On the Resource Allocation for Multi-relay Cognitive Radio Systems

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***Abstract*—In this paper, we investigate the resource allocation problem in an OFDM-based cognitive radio system with multiple relays. We address the formulated intractable problem by a two-stage procedure which corresponds to two subproblems: subchannel allocation and power distribution. For each subproblem, a fast algorithm is proposed by exploiting its special structure. Our proposed algorithms can achieve performance close to the upper bound. The remarkable characteristic of the proposed algorithm is that the approximate linear complexity is significantly lower than that of standard techniques, making it promising for applications. Simulation results also validate the superior performance of the proposed algorithm.**

I. Introduction

Cognitive Radio (CR) [1] is deemed as a potential technique to alleviate the looming spectrum scarcity. In CR systems, CR users (also referred to as Secondary Users, SUs) sense the spectrum registered by the licensed Primary Users (PUs) and opportunistically access the idle part of the spectrum. Since the access of the SUs may degrade the performance of the PUs operating in the adjacent bands even with perfect spectrum sensing, the interference introduced to the PUs should be carefully considered and always kept within a tolerable range defined by the PUs [2].

Nevertheless, reliable transmission between a certain pair of CR nodes may require high transmission power, leading to heavy interference to the PUs. Consequently, conventional end-to-end transmission might not be always available in CR systems [3]. Instead, cooperative relay has emerged as a key spatial diversity technique to overcome channel fading and boost the overall performance of wireless systems. For a CR network, cooperative relay can not only weaken the effect of multipath fading, but also reduce the required transmission power by generating alternative paths between source and destination via relays [4]. So the interference to the PUs can be mitigated to an acceptable degree in this way.

Recently, adaptive resource allocation (RA) has drawn significant attention for the arising relaying CR networks. Generally, relay selection and power allocation are significantly important to enhance the throughput of the system. In [5], the authors study the power allocation problem in CR networks with cooperative relay. A simplified relay and power allocation algorithm is proposed under the interference constraints in [6]. For multi-relay aided CR systems with single pair of transceiver nodes, relay selection and power allocation are performed separately in [7], while [8] proposes a joint relay and optimal power allocation scheme. For both spectrum sensing and spectrum sharing cases, cooperative communications for CR networks are discussed intensively in [9]. In [10], the authors present a joint subcarrier and power allocation scheme with the iterative partitioned water-filling algorithm, by simplifying the three-node relay network into a two-node network.

In this paper, we consider a signal antenna OFDM-based CR system, where the channel between source and destination suffers deep fade and the direct link is not suitable for transmission. Multiple relays, operating with Decode-and-Forward (DF) protocol, are constructed to assist the end-to-end transmission. The CR system adopts OFDM modulation. We do not assume the licensed system also adopts OFDM. We aim to maximize the sum capacity of the relaying CR system under the constraints of total transmission power budget and the interference limitation of the PUs. We analyze the formulated optimization problem intensively and develop a fast algorithm to work out the optimal solution with approximated linear computational complexity, which is much lower than standard convex optimization techniques as proposed in [11].

II. System Model and Problem Formulation

A. System Model

Consider a single antenna OFDM-based CR network enhanced by multiple relays, coexisting with a primary system, where the Destination Node (DN) receives the signals without diversity and the signal from the direct path is not taken into account [12]. There are K DF relays, denoted by $\mathcal{K} = \{1, 2, \ldots, K\}$, assisting the transmission between the Source Node (SN) and the DN. Each transmission frame from the SN and the DN consists of two time slots with the same duration. In the first time slot, the SN transmits while the relays listen. Then the relays are half-duplex, they receive data in the first time slot and transfer them in the second time slot.

The available spectrum is equally divided into N OFDM subchannels with bandwidth B, denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$. The nominal bandwidth of subchannel n spans from $f_s + (n-1)B$ to $f_s + nB$, where f_s is the starting frequency. There are L PUs and the bandwidth of PU l ranges from f_l to $f_l + B_l$, where f_l and B_l are PU l's starting frequency and bandwidth, respectively. The interference to PU l band by the CR SN in the first time slot with unit

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transmission power on subchannel n is given by

$$I_{n,l}^{SP} = \int_{f_l - f_s - (n-1/2B)}^{f_l + B_l - f_s - (n-1/2B)} |g_{n,l}^{SP}|^2 \phi(f) df, \quad (1)$$

where $g_{n,l}^{SP}$ is the channel gain from the CR SN to the lth PU's receiver on the nth subchannel. $\phi(f)$ is the baseband power spectral density (PSD) of OFDM signals with $\phi(f) = T(\frac{\sin \pi f T}{\pi f T})$, where T is the OFDM symbol duration. Similarly, the interference to PU l by the kth relay in the second time slot with unit power on subchannel n is

$$I_{k,n,l}^{RP} = \int_{f_l - f_s - (n-1/2B)}^{f_l + B_l - f_s - (n-1/2B)} |g_{k,n,l}^{RP}|^2 \phi(f) df, \quad (2)$$

where $g_{k,n,l}^{RP}$ is the channel gain from the kth CR relay to PU l's receiver on subchannel n.

Denote $h_{k,n}^{SR}$ and $h_{k,n}^{RD}$ as the channel gain from the CR SN to the kth relay and the kth relay to the CR DN. In the first slot, the SN transmits signals to the kth relay on the nth subchannel with power $p_{k,n}^S$, while the kth relay transmits signals to the DN on the nth subchannel with power $p_{k,n}^R$ in the second slot. Thus, the capacity of each transmission slot is given by

$$\begin{aligned} C_{k,n}^{SR} &= \log\left(1 + \frac{p_{k,n}^S |h_{k,n}^{SR}|^2}{\sigma_k^2 + \sum_{l=1}^L J_{k,n}^l}\right), \\ C_{k,n}^{RD} &= \log\left(1 + \frac{p_{k,n}^R |h_{k,n}^{RD}|}{\sigma^2 + \sum_{l=1}^L J_n^l}\right), \end{aligned} \quad (3)$$

where σ_k^2 and σ^2 are the variance of additive white Gaussian noise at the kth relay and DN. The interference cast by the lth PU's signal to the kth relay and the DN on the nth subchannel are denoted by $J_{k,n}^l$ and J_n^l, which can be regarded as noise and measured by the receivers of relays and DN. For notion brevity, let $H_{1,k,n} = \frac{|h_{k,n}^{SR}|^2}{\sigma_k^2 + \sum_{l=1}^L J_{k,n}^l}$ and $H_{2,k,n} = \frac{|h_{k,n}^{RD}|^2}{\sigma^2 + \sum_{l=1}^L J_n^l}$ denote the SNR at the kth relay and the DN on the nth subchannel, respectively. Hence, the transmission rate from the SN to the DN aided by the kth relay on the nth subchannel is limited by the minimum of the two hops [12],

$$C_{k,n} = \min\{C_{k,n}^{SR}, C_{k,n}^{RD}\}. \quad (4)$$

B. Problem Formulation

We try to maximize the sum capacity of the CR system and the optimization problem can be expressed as follows,

$$\begin{aligned} \max_{\rho_{k,n}, p_{k,n}} \quad & \sum_{n=1}^N \sum_{k=1}^K \rho_{k,n} \min\{C_{k,n}^{SR}, C_{k,n}^{RD}\} \\ s.t. \quad & C1: \sum_{l=1}^L \sum_{k=1}^K \sum_{n=1}^N \rho_{k,n} I_{n,l}^{SP} p_{k,n}^S \le I_{th} \\ & C2: \sum_{l=1}^L \sum_{k=1}^K \sum_{n=1}^N \rho_{k,n} I_{k,n,l}^{RP} p_{k,n}^R \le I_{th} \\ & C3: \sum_{k=1}^K \sum_{n=1}^N \rho_{k,n} (p_{k,n}^S + p_{k,n}^R) \le P_t \\ & C4: p_{k,n}^S \ge 0, \ p_{k,n}^R \ge 0, \forall k, n \\ & C5: \sum_{k=1}^K \rho_{k,n} = 1, \forall n \\ & C6: \rho_{k,n} \in \{0, 1\}, \forall k, n, \end{aligned} \quad (5)$$

where P_t is the total transmission power limit for SN and relays. C1 and C2 indicate that the interference to the PU band can not exceed I_{th} in the each transmission slot. C5 and C6 declare that each subchannel cannot be shared by multiple relays.

It has been shown the minimum of the capacities in the objective in (5) is maximized if the SNR at relay and DN are equal for the total power constraint [13]. Although there are additional interference constraints in (5), we can also adopt the same approach proposed in [11] to give an approximation of (5) as $H_{1,k,n} p_{k,n}^S = H_{2,k,n} p_{k,n}^R$.

Hence, Eq.(5) can be simplified as follows: let $p_{k,n} = p_{k,n}^S + p_{k,n}^R$, we have $p_{k,n}^S = \frac{H_{2,k,n} p_{k,n}}{H_{1,k,n} + H_{2,k,n}}$ and $p_{k,n}^S = \frac{H_{1,k,n} p_{k,n}}{H_{1,k,n} + H_{2,k,n}}$. Instead of optimizing $\rho_{k,n}$, $p_{k,n}^S$ and $p_{k,n}^R$, only $\rho_{k,n}$ and $p_{k,n}$ are involved in the formulated problem. The optimization problem (5) can be reformulated as

$$\begin{aligned} \max_{\rho_{k,n}, p_{k,n}} \quad & \sum_{n=1}^N \sum_{k=1}^K \rho_{k,n} \log(1 + H_{k,n} p_{k,n}) \\ s.t. \quad & C1: \sum_{l=1}^L \sum_{n=1}^N \sum_{k=1}^K \rho_{k,n} A_{k,n,l} p_{k,n} \le I_{th} \\ & C2: \sum_{l=1}^L \sum_{n=1}^N \sum_{k=1}^K \rho_{k,n} B_{k,n,l} p_{k,n} \le I_{th} \\ & C3: \sum_{k=1}^K \sum_{n=1}^N \rho_{k,n} p_{k,n} \le P_t \\ & C4: p_{k,n} \ge 0, \forall k, n \\ & C5: \sum_{k=1}^K \rho_{k,n} = 1, \forall n \\ & C6: \rho_{k,n} \in \{0, 1\}, \forall k, n, \end{aligned} \quad (6)$$

where $p_{k,n} = p_{k,n}^S + p_{k,n}^R$, $H_{k,n} = \frac{H_{1,k,n} H_{2,k,n}}{H_{1,k,n} + H_{2,k,n}}$, $A_{k,n,l} = I_{n,l}^{SP} H_{k,n} / H_{1,k,n}$ and $B_{k,n,l} = I_{k,n,l}^{RP} H_{k,n} / H_{2,k,n}$.

III. The Proposed Algorithms

Eq.(6) defines a mixed integer programming problem which is NP-hard and generally intractable. In this paper, we develop a two-stage approach: subchannels allocation and power distribution among subchannels. By exploiting the structures of the formulated problems, our proposed algorithms can work out solutions close to the upper bound with reasonable complexity.

A. Subchannel Allocation

By assuming equal power allocation among subchannels (i.e. $p_{k,n} = P_t/N$) as proposed in [11], we can design a subchannel assignment scheme with time-sharing strategy, which moves the integer constraints by relaxing the integer variables [11]. Redefine $\rho \in [0, 1]$ as the faction of the nth subchannel allocated to the kth relay, the relaxation form of subchannel allocation is as follows,

$$\begin{aligned}
\max_{\rho_{k,n}} \quad & \sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n}\log(1+H_{k,n}P_t/N) \\
s.t. \quad & C1: \sum_{l=1}^{L}\sum_{n=1}^{N}\sum_{k=1}^{K} P_t A_{k,n,l}\rho_{k,n} \le NI_{th} \\
& C2: \sum_{l=1}^{L}\sum_{n=1}^{N}\sum_{k=1}^{K} P_t B_{k,n,l}\rho_{k,n} \le NI_{th} \\
& C3: \sum_{k=1}^{K}\rho_{k,n} = 1, \forall n \\
& C4: \rho_{k,n} \ge 0, \forall k, n.
\end{aligned} \tag{7}$$

Note that (7) is a linear program (LP) problem. LP is a convex optimization problem which can be solved by interior point method, such as the barrier method. The complexity to solve (7) is generally $O(K^3N^3)$ [11, 14], which is still too high and unacceptable for practical wireless systems because there are usually thousands of subchannels.

Motivated by the method proposed in [15–17], we exploit the structure of the (6) and develop a fast barrier method to work out the optimal solution. The barrier method moves all inequality constraints into a logarithmic barrier function and converts the original problem into a set of minimization problems with a given parameter t. Particularly, each minimization problem associated to the parameter t can be solved by Newton method, and the solution of each minimization problem is called a central point in the central path to the optimal solution of the original problem. As t increases, the central point will be more and more accurate.

First, we reformulate the (7) into a set of minimization problems, making all inequality constraints implicit in an objective function. The logarithmic barrier function is following [14],

$$\begin{aligned}
\phi(x) = &-\log(P_{th} - \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N} A_{k,n,l}\rho_{k,n}) \\
&-\log(P_{th} - \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N} B_{k,n,l}\rho_{k,n}) - \sum_{k=1}^{K}\sum_{n=1}^{N}\log(\rho_{k,n}),
\end{aligned} \tag{8}$$

where $P_{th} = NI_{th}/P_t$ and $x = (\rho_{1,1}, \rho_{1,2}, \ldots, \rho_{K,N})^T$. Denote $w_{k,n} = \log(1 + P_tH_{k,n}/N)$ and $f(x) = \sum_{k=1}^{K}\sum_{n=1}^{N} w_{k,n}\rho_{k,n}$, the optimal solution of the (7) can be approximated by solving the following minimization problem,

$$\begin{aligned}
\min \quad & \psi_t(x) = -tf(x) + \phi_(x) \\
s.t. \quad & Ax = 1.
\end{aligned} \tag{9}$$

where $A \in \Re^{N\times KN}$ with $A_{n,(k-1)N+n} = 1, \forall k, n$ and the other elements are all 0. In each centering step of the barrier method, we adopt Newton method to compute the central point for a given parameter t. The Newton step at x, denoted by Δ_x, and the associated dual variables ν are given by

$$\begin{bmatrix} \nabla^2\psi_t(x) & A^T \\ A & 0_n \end{bmatrix} \begin{bmatrix} \Delta x \\ \nu \end{bmatrix} = \begin{bmatrix} -\nabla\psi_t(x) \\ 0_v \end{bmatrix}. \tag{10}$$

where $0_n \in \Re^{N\times N}$ and $0_v \in \Re^{N\times 1}$. $\nabla^2\psi_t(x)$ and $\nabla\psi_t(x)$ are the Hessian and the gradient of $\psi_t(x)$ respectively,

$$\begin{aligned}
\nabla^2\psi_t(x) &= D + \frac{\nabla f_1 \nabla f_1^T}{f_1^2} + \frac{\nabla f_2 \nabla f_2^T}{f_2^2} \\
\nabla\psi_t(x) &= -tw_{k,n} - \frac{\nabla f_1}{f_1} - \frac{\nabla f_2}{f_2} - \frac{1}{\rho_{k,n}}
\end{aligned} \tag{11}$$

where $D = \text{diag}(1/\rho_{1,1}^2, 1/\rho_{1,2}^2, \ldots, 1/\rho_{K,N}^2)$, $f_1 = P_{th} - \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N} A_{k,n,l}\rho_{k,n}$ and $f_2 = P_{th} - \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N} B_{k,n,l}\rho_{k,n}$.

Since the Hessian is positive definite and A is a full row rank matrix, the KKT matrix on the left of the (10) is invertible. Rather than computing Δx in the (10) by matrix inversion with complexity $O(K^3N^3)$, which is the main computation burden of the barrier method, we exploit its structure and develop a fast algorithm to calculate the Newton step with low complexity. Based on the (11), we can rewrite the KKT matrix in the (10) as

$$H = \begin{bmatrix} D & A^T \\ A & 0_n \end{bmatrix} + \sum_{i=1}^{2} G_iG_i^T, \tag{12}$$

where $G_i = \begin{bmatrix} \nabla f_i/f_i \\ 0_v \end{bmatrix}, i = 1, 2$. Then matrix H is the sum of a sparse matrix and two rank-one matrices. Let $G_0 = \begin{bmatrix} -\nabla\phi_t(x) \\ 0_v \end{bmatrix}$. We can derive an efficient method the calculate the Newton step as follows,

Step 1 $H = H_1 + G_1G_1^T$,
where $H_1 = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix} + G_2G_2^T$
Particularly we have $\tilde{x} = v_1^1 - \frac{G_1v_1^1}{1+G_1v_2^1}v_2^1$,
Where $H_1v_1^1 = G_0$ and $H_1v_2^1 = G_1$.

Step 2 $H_1 = H_2 + G_2G_2^T$,
where $H_2 = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix}$
Similarly, $v_i^1 = v_i^2 - \frac{G_2v_i^2}{1+G_2v_3^2}v_3^2, i = 1, 2,$
And $H_2v_i^2 = G_{i-1}, i = 1, 2, 3.$

To calculate the Newton step, we just need to solve the three equations listed in Step 2. Now we consider the equations in a unified form as follows,

$$\begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} G \\ \mathbf{0}_v, \end{bmatrix}. \tag{13}$$

where $u \in \mathcal{R}^{KN}$ and $v \in \mathcal{R}^N$. And we have

$$\begin{aligned}
& D_{k,n}u_{(k-1)N+n} + v_n = G_{(k-1)N+n}, \forall k, n \\
& \sum_{k=1}^{K} u_{(k-1)N+n} = 0, \forall n
\end{aligned} \tag{14}$$

We can first obtain all v_n's according to (14),

$$v_n = (\sum_{k=1}^{K} D_{k,n}^{-1}G_{(k-1)N+n})/(\sum_{k=1}^{K} D_{k,n}^{-1}). \tag{15}$$

Substituting the (15) back in the (14), then u can be worked out. After solving the LP problem (7), the nth subchannel is assigned to the kth relay with the highest $\rho_{k,n}$.

B. Power Distribution

Given a subchannel assignment, the binary variables $\rho_{k,n}$'s in the (7) are fixed to 0 or 1, the integer constraints vanish and power distribution across subchannels follows. Let Ω_k be the set of subchannels allocated to the kth relay, the power distribution problem can be given by

$$\begin{aligned} \max_{\rho_{k,n}} \quad & \sum_{k=1}^{K} \sum_{n\in\Omega_k} \log(1+H_n p_n) \\ s.t. \quad & C1: \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n\in\Omega_k} A_{n,l}p_n \le I_{th} \\ & C2: \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n\in\Omega_k} B_{n,l}p_n \le I_{th} \\ & C3: \sum_{k=1}^{K} p_{k,n} \le P_t \\ & C4: p_n \ge 0, \forall k,n. \end{aligned} \tag{16}$$

Obviously, the (16) defines a convex optimization problem because the objective function and constraints are all convex. It can also be solved by the barrier method proposed in Section III-A. Furthermore, we find the proposed fast barrier method to solve the (7) is also applicable for the (16) because of the same structure of the two optimization problems. The derivation is not exhibited in detail because of space limitation.

C. On the Computational Complexity

As mentioned above, the main computational cost of the fast barrier method lies in computing the Newton step. It consumes two step of decomposition for speedup computation the Newton step, while each decomposition yields an additional equation. For executing the reverse substitution, we need to first solve three matrix systems in the last step of decomposition. During the subchannel allocation procedure, the computational complexity for solving each system is measured by $O(KN)$. Hence, the total computational cost for working out the optimal solution is $O(KN)$, which is much lower than $O(K^3N^3)$ if using matrix inversion directly. Similarly, we can analyze the computational cost for the optimal power allocation is $O(N)$.

IV. Simulation Results

Consider an OFDM based CR system with single antennae co-located with a licensed systems. There are 2 PUs in the licensed system with bandwidth 1MHz and 2MHz, respectively. The values of σ^2, σ_k^2 are set to 10^{-3}W, while the interferences $J_{k,n}^l$, J_n^l are set to 10^{-6}W. The channel gains $g_{n,l}^{SP}$, $g_{k,n,l}^{RP}$, $h_{k,n}^{SR}$, $h_{k,n}^{RD}$ follow Reyleigh fading with a mean power gain of -10dB. The interference threshold is 10^{-6}W. All simulation results are obtained by 10000 Monte Carlo simulations.

To evaluate the sum capacity, we compare our proposed algorithm with the standard interior point method used in [11] (Standard) and the Upper Bound. The upper bound is obtained by relaxing the binary variables in Eq.(5), which can be solved by commercial software[1]. Fig.1 depicts the average number

[1]Note that the upper bound cannot be a feasible solution because the relaxed form of the original problem ignores the integer constraints.

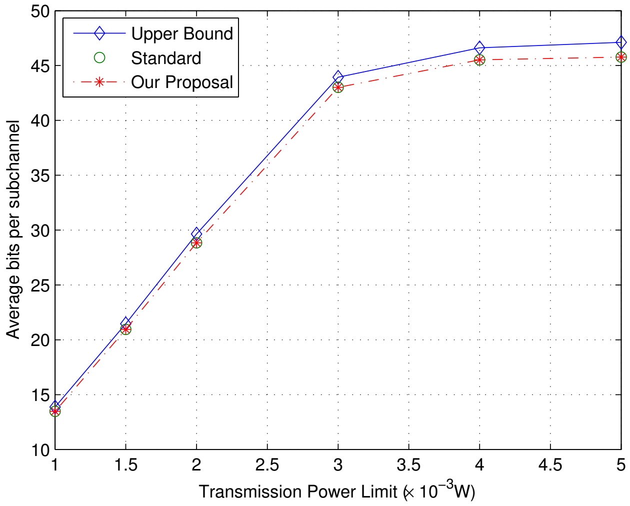

Fig. 1. Average capacity per subchannel as a function of the transmission power limit with $K = 3$

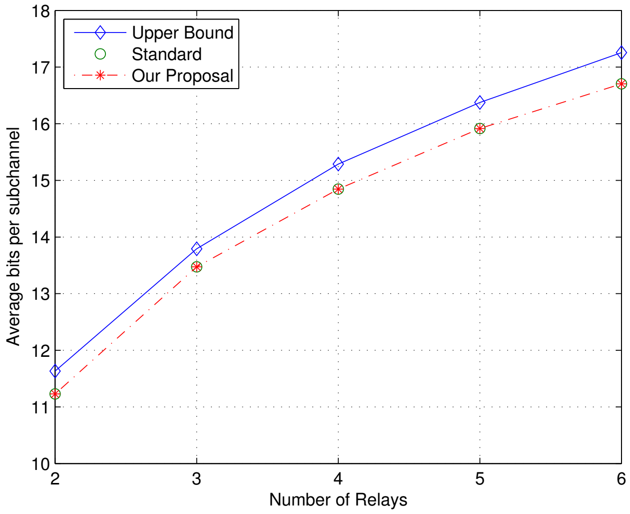

Fig. 2. Average capacity per subchannel as a function of the number of relays with $Pt = 10^{-3}W$

of bits per subchannel versus the transmission power limit, where the number of relays are fixed to 3. Fig.1 shows that the sum capacity grows with the increase of power budget, until the interference threshold is sufficiently saturated. It can be observed that our proposed algorithm and the Standard can always work out the same solution, which achieves more than 95% of the upper bound.

We verify the effect of spatial diversity for the CR relaying network. Fig.2 shows the sum capacity as a function of the number of relays, where the transmission power limit is set to $10^{-3}W$. As the number of relays becomes larger, the sum capacity of the CR system increases. We can explain this phenomenon as a result of spatial diversity. When there are more relays, there are more chances for a subchannel to be

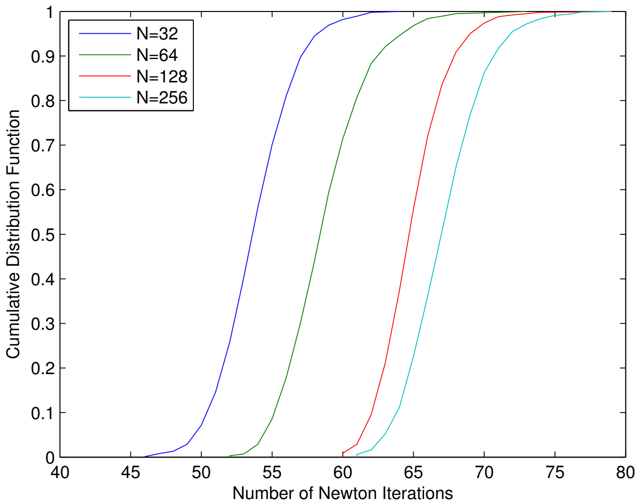

Fig. 3. CDF of the number of Newton iterations required for convergence over 1000 channel realizations.

TABLE I
NORMALIZED TIME COST OF SOME CASES WITH $P_t = 10^{-3}W$

K	3	3	3	6	6	6
N	64	128	256	64	128	256
Standard	5.710	23.851	101.440	23.107	87.308	694.079
This Work	0.118	0.162	0.256	0.156	0.202	0.542

allocated to a link with relatively high channel gain, and then more spatial diversity is gained. That is to say, the CR system is able to benefit from cooperative relays. Meanwhile, the capacity gap between our proposal and the upper bound is less than 5%.

In Fig.3, we investigate the convergence of our proposed barrier method. As discussed in Section III-C, the computational load of the barrier method mainly lies in the computation of Newton step. Fig.3 gives the cumulative distribution function (CDF) of the number of Newton iterations for onekick resource allocation, including the subchannel assignment and power distribution. It shows the the number of Newton iterations varies in a narrow range with a given N and increases slowly when N gets larger, which validates the efficiency and effectiveness of our proposed algorithm.

Finally, we examine the computation efficiency of our proposed fast barrier method. We compare the time cost of our proposed algorithm with the standard interior point method used in [11]. The elapsed time is counted by the inbuilt function $tic - toc$ in *Matlab*. Table I gives the time cost when both of the algorithms converge for cases with $P_t = 10^{-3}$W. We can see that the elapsed time cost of our proposal is much lower than that of the method in [11]. Moreover, the standard method consumes more and more time as the number of subchannels and relays increases, while the time cost of our proposal increases slowly and stably when N and K become larger. Such results is consistent with the complexity analysis in Section III-C.

V. CONCLUSION

We proposed an efficient algorithm to achieve the resource allocation for a signal antennae CR network enhanced by multiple relays. Since the subchannel allocation and power loading processes were addressed separately, our proposed method can always work out the optimal solution for each subproblem. By speedup the computation of Newton step in an ingenious way, our proposal can significantly lower the complexity compared to the standard method. Simulation results also verified the effectiveness and the efficiency of our proposed algorithm, indicating that it is promising for applications.

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