation can be obtained by searching the optimal dual point via bisection.

Resource sharing scheme for a cellular network, including multiple D2D pairs and multiple CUs, is investigated in [11] and [12]. The objective is to maximize the sum rate of both D2D pairs and CUs while satisfying all users' rate requirements. Each D2D pair can only reuse one subchannel. Meanwhile, each CU shares the subchannel with at most one D2D pair. In [11], the authors propose a suboptimal subchannel allocation, which can roughly guarantee each user's required rate with using its maximum transmission power. In [12], the optimal power allocation for each D2D pair reusing an arbitrary subchannel can be searched in a set of possible solutions. By using the information, the original problem reduces to an assignment problem, which can be solved by Hungarian method and Kuhn-Munkres algorithm [13, 14].

It is noteworthy that most of previous research focus on the case of each user occupies at most one subchannel [3–12]. Power allocation for the case of each user occupies multiple subchannels is not studied extensively in the literature. In this paper, we study the intracell power allocation for the D2D uplink underlaying cellular networks, where our objective is to maximize the total transmission rate of the system. The formulated problem is nonconcave, which is generally very hard to find the global optimum. We propose a linear complexity iterative algorithm to work out an approximation solution of the original problem, where a concave lower bound of the total transmission rate is maximized at each iteration. The convergence of the proposed algorithm can be guaranteed and we give the proof. Furthermore, we also show the lower

* This work was partially supported by JiangsuSF (BK2011051).

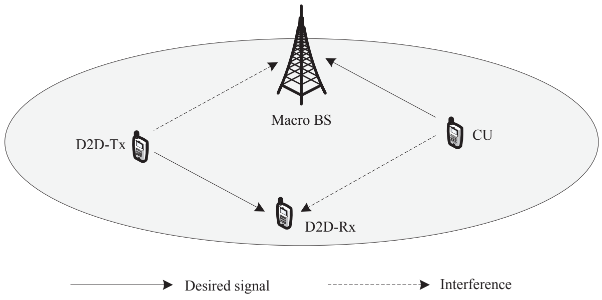

Fig. 1. Illustration of D2D communication uplink underlaying cellular network. The D2D link uses cellular uplink resources. Intracell interference happens when cellular and D2D communications are sharing the same resources.

bound of the total transmission is tight when the algorithm converges. To solve the approximation problem efficiently, we develop a fast barrier-based method with linear complexity by exploiting the structure of the problem. Simulation results show that our proposed algorithm converges quickly and stably.

The remainder of this paper is organized as follows. In Section II, we present the system model and formulate the target problem. In Section III, the power allocation algorithm is proposed in detail. In Section IV, numerical results are reported with discussions. Conclusion is drawn in Section V.

II. System Model and Problem Formulation

Consider the uplink direction of a D2D communication underlaying cellular network as illustrated in Fig. 1, where a macro BS, a CU, and two D2D users coexist. The CU is associated with the macro BS, while the D2D transmitter (D2D-Tx) sends information to the D2D receiver (D2D-Rx) directly under the control of BS. The spectrum allocated by the macro BS is divided into N subchannels which are denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$. Each subchannel is shared by both cellular link and D2D link. Let $p_{n,1}$ and $p_{n,2}$ be the transmission power of the cellular link and the D2D link on subchannel n, respectively. We collect the variables into one vector $\boldsymbol{p} \in \mathbf{R}^{2N\times 1}$, $\boldsymbol{p} = (p_{1,1}, p_{1,2}, \ldots, p_{n,1}, p_{n,2}, \ldots, p_{N,1}, p_{N,2})^T$. Denote $H_{n,1}$ and $H_{n,2}$ as the corresponding channel gain of the cellular link and the D2D link on subchannel n, respectively. Let $\widetilde{H}_{n,2}$ be the channel gain of interference link from the D2D-Tx to the macro BS and let $\widetilde{H}_{n,1}$ be the channel gain of interference link from the CU to the D2D-Rx. We assume that intercell interference is properly managed by the cellular system.

For the cellular link, the signal to interference plus noise ratio (SINR) on subchannel n can be written as

$$\gamma_{n,1}(\boldsymbol{p}) = \frac{p_{n,1}H_{n,1}}{p_{n,2}\widetilde{H}_{n,2} + \sigma_n^2},$$

where σ_n^2 is the noise power on subchannel n. For the D2D link, the SINR on subchannel n can be written as

$$\gamma_{n,2}(\boldsymbol{p}) = \frac{p_{n,2}H_{n,2}}{p_{n,1}\widetilde{H}_{n,1} + \sigma_n^2}.$$

Let $R_n(\boldsymbol{p})$ be the total achievable transmission rate on subchannel n, which can be calculated by

$$R_n(\boldsymbol{p}) = \log_2(1 + \gamma_{n,1}(\boldsymbol{p})) + \log_2(1 + \gamma_{n,2}(\boldsymbol{p})).$$

We try to maximize the sum rate of the system under power limitation, the optimization problem can be formulated as follows:

$$\begin{aligned} &\max_{\boldsymbol{p}} && \sum_{n\in\mathcal{N}} R_n(\boldsymbol{p}) \\ &s.t.\ C_1: && \sum_{n=1}^{N} p_{n,1} \le P_1^{max}, \\ &\quad\ \ C_2: && \sum_{n=1}^{N} p_{n,2} \le P_2^{max}, \\ &\quad\ \ C_3: && p_{n,1} \ge 0, p_{n,2} \ge 0, \forall n \in \mathcal{N}, \end{aligned} \tag{1}$$

where P_1^{max} and P_2^{max} are the maximum transmission power of the CU and the D2D-Tx, respectively.

III. Low Complexity Iterative Power Allocation

The objective function of Eq.(1) is nonconcave for both $p_{n,1}$ and $p_{n,2}$, which implies that there exist multiple local optima [15]. Therefore, finding the global optimum is usually very difficult and time consuming. One approach for such kind of problems is to obtain a suboptimal solution by solving a sequence of concave optimization problems [16, 17]. The optimal solutions to the concave problems can be obtained at a high computation cost. We propose a linear complexity iterative power allocation for Eq.(1) in this work.

A. A Tight Lower Bound of Sum Rate

Denote $P_{n,1}(\boldsymbol{p})$ as the received power of the macro BS on subchannel n together with $I_{n,1}(\boldsymbol{p})$ as the interference plus noise power, and they could be presented as follows,

$$P_{n,1}(\boldsymbol{p}) = H_{n,1}p_{n,1} + \widetilde{H}_{n,2}p_{n,2} + \sigma_n^2,$$

$$I_{n,1}(\boldsymbol{p}) = \widetilde{H}_{n,2}p_{n,2} + \sigma_n^2.$$

Similarly, denote $P_{n,2}(\boldsymbol{p})$ and $I_{n,2}(\boldsymbol{p})$ as the received power and the interference plus noise power of the D2D-Rx on subchannel n,

$$P_{n,2}(\boldsymbol{p}) = H_{n,2}p_{n,2} + \widetilde{H}_{n,1}p_{n,1} + \sigma_n^2,$$

$$I_{n,2}(\boldsymbol{p}) = \widetilde{H}_{n,1}p_{n,1} + \sigma_n^2.$$

Then, $R_n(\boldsymbol{p})$ can be rewritten as follows,

$$\begin{aligned} R_n(\boldsymbol{p}) \ = \ & \log_2 P_{n,1}(\boldsymbol{p}) + \log_2 P_{n,2}(\boldsymbol{p}) - \log_2 I_{n,1}(\boldsymbol{p}) \\ & - \log_2 I_{n,2}(\boldsymbol{p}). \end{aligned}$$

We define a tight concave lower bound of $R_n(\boldsymbol{p})$, which is parameterized by a given power allocation $\boldsymbol{p}^* \in \mathbf{R}^{2N\times 1}$,

TABLE I
ALGORITHM 1: OVERALL POWER ALLOCATION

1: *Initialization:* $l = 0$, $\boldsymbol{p}', \boldsymbol{p}^* \in \mathbf{R}^{2N\times 1}$, $\boldsymbol{p}' = \boldsymbol{p}^{(0)}$, $\boldsymbol{p}^* = \boldsymbol{0}$.
2: **while** $||\text{vec}(\boldsymbol{p}' - \boldsymbol{p}^*)|| \geq \epsilon$ and $l < L$
3: $l = l + 1$;
4: $\boldsymbol{p}^* = \boldsymbol{p}'$;
5: Obtain the optimal solution $\boldsymbol{p}'$ to Eq.(3) by using the proposed fast barrier method;
6: **end while**
7: **return** $\boldsymbol{p}^*$

$\boldsymbol{p}^* = (p^*_{1,1}, p^*_{1,2}, \ldots, p^*_{n,1}, p^*_{n,2}, \ldots, p^*_{N,1}, p^*_{N,2})^T$:

$$\begin{aligned} R^*_n(\boldsymbol{p},\boldsymbol{p}^*) = \;& \log_2 P_{n,1}(\boldsymbol{p}) - \log_2(\widetilde{H}_{n,2}p^*_{n,2} + \sigma^2_n) \\ & + \log_2 P_{n,2}(\boldsymbol{p}) - \log_2(\widetilde{H}_{n,1}p^*_{n,1} + \sigma^2_n) \\ & - \frac{\widetilde{H}_{n,2}(p_{n,2} - p^*_{n,2})}{\ln 2 \cdot (\widetilde{H}_{n,2}p^*_{n,2} + \sigma^2_n)} \\ & - \frac{\widetilde{H}_{n,1}(p_{n,1} - p^*_{n,1})}{\ln 2 \cdot (\widetilde{H}_{n,1}p^*_{n,1} + \sigma^2_n)}. \end{aligned} \tag{2}$$

Theorem 1. *Given any power allocation $\boldsymbol{p}^*$, it always holds that*

$$R_n(\boldsymbol{p}) \geq R^*_n(\boldsymbol{p},\boldsymbol{p}^*).$$

Proof: The proof is presented in Appendix A. ∎

According to Theorem 1, Eq.(2) is a concave function of $p_{n,1}$ and $p_{n,2}$, as well as a lower bound of $R_n(\boldsymbol{p})$. Note that $R^*_n(\boldsymbol{p}^*,\boldsymbol{p}^*) = R_n(\boldsymbol{p}^*)$, which leads to the fact that the bound is tight when $\boldsymbol{p} = \boldsymbol{p}^*$.

B. Iterative Procedure for Power Allocation

For obtaining a tight lower bound of Eq.(1), we formulate the optimization problem with a given power allocation $\boldsymbol{p}^*$,

$$\begin{aligned} \max_{\boldsymbol{p}} \quad & \sum_{n\in\mathcal{N}} R^*_n(\boldsymbol{p},\boldsymbol{p}^*) \\ s.t. \quad & C_1 \sim C_3 \text{ of Eq.(1)}. \end{aligned} \tag{3}$$

Obviously, Eq.(3) defines a convex optimization problem because the objective function is concave and the constraints are all affine. The power allocation algorithm (Algorithm 1) is described in Table I, where L is the maximum number of iterations, $\boldsymbol{p}^{(0)}$ is the initial power allocation and $\boldsymbol{0} \in \mathbf{R}^{2N\times 1}, \boldsymbol{0} = (0, 0, \ldots, 0)^T$. Generally, the initial power allocation can be arbitrary feasible solution to Eq.(1). We use the uniform power allocation for initialization.

Theorem 2. *An improved solution to Eq.(1) can be found at each iteration until Algorithm 1 converges.*

Proof: The proof is presented in Appendix B. ∎

Notice that when Algorithm 1 converges, we have $\boldsymbol{p}' = \boldsymbol{p}^*$. Therefore, a tight lower bound can be obtained by Algorithm 1.

C. Fast Barrier Method to Solve Eq.(3)

At each iteration, we need to find the optimal solution to Eq.(3). As mentioned above, Eq.(3) defines a convex optimization problem, barrier method is a general convex optimization algorithm [15] and can be employed to solve Eq.(3). In this paper, we develop a fast barrier method by exploiting the special structure of Eq.(3) as suggested in [18–20]. For simplicity, denote

$$\begin{aligned} f_1 &= P^{max}_1 - \sum_{n=1}^{N} p_{n,1}, \\ f_2 &= P^{max}_2 - \sum_{n=1}^{N} p_{n,2}. \end{aligned}$$

First, we convert all inequality constraints into a logarithmic barrier function $\phi(\boldsymbol{p})$,

$$\phi(\boldsymbol{p}) = -\ln f_1 - \ln f_2 - \sum_{n\in\mathcal{N}} \ln p_{n,1} - \sum_{n\in\mathcal{N}} \ln p_{n,2}.$$

The original optimization task can be converted into a sequence of minimization problems by introducing a logarithmic barrier function with a parameter t. The optimal solution to Eq.(3) can be approximated by solving the following minimization problem:

$$\min_{\boldsymbol{p}} \quad \psi_t(\boldsymbol{p}) = -t \sum_{n\in\mathcal{N}} R^*_n(\boldsymbol{p}.\boldsymbol{p}^*) + \phi(\boldsymbol{p}).$$

As t increases, such an approximation becomes more and more close to the optimal solution to Eq.(3) .

Second, Newton method is adopted to find the central point which is the solution of the minimization problem. For a given parameter t, Newton step $\triangle\boldsymbol{p}$ can be worked out by solving the following equation,

$$\nabla^2\psi_t(\boldsymbol{p})\triangle\boldsymbol{p} = -\nabla\psi_t(\boldsymbol{p}),$$

where $\nabla^2\psi_t(\boldsymbol{p})$ and $\nabla\psi_t(\boldsymbol{p})$ are the Hessian and the gradient of $\psi_t(\boldsymbol{p})$, respectively.

Matrix inversion is needed to compute Newton step, which generates a complexity of $O(N^3)$ for our considered problem. It is too high for such a real time problem. We propose a fast algorithm that has a complexity of only $O(N)$ by exploiting the structure of the optimization problem.

Fact 1. *Given a nonsingular matrix $A \in \mathbf{R}^{2N\times 2N}$, vectors $f, b \in \mathbf{R}^{2N\times 1}$, where f satisfies $1 + f^T A^{-1} f \neq 0$, if*

$$Ax = b, \; (A + ff^T)\tilde{x} = b,$$

it always holds

$$\tilde{x} = x - \frac{f^T x}{1 + f^T g} g,$$

where $g = A^{-1}f, g \in \mathbf{R}^{2N\times 1}$.

Based on Fact 1, we can calculate Newton step $\triangle\boldsymbol{p}$ efficiently. Note that the Hessian of $\psi_t(\boldsymbol{p})$ can be written as

$$\nabla^2\psi_t(\boldsymbol{p}) = \begin{bmatrix} D_1 & & \\ & \ddots & \\ & & D_N \end{bmatrix} + \frac{\nabla f_1 \nabla f_1^T}{f_1^2} + \frac{\nabla f_2 \nabla f_2^T}{f_2^2},$$

TABLE II
THE BARRIER METHOD

Initialization: $\boldsymbol{p} = \boldsymbol{p}^{(0)}$, $\epsilon_b > 0, \epsilon_n > 0$, $\alpha \in (0, 1/2), \beta \in (0, 1)$, $t = t^{(0)}, \mu > 1$.
while $(2N + 2)/t \geq \epsilon_b$
    while true
        Compute $\triangle \boldsymbol{p}$ by using Eq.(4); $s = 1$; $\lambda^2 = \nabla\psi_t(\boldsymbol{p})^T \triangle \boldsymbol{p}$;
        if $\lambda^2/2 < \epsilon_n$
            break;
        end if
        while $\psi_t(\boldsymbol{p} + s\triangle \boldsymbol{p}) > \psi_t(\boldsymbol{p}) - \alpha s \lambda^2$
            $s = \beta s$;
        end while
        $\boldsymbol{p} = \boldsymbol{p} + s\triangle \boldsymbol{p}$;
    end while
    $t = \mu t$;
end while
return $\boldsymbol{p}$

where $D_n \in \mathbf{R}^{2\times 2}$ and

$$D_n = \begin{bmatrix} D_{n,1} & D_{n,2} \\ D_{n,3} & D_{n,4} \end{bmatrix},$$

$$D_{n,1} = \frac{t}{\ln 2} \cdot \left(\frac{H_{n,1}^2}{P_{n,1}(\boldsymbol{p})^2} + \frac{\widetilde{H}_{n,1}^2}{P_{n,2}(\boldsymbol{p})^2} \right) + \frac{1}{p_{n,1}^2},$$

$$D_{n,2} = D_{n,3} = \frac{t}{\ln 2} \cdot \left(\frac{H_{n,1}\widetilde{H}_{n,2}}{P_{n,1}(\boldsymbol{p})^2} + \frac{H_{n,2}\widetilde{H}_{n,1}}{P_{n,2}(\boldsymbol{p})^2} \right),$$

$$D_{n,4} = \frac{t}{\ln 2} \cdot \left(\frac{\widetilde{H}_{n,2}^2}{P_{n,1}(\boldsymbol{p})^2} + \frac{H_{n,2}^2}{P_{n,2}(\boldsymbol{p})^2} \right) + \frac{1}{p_{n,2}^2}.$$

We first solve the following equations: $Dx_0 = -\nabla\psi_t(\boldsymbol{p})$, $Dx_1 = \frac{\nabla f_1}{f_1}$, and $Dx_2 = \frac{\nabla f_2}{f_2}$. Since D is a diagonal matrix, we can easily obtain x_0, x_1 and x_2 by matrix inversion with a complexity of $O(N)$. Since x_0^* and x_2^* satisfy the following equations:

$$(D + \frac{\nabla f_1 \nabla f_1^T}{f_1^2})x_0^* = -\nabla\psi_t(\boldsymbol{p}),$$

$$(D + \frac{\nabla f_1 \nabla f_1^T}{f_1^2})x_2^* = \frac{\nabla f_2}{f_2},$$

we can work them out with a complexity of $O(N)$:

$$x_0^* = x_0 - \frac{\nabla f_1^T x_0}{f_1 + \nabla f_1^T x_1} x_1,$$

$$x_2^* = x_2 - \frac{\nabla f_1^T x_2}{f_1 + \nabla f_1^T x_1} x_1.$$

Finally, Newton step $\triangle\boldsymbol{p}$ can be worked out as follows,

$$\triangle\boldsymbol{p} = x_0^* - \frac{\nabla f_2^T x_0^*}{f_2 + \nabla f_2^T x_2^*} x_2^*. \quad (4)$$

The outline of the barrier method is summarized in Table II, where ϵ_b and ϵ_n are the tolerances of the barrier method and the Newton method, respectively. α and β are two constants utilized in backtracking line search with $\alpha \in (0, 0.5)$ and $\beta \in (0, 1)$. The step size of the backtracking line search is s with $s > 0$. t and μ are parameters associated with a tradeoff between outer iterations and inner iterations.

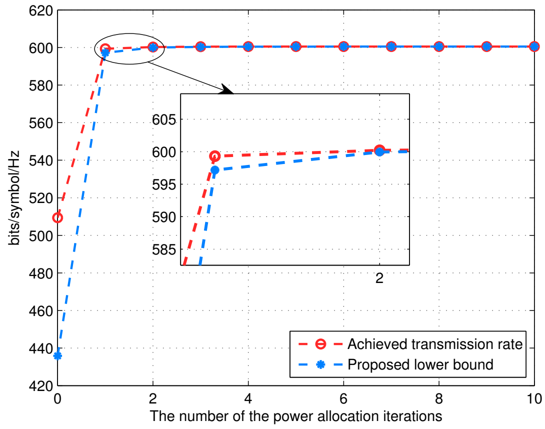

Fig. 2. The gap between our proposed lower bound and the total transmission rate over 100 channel realizations. The number of subchannels is 64 and the parameter ϵ is set to 10^{-3}.

IV. SIMULATION RESULTS

Experiments are performed to evaluate the performance of the proposed algorithm. Consider a D2D communication underlaying cellular network, where the CU and the D2D-Tx are uniformly distributed in a circle within 300m from the macro BS. The D2D-Rx is uniformly distributed in a circle within 100m from the D2D-Tx. The path loss model (in dB) is $15.3 + 37.6 \log_{10} D$ for distance D (in m). The variance of shadowing is 10dB and the amplitude of multipath fading is Rayleigh. The noise power on each subchannel is -90dBm. The maximal transmission power of the CU is 20dBm, as well as that of the D2D-Tx.

First, we investigate the convergence of our proposed power allocation algorithm over 100 random instances, which is illustrated by Fig. 2 and Fig. 3. The parameter ϵ is set to 10^{-3}. Fig. 2 shows the gap between our proposed lower bound (the objective of Eq.(3)) and the achieved transmission rate (the objective of Eq.(1)) for the case of $N = 64$. If the lower bound is less that zero, we set it zero. From Fig. 2 we can observe that the gap decreases as the increase of iterations. After about two iterations, the lower bound is very close to the total transmission rate. The number of iterations for the Algorithm 1 to converge (when L is set to a large number) is shown in Fig.3, where the average number of iterations is marked with dashed line. Notice that the performance of the convergence is very close for the different number of subchannels. Again, we can observe that the number of iterations varies in a narrow range. 80% of iterations are less than 5 and 97% of iterations are less than 10 for the case of $N = 256$.

The performance of the convergence of our proposed barrier methodis shown in Fig.4, which is averaged over 1000 random instances. The computational load of the barrier method mainly lies in the computation of Newton step. If the number

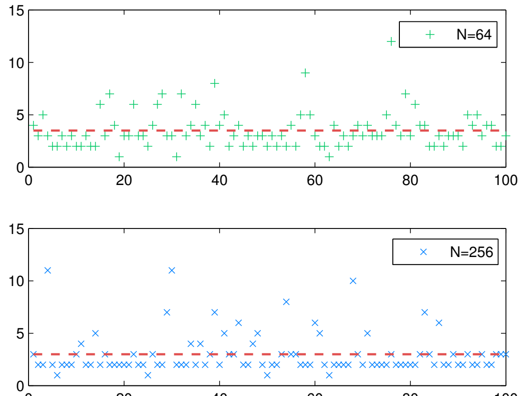

Fig. 3. Number of iterations required for convergence of Algorithm 1 over 100 channel realizations. The parameter ϵ is set to 10^{-3}.

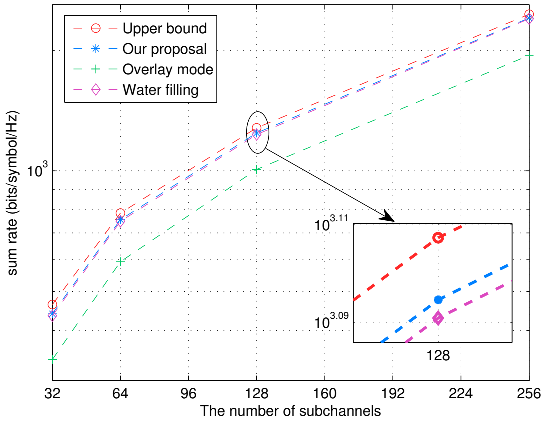

Fig. 5. The achievable sum rate versus the number of subchannels in the scenario that mutual interference is slight.

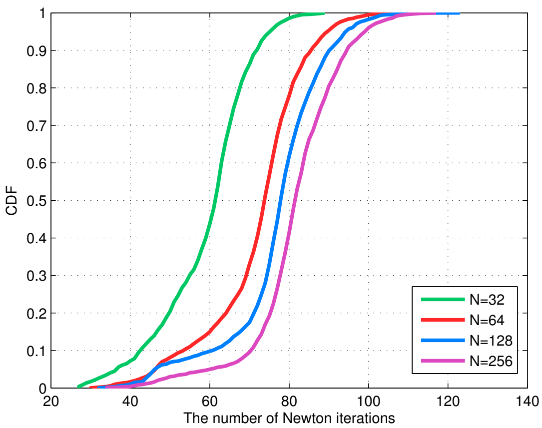

Fig. 4. CDF of the number of Newton iterations required for convergence over 1000 channel realizations. The parameters of the barrier method are set as follows, $t^{(0)} = 0.1, \epsilon_b = \epsilon_n = 10^{-3}, \mu = 10, \alpha = 0.01, \beta = 0.1$.

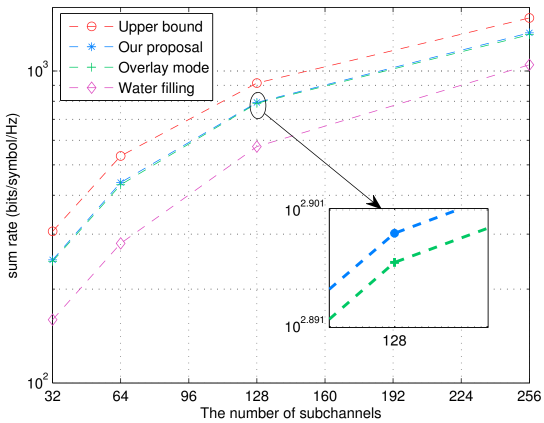

Fig. 6. The achievable sum rate versus the number of subchannels in the scenario that mutual interference is heavy.

of Newton iterations is large or varies in a wide range, the proposed algorithm would be difficult to be applied. Fig.4 gives the cumulative distribution function (CDF) of the number of Newton iterations. Parameters of the barrier method are set as follows, $t^{(0)} = 0.1, \epsilon_b = \epsilon_n = 10^{-3}, \mu = 10, \alpha = 0.01, \beta = 0.1$. The number of subchannels varies from 32 to 256. Fig.4 shows that the number of Newton iterations also varies in a narrow range. So we can conservatively conclude that our proposed method is effective and efficient.

Finally, we investigate the achievable transmission rate as a function of the number of subchannels, which is illustrated in Fig.5 and Fig.6. We compare our proposed scheme with the following three schemes,

Water filling: The power allocation is obtained by water filling algorithm without consideration of mutual interference between D2D link and cellular link.

Upper bound: The power allocation is same as water filling scheme. Interference is neglected in the computation of SINR.

Overlay mode: Each subchannel is allocated to one and only one link. The optimal subchannel allocation is obtained by Karush-Kuhn-Tucker (KKT) conditions and the power allocation is obtained by water filling algorithm.

The number of subchannels varies from 32 to 256. Two scenarios are considered: Slight interference case which is shown in Fig.5; Heavy interference case which is shown in Fig.5. From Fig.5 we can observe that the performance of our algorithm is close to the upper bound. In such a scenario, water filling is also close to the upper bound since the effect of mutual interference is negligible. However, the gap between overlay mode and the upper bound is large that 20%. From Fig.6 we can see that the performance of our algorithm is close to the overlay mode scheme and outperform the water filling. The gap between our algorithm and the upper bound is less then 10% when $N \geq 64$. It is noteworthy that the upper bound is not tight in this scenario, which indicates that the performance of our algorithm is still very close to the optimal solution. Thus, our proposal can be widely applicable for different network scenarios.

V. Conclusion

In this paper, we studied the power allocation for a D2D communication uplink underlaying cellular network. The optimization target is to maximize the sum rate of the system with the constraints of the maximum power budgets. Since the formulated problem is a general nonconcave one, we proposed a iterative algorithm to work out a suboptimal solution with low complexity. Furthermore, we proved that the convergence of the proposed algorithm can be guaranteed. Numerical simulations validated the effectiveness and efficiency of our proposal.

Appendix A
Proof of Theorem 1

Define

$$F(p_{n,2}) = \log_2 I_{n,1}(\boldsymbol{p}) - \log_2(\widetilde{H}_{n,2}p^*_{n,2} + \sigma^2_n) - \frac{\widetilde{H}_{n,2}(p_{n,2} - p^*_{n,2})}{\ln 2 \cdot (\widetilde{H}_{n,2}p^*_{n,2} + \sigma^2_n)}.$$

Note that $F(p^*_{n,2}) = 0$ and

$$F'(p_{n,2}) = \frac{\widetilde{H}_{n,2}}{\ln 2 \cdot (\widetilde{H}_{n,2}p_{n,2} + \sigma^2_n)} - \frac{\widetilde{H}_{n,2}}{\ln 2 \cdot (\widetilde{H}_{n,2}p^*_{n,2} + \sigma^2_n)}.$$

We can obtain

$$F'(p_{n,2}) \begin{cases} > 0, & \text{if } p_{n,2} < p^*_{n,2}, \\ = 0, & \text{if } p_{n,2} = p^*_{n,2}, \\ < 0, & \text{if } p_{n,2} > p^*_{n,2}. \end{cases}$$

Therefore, $p^*_{n,2}$ is the maximal point of $F(p_{n,2})$ and it always holds $F(p_{n,2}) \leq F(p^*_{n,2}) = 0$. Then, we have

$$\log_2 I_{n,1}(\boldsymbol{p}) \leq \log_2(\widetilde{H}_{n,2}p^*_{n,2} + \sigma^2_n) + \frac{\widetilde{H}_{n,2}(p_{n,2} - p^*_{n,2})}{\ln 2 \cdot \widetilde{H}_{n,2}p^*_{n,2} + \sigma^2_n}.$$

Similarly, we can also obtain

$$\log_2 I_{n,2}(\boldsymbol{p}) \leq \log_2(\widetilde{H}_{n,1}p^*_{n,1} + \sigma^2_n) + \frac{\widetilde{H}_{n,1}(p_{n,1} - p^*_{n,1})}{\ln 2 \cdot \widetilde{H}_{n,1}p^*_{n,1} + \sigma^2_n}.$$

Thus, $R_n(\boldsymbol{p}) \geq R^*_n(\boldsymbol{p}, \boldsymbol{p}^*)$.

Appendix B
Proof of Theorem 2

At each iteration, we obtain the $\boldsymbol{p}'$ which is the optimal solution to Eq.(3) with given $\boldsymbol{p}^*$. According to Theorem 1, it always holds that

$$\begin{aligned} \sum_{n\in\mathcal{N}} R_n(\boldsymbol{p}') \geq & \sum_{n\in\mathcal{N}} R^*_n(\boldsymbol{p}', \boldsymbol{p}^*) \\ \geq & \sum_{n\in\mathcal{N}} R^*_n(\boldsymbol{p}^*, \boldsymbol{p}^*) \\ = & \sum_{n\in\mathcal{N}} R_n(\boldsymbol{p}^*). \end{aligned}$$

Moreover, $\sum_{n\in\mathcal{N}} R_n(\boldsymbol{p}')$ is bounded by the optimal value of Eq.(1), which leads to the fact that the convergence of Algorithm 1 is guaranteed.

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