Efficient Resource Allocation for OFDMA-based Device-to-Device Communication Underlaying Cellular Networks

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***Abstract*—In this paper, we investigate the resource allocation problem in OFDMA-based device-to-device (D2D) communication underlaying cellular networks, where we aim to maximize the sum rate of the cellular system. The formulated challenging optimization task is divided into two subproblems: D2D pair assignment and power distribution. We show that the D2D pair association falls into the maximum assignment problem class and develop an efficient time-sharing method to address it. A low complexity iterative algorithm is developed to deal with the power allocation problem, where the locally optimal solution is approached by solving a series of convex optimization problems that can yield a tight lower bound of the sum rate. The convergence of our proposed algorithm is proved and confirmed by numerical results.**

I. Introduction

Device-to-device (D2D) communication, which is deemed as a promising technique to improve spectrum efficiency and offload cellular traffic, has received significant attention recently. As the proximity of users allows for high transmission rate, low latency and power consumption, D2D users can communicate with each other under the control of base stations (BSs) [1]. In a D2D communication underlaying cellular network, cellular users (CUs) share the same spectrum with D2D pairs to improve spectrum efficiency. However, there inevitably exists mutual interference between CUs and D2D pairs, resulting that radio recourse allocation becomes a crucial issue [2].

The selection of modulation pattern is also an important issue in D2D communication. Currently, 3GPP Long Term Evolution (LTE) uses single-carrier frequency division multiple access (SC-FDMA) in the uplink and orthogonal frequency division multiple access (OFDMA) in the downlink. As a result, SC-FDMA and OFDMA are considered as two promising air interfaces for D2D communication systems [3]. Compare with OFDMA system, users in SC-FDMA systems use subcarriers sequentially, rather than in parallel. In other words, each user transmits signal using single channel from the viewpoint of SC-FDMA systems [4].

Most of previous researches focus on the case that each user occupies at most one channel, which corresponds with SC-FDMA systems. E.g., the case that one D2D pair shares a single channel with one CU is investigated in [5, 6]. The key issue is to design an efficient radio resource sharing mode to improve the throughput and control the interference between the CU and the D2D pair. In [7–9], multiuser diversity gain is achieved by properly pairing multiple CUs and D2D pairs, where each user is constrained to use only one channel. Obviously, it is not spectrum-efficient for OFDMA-based systems because the system capacity can be further improved if each user can use multiple channels [10]. In [11, 12], each D2D pair can use multiple channels and each CU occupies only one channel. However, it still cannot achieve an optimal resource allocation from the viewpoint of OFDMA-based D2D communication underlaying cellular networks.

In this paper, we study the resource allocation for OFDMA-based cellular networks with D2D communication enhancement, where each user can use multiple subcarriers. Our goal is to maximize the sum rate among all CUs and D2D pairs. We propose an efficient algorithm to address the formulated problem. Specifically, we first convert the problem into an equivalent problem, which falls into maximum assignment problem class that can be solved by Hungarian method with high complexity. To reduce the computational load, we give the relaxation of the equivalent problem and propose a D2D pair assignment scheme by exploiting the optimal solutions to the relaxation. Finally, we show that the power distribution task is a D.C. (Difference of Convex) programming problem, and then develop a low complexity iterative algorithm. Simulation results validate the effectiveness and efficiency of our proposed scheme.

The remainder of this paper is organized as follows. In Section II, we present the system model and formulate the optimization problem. In Section III, the resource allocation algorithm is proposed in detail. In Section IV, the simulation results are reported with discussions. Conclusion is drawn in Section V.

II. System Model and Problem Formulation

Consider an OFDMA-based D2D communication underlaying cellular network, including one BS, N CUs and M D2D pairs. The D2D pairs use the uplink resource of the cellular

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system. The sets of the CUs and the D2D pairs are denoted by $\mathcal{N}$ and $\mathcal{M}$, respectively. The available spectrum is divided into K subcarriers. Denote $\mathcal{K}_n$ as the set of the subcarriers which are assigned to CU n. We use $\mathcal{K} = \cup_{n\in\mathcal{N}}\mathcal{K}_n$ to denote the set of all subcarriers. Let $K = |\mathcal{K}|$ be the number of all subcarriers. The same as assumed in [7–9], we assume that $\mathcal{K}_n$ is predetermined by the cellular system and at most one D2D pair can use the subcarriers assigned to the CU n.

Let $p_{n,k}$ and $p_{m,k}$ be the powers of the CU n and the D2D transmitter (D2D-Tx) m on the kth subcarriers. We collect all $p_{n,k}$'s and $p_{m,k}$'s into one vector $\bar{p}$. Let $H_{n,k}$ and $H_{m,k}$ be the channel gains of the cellular link and the D2D link on the kth subcarrier, respectively. Denote $\widetilde{H}_{m,k}$ and $\widetilde{H}^m_{n,k}$ as the power gains of interference link from the D2D-Tx m to the BS and from the CU n to the D2D receiver (D2D-Rx) m on the kth subcarrier, respectively. For the CU n, the signal to interference plus noise ratio (SINR) on the subcarrier $k \in \mathcal{K}_n$ can be written as

$$\gamma^C_{n,k} = \frac{p_{n,k}H_{n,k}}{\sum\limits_{m\in\mathcal{M}} \rho_{m,n}p_{m,k}\widetilde{H}_{m,k} + \sigma_0^2}, \tag{1}$$

where σ_0^2 is the noise power and $\rho_{m,n}$ informs whether the D2D pair m uses the subcarriers $\mathcal{K}_n$ or not: $\rho_{m,n} = 1$, the D2D pair m uses the subcarriers $\mathcal{K}_n$; $\rho_{m,n} = 0$, otherwise. We collect all $\rho_{m,n}$'s into one vector $\bar{\rho}$. For the D2D pair m, the SINR of the D2D-Rx on the kth subcarrier can be written as

$$\gamma^D_{m,k} = \frac{p_{m,k}H_{m,k}}{p_{n,k}\widetilde{H}^m_{n,k} + \sigma_0^2}. \tag{2}$$

Then, the sum rate on the kth subcarrier can be calculated by

$$R_k = \log_2(1+\gamma^C_{n,k}) + \sum_{m\in\mathcal{M}} \rho_{m,n}\log_2(1+\gamma^D_{m,k}). \tag{3}$$

Our optimization task is to maximize the sum rate of the system, which can be mathematically formulated as the following problem:

$$\begin{aligned} \max_{\bar{p},\bar{\rho}} \quad & \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}_n} R_k \\ s.t. \quad C_1: \quad & \sum_{k\in\mathcal{K}} p_{m,k} \le P^D_m, \forall m \in \mathcal{M}, \\ C_2: \quad & \sum_{k\in\mathcal{K}_n} p_{n,k} \le P^C_n, \forall n \in \mathcal{N}, \\ C_3: \quad & \rho_{m,n}P^C_m \ge p_{m,k} \ge 0, \forall m \in \mathcal{M}, k \in \mathcal{K}_n, \\ C_4: \quad & p_{n,k} \ge 0, \forall n \in \mathcal{N}, \\ C_5: \quad & \rho_{m,n} \in \{0,1\}, \forall m \in \mathcal{M}, n \in \mathcal{N}, \\ C_6: \quad & \sum_{m\in\mathcal{M}} \rho_{m,n} \le 1, \forall n \in \mathcal{N}, \\ C_7: \quad & \sum_{n\in\mathcal{N}} \rho_{m,n} \le 1, \forall m \in \mathcal{M}, \end{aligned} \tag{4}$$

where P^D_m and P^C_n are the maximum powers of the D2D-Tx m and the CU n, respectively. C_1 and C_2 are the transmission power budgets for the D2D pairs and CUs. C_3 means that $p_{m,k} = 0$ if $\rho_{m,n} = 0$ and $P^C_m \ge p_{m,k} \ge 0$ if $\rho_{m,n} = 1$. C_4 is intuitive. $C_5 \sim C_7$ ensure that each D2D pair can use at most one subset of subcarriers and the D2D pairs cannot share the same subcarriers at the same time.

III. Efficient Resource Allocation Algorithm

Problem (4) is a mixed integer programming (MIP) problem, which is hard to tackle. Even if we relax the integer variables, it is still hard to obtain the globally optimal solution because the relaxed form is non-concave for both $\rho_{m,n}$ and $p_{m,n}$ [13]. We propose an efficient resource allocation algorithm to address problem (4).

A. D2D Pair Assignment

We first convert problem (4) into an equivalent form to make the problem more trackable. We use R^*_n to denote the maximum sum rate over the subcarriers $\mathcal{K}_n$ for the case that no D2D pair shares the subcarriers. Denote $R^*_{m,n}$ as the maximum sum rate over the subcarriers $\mathcal{K}_n$ for the case that the D2D pair m is assigned to use them. Based on the conclusion in [7], problem (4) can be converted into the following equivalent form:

$$\begin{aligned} \max_{\bar{\rho}} \quad & \sum_{n\in\mathcal{N}} \rho_{m,n}(R^*_{m,n} - R^*_n) \\ s.t. \quad C_1: \quad & \sum_{m\in\mathcal{M}} \rho_{m,n} \le 1, \forall n \in \mathcal{N}, \\ C_2: \quad & \sum_{n\in\mathcal{N}} \rho_{m,n} \le 1, \forall m \in \mathcal{M}, \\ C_3: \quad & \rho_{m,n} \in \{0,1\}, \forall m \in \mathcal{M}, n \in \mathcal{N}, \end{aligned} \tag{5}$$

where $R^*_{m,n} - R^*_n$ is the capacity gain yielded by the D2D pair m over the subcarriers $\mathcal{K}_n$. Problem (5) is a maximum assignment problem [14] that can be solved by classic Hungarian method with the complexity of $O(N^3)$ [15].

To reduce the computational load, we develop a time-sharing method to tackle (5). The relaxation of (5) is

$$\begin{aligned} \max_{\bar{\rho}} \quad & \sum_{n\in\mathcal{N}} \rho_{m,n}(R^*_{m,n} - R^*_n) \\ s.t. \quad & 1 \ge \rho_{m,n} \ge 0, \forall m \in \mathcal{M}, n \in \mathcal{N}, \\ & C_1 \sim C_2 \text{ in (5).} \end{aligned} \tag{6}$$

Note that (6) defines a linear programming problem which can be solved by using standard algorithms or solvers efficiently, such as CVX [16]. Since the solution to (6) contains continuous variables $\rho_{m,n}$'s, rounding is necessary to obtain a feasible solution to the original problem (6). It is straightforward to allocate the D2D pair m who has the largest $\rho_{m,n}$ to share the subcarriers $\mathcal{K}_n$ with the CU n, that is,

$$\rho_{m,n} = \begin{cases} 1 & m = \arg\max\limits_{m^*\in\mathcal{M}} \rho_{m^*,n}, \\ 0 & \text{otherwise}, \end{cases} \quad \forall n \in \mathcal{N}. \tag{7}$$

B. Power Distribution

To apply the method proposed in the previous subsection, we need to figure out the optimal power allocation for the cases that no D2D pair uses $\mathcal{K}_n$ and the D2D pair m uses the $\mathcal{K}_n$. For the former case, the optimal power allocation to the CU n over the subcarriers $\mathcal{K}_n$ has a water-filling structure [17]. Thus we can obtain the R^*_n efficiently by using standard water-filling algorithm. For the latter one, we need to find the optimal power allocation for the D2D pair m and the CU n over the subcarriers $\mathcal{K}_n$.

Define $\bar{p}_k = (p_{n,k}, p_{m,k}), k \in \mathcal{K}_n$, $R^*_{m,n}$ is the optimal value to the following problem:

$$\begin{aligned} \max_{\{\bar{p}_k\}_{k\in\mathcal{K}_n}} \quad & \sum_{k\in\mathcal{K}_n} R_k(\bar{p}_k) \\ s.t. \quad C_1: \quad & \sum_{k\in\mathcal{K}_n} p_{m,k} \le P^D_m, \\ C_2: \quad & \sum_{k\in\mathcal{K}_n} p_{n,k} \le P^C_n, \\ C_3: \quad & p_{m,k} \ge 0, p_{n,k} \ge 0, \forall k \in \mathcal{K}_n, \end{aligned} \tag{8}$$

where

$$\begin{aligned} R_k(\bar{p}_k) = & \log_2 \Big(1 + \frac{p_{n,k} H_{n,k}}{p_{m,k}\widetilde{H}_{m,k} + \sigma_0^2}\Big) \\ & + \log_2 \Big(1 + \frac{p_{m,k} H_{m,k}}{p_{n,k}\widetilde{H}^m_{n,k} + \sigma_0^2}\Big). \end{aligned} \tag{9}$$

Note that we use $R_k(\bar{p}_k)$ as a replacement of R_k to highlight the sum rate on the kth subcarrier is only related to $\bar{p}_k$ in the this case. $R_k(\bar{p}_k)$ can be rewritten as the following form:

$$\begin{aligned} R_k(\bar{p}_k) = & \log_2 P_{n,k}(\bar{p}_k) + \log_2 P_{m,k}(\bar{p}_k) \\ & - \log_2 I_{n,k}(p_{m,k}) - \log_2 I_{m,k}(p_{n,k}). \end{aligned} \tag{10}$$

In (10), $P_{n,k}(\bar{p}_k) = p_{n,k}H_{n,k} + p_{m,k}\widetilde{H}_{m,k} + \sigma_0^2$ is the BS's the received power on the kth subcarrier. $P_{m,k}(\bar{p}_k) = p_{m,k}H_{m,k} + p_{n,k}\widetilde{H}^m_{n,k} + \sigma_0^2$ is the received power of the D2D-Rx m on the kth subcarrier. $I_{n,k}(p_{m,k}) = p_{m,k}\widetilde{H}_{m,k} + \sigma_0^2$ and $I_{m,k}(p_{n,k}) = p_{n,k}\widetilde{H}^m_{n,k} + \sigma_0^2$ are the BS's received interference plus noise power and the received interference plus noise power of the D2D-Rx m on the kth subcarrier, respectively. Since $\log_2 P_{n,k}(\bar{p}_k), \log_2 P_{m,k}(\bar{p}_k), \log_2 I_{n,k}(p_{m,k})$ and $\log_2 I_{m,k}(p_{n,k})$ are concave for both $p_{m,k}, p_{n,k}$, (10) is a D.C. function [18].

One approach to address D.C. programming problems is to solve a sequence of concave optimization problems [19, 20]. As shown in (11), we define a concave lower bound of $R_k(\bar{p}_k)$, which is parameterized by given $\bar{p}^*_k$. We have the following theorem:

Theorem 1. *Given any $\bar{p}_k$ and $\bar{p}^*_k$ that satisfy the $C_1 \sim C_3$ in (8), we have $R_k(\bar{p}_k) \ge \underline{R}_k(\bar{p}_k, \bar{p}^*_k), \forall k \in \mathcal{K}_n$.*

Proof: As we mentioned before, $\log_2 I_{n,k}(p_{m,k})$ and $\log_2 I_{m,k}(p_{n,k})$ are concave functions. According to the first-order condition for a concave function [13], we have

$$\log_2 I_{n,k}(p_{m,k}) \le \log_2 I_{n,k}(p^*_{m,k}) + \frac{\widetilde{H}_{m,k}(p_{m,k} - p^*_{m,k})}{\ln 2 \cdot I_{n,k}(p_{m,k})} \tag{12}$$

and

$$\log_2 I_{m,k}(p_{n,k}) \le \log_2 I_{m,k}(p^*_{n,k}) + \frac{\widetilde{H}^m_{n,k}(p_{n,k} - p^*_{n,k})}{\ln 2 \cdot I_{m,k}(p^*_{n,k})}. \tag{13}$$

Thus, $R_k(\bar{p}_k) \ge \underline{R}_k(\bar{p}_k, \bar{p}^*_k), \forall k \in \mathcal{K}_n$. ■

According to Theorem 1, $\underline{R}_k(\bar{p}_k, \bar{p}^*_k)$ is a concave function of $p_{n,k}$ and $p_{m,k}$, as well as a lower bound of $R_k(\bar{p}_k)$. Note that, we have $R_k(\bar{p}^*_k) = \underline{R}_k(\bar{p}^*_k, \bar{p}^*_k), \forall k \in \mathcal{K}_n$. Thus, $\underline{R}_k(\bar{p}_k, \bar{p}^*_k)$ is a tight lower bound of (10).

For given $\{\bar{p}^*_k\}_{k\in\mathcal{K}_n}$, consider the following optimization problem,

$$\begin{aligned} \max_{\{\bar{p}_k\}_{k\in\mathcal{K}_n}} \quad & \sum_{k\in\mathcal{K}_n} \underline{R}_k(\bar{p}_k, \bar{p}^*_k) \\ s.t. \quad & C_1 \sim C_3 \text{ in (8)}. \end{aligned} \tag{14}$$

Since $\underline{R}_k(\bar{p}_k, \bar{p}^*_k)$ is concave, (14) defines a convex optimization problem. Intuitively, if we only optimize (14) and obtain a solution, the quality of the solution varies from case to case due to different selections of $\{\bar{p}^*_k\}_{k\in\mathcal{K}_n}$. It can be proved that the quality of the new obtained solution is always no worse than $\{\bar{p}^*_k\}_{k\in\mathcal{K}_n}$. We have the following theorem:

Theorem 2. *Denote $\{\bar{p}^{opt}_k\}_{k\in\mathcal{K}_n}$ as the optimal solution to (14), we have*

$$\sum_{k\in\mathcal{K}_n} R_k(\bar{p}^{opt}_k) \ge \sum_{k\in\mathcal{K}_n} R_k(\bar{p}^*_k). \tag{15}$$

Proof: As mentioned before, $R_k(\bar{p}^{opt}_k) = \underline{R}_k(\bar{p}^{opt}_k, \bar{p}^{opt}_k)$ and $R_k(\bar{p}^*_k) = \underline{R}_k(\bar{p}^*_k, \bar{p}^*_k)$. We can obtain

$$\sum_{k\in\mathcal{K}_n} \underline{R}_k(\bar{p}^{opt}_k, \bar{p}^{opt}_k) \ge \sum_{k\in\mathcal{K}_n} \underline{R}_k(\bar{p}^{opt}_k, \bar{p}^*_k) \ge \sum_{k\in\mathcal{K}_n} \underline{R}_k(\bar{p}^*_k, \bar{p}^*_k), \tag{16}$$

where the first inequality follows by Theorem 1 and the second inequality follows that $\bar{p}^{opt}_k$ is the optimal power allocation to (14) on the kth subcarrier. Thus, (15) holds. ■

Since an improved solution to (4) can be obtained by maximizing the lower bound, we develop an iterative algorithm. First, we initialize $\bar{p}^*_k$ as a feasible power allocation $\bar{p}^{(0)}_k$. A

TABLE I
ALGORITHM 1: ITERATIVE ALGORITHM FOR POWER ALLOCATION

1: *Initialization*: $l = 0$, $\bar{p}^{(0)}_k$;
2: **repeat**
3: $\quad \bar{p}^*_k = \bar{p}^{(l)}_k, \forall k \in \mathcal{K}_n$;
4: $\quad$ Obtain the optimal solution to (14) by the fast barrier method;
5: $\quad l = l + 1$;
6: $\quad$ Update $\bar{p}^{(l)}_k, \forall k \in \mathcal{K}_n$ with the optimal solution;
7: **until** $\sum_{k\in\mathcal{K}_n} ||\bar{p}^{(l)}_k - \bar{p}^{(l-1)}_k||^2_2 \le \epsilon$ or $l \ge L$

$$\underline{R}_k(\bar{p}_k, \bar{p}^*_k) = \log_2 P_{n,k}(\bar{p}_k) + \log_2 P_{m,k}(\bar{p}_k) - \log_2 I_{n,k}(p^*_{m,k}) - \log_2 I_{m,k}(p^*_{n,k}) - \frac{\widetilde{H}_{m,k}(p_{m,k} - p^*_{m,k})}{\ln 2 \cdot I_{n,k}(p^*_{m,k})} - \frac{\widetilde{H}^m_{n,k}(p_{n,k} - p^*_{n,k})}{\ln 2 \cdot I_{m,k}(p^*_{n,k})} \tag{11}$$

possible initialization of $\bar{p}_k^{(0)}$ is to set $p_{n,k} = P_n^C/|\mathcal{K}_n|$ and $p_{m,k} = P_m^D/|\mathcal{K}_n|$. Then, we solve the problem in (14) to work out a new solution and update $\bar{p}_k^*$'s. The procedure repeats until the stopping criteria is met. The iterative algorithm is summarized in Table I. The convergence of Algorithm 1 is also guaranteed since an improved solution to (8) can be obtained at each iteration using Theorem 2. Furthermore, at least a locally optimal solution can be found since Algorithm 1 belongs to the class of iterative coordinate ascent methods [21]. Notice that the algorithm usually converges within a few tens of iterations. The improvement is quite small after 5 iterations, so we can set a small number of iterations (i.e. L in the stopping criteria) without degenerating the capacity performance of our proposed algorithm.

C. Fast Barrier Method

At each iteration, we need to obtain the optimal solution to (14). Based on the Karush-Kuhn-Tucker (KKT) conditions, the optimal power allocation to the CU n on each subcarrier is related to the optimal power allocation to the D2D pair m on the same subcarrier, as well as dual optimal points for all constraints. Some well-known methods, e.g., multilevel water-filling method and sub-gradient algorithm, cannot be applied to address the problem. Barrier method, which has rapid convergence, is one of the convex optimization algorithms [13]. The main disadvantage of barrier method is that the cost of storing Hessian and computing Newton step is high. For the problem (14), the storage complexity and computational complexity of standard barrier method are $O(|\mathcal{K}_n|^2)$ and $O(|\mathcal{K}_n|^3)$, respectively. We develop a fast barrier method by exploiting the special structure of (14) to substantially reduce the storage complexity and computational complexity.

For simplicity, we collect all $p_{m,k}$'s and $p_{n,k}$'s into one vector $\tilde{p}_n$. We convert all inequality constraints into a logarithmic barrier function $\phi(\tilde{p}_n)$, that is,

$$\phi(\tilde{p}_n) = -\sum_{i=1}^{2} \ln f_i - \sum_{k\in\mathcal{K}_n} \ln p_{n,k} - \sum_{k\in\mathcal{K}_n} \ln p_{m,k}, \quad (17)$$

where

$$\begin{aligned} f_1 &= P_m^D - \sum_{k\in\mathcal{K}_n} p_{m,k}, \\ f_2 &= P_n^C - \sum_{k\in\mathcal{K}_n} p_{n,k}. \end{aligned} \quad (18)$$

The original problem can be converted into a sequence of minimization problems, by introducing a logarithmic barrier function with a parameter t. The optimal solution to (14) can be approximated by solving the following problem:

$$\min_{\tilde{p}_n} \quad \psi_t(\tilde{p}_n) = -t \sum_{k\in\mathcal{K}_n} \underline{R}_k(\bar{p}_k, \bar{p}_k^*) + \phi(\tilde{p}_n). \quad (19)$$

With t increases, such approximation becomes more and more close to the optimal solution to (14). Note that (19) is an unconstrained minimization problem, which can be addressed by Newton method. For a given parameter t, the Newton step $\triangle\tilde{p}_n$ can be given by following equation:

$$\nabla^2\psi_t(\tilde{p}_n)\triangle\tilde{p}_n = -\nabla\psi_t(\tilde{p}_n), \quad (20)$$

TABLE II

BARRIER METHOD

Initialization: $\tilde{p}_n$, $t = t^{(0)}$, $\mu > 1$, $\alpha \in (0, 1/2), \beta \in (0, 1)$;
while $(2\mathcal{K}_n + 2)/t > \epsilon_b$
  while $\lambda^2/2 \le \epsilon_n$
    Calculate $\triangle\tilde{p}_n$ and $\lambda = \nabla\psi_t(\tilde{p}_n)\triangle\tilde{p}_n$;
    $s = 1$;
    while $\psi_t(\tilde{p}_n + s\triangle\tilde{p}_n) > \psi_t(\tilde{p}_n) - \alpha s\lambda^2$
      $s = \beta s$;
    end while
    Update $\tilde{p}_n = \tilde{p}_n + s\triangle\tilde{p}_n$;
  end while
  $t = \mu t$;
end while

where $\nabla^2\psi_t(\tilde{p}_n)$ and $\nabla\psi_t(\tilde{p}_n)$ are the Hessian and the gradient of $\psi_t(\tilde{p}_n)$, respectively. Now, we give a simple version of barrier method, as presented in Table II.

Note that the Hessian of $\psi_t(\tilde{p}_n)$ can be written as

$$\begin{aligned} \nabla^2\psi_t(\tilde{p}_n) &= D + \sum_{i=1}^{2} \lambda_i\lambda_i^T \\ &= diag(D_1, ..., D_{|\mathcal{K}_n|}) + \sum_{i=1}^{2} \lambda_i\lambda_i^T, \end{aligned} \quad (21)$$

where

$$\begin{aligned} D_k &= -t \cdot \begin{bmatrix} \dfrac{\partial^2 \underline{R}_k(\bar{p}_k, \bar{p}_k^*)}{\partial p_{n,k}^2} & \dfrac{\partial^2 \underline{R}_k(\bar{p}_k, \bar{p}_k^*)}{\partial p_{n,k}\partial p_{m,k}} \\ \dfrac{\partial^2 \underline{R}_k(\bar{p}_k, \bar{p}_k^*)}{\partial p_{m,k}\partial p_{n,k}} & \dfrac{\partial^2 \underline{R}_k(\bar{p}_k, \bar{p}_k^*)}{\partial p_{m,k}^2} \end{bmatrix} \\ &\quad + \begin{bmatrix} \dfrac{1}{p_{n,k}^2} & 0 \\ 0 & \dfrac{1}{p_{m,k}^2} \end{bmatrix} \end{aligned} \quad (22)$$

and $\lambda_i = \nabla f_i/f_i$. We can calculate the Newton step $\triangle\tilde{p}_n$ efficiently by using the following mathematical fact:

Fact 1. *Given a nonsingular matrix $A \in \mathbf{R}^{2|\mathcal{K}_n|\times 2|\mathcal{K}_n|}$, vectors $f, b \in \mathbf{R}^{2|\mathcal{K}_n|\times 1}$, where f satisfies $1+f^TA^{-1}f \neq 0$. Then, if $Ax = b, (A+ff^T)\tilde{x} = b$, it always holds $\tilde{x} = x - \frac{f^Tx}{1+f^Tg}g$, where $g = A^{-1}f, g \in \mathbf{R}^{2|\mathcal{K}_n|\times 1}$.*

The details of the proof of Fact 1 can be referred to the Appendix C in [13]. Since D is a diagonal matrix, we can obtain the solutions of $Dx_i = \lambda_i, i = 1, 2$ and $Dx_3 = -\nabla\psi_t(\tilde{p}_n)$ by matrix inversion with complexity of $O(|\mathcal{K}_n|)$. Then, we work out the solutions of $(D + \lambda_1\lambda_1^T)x_2 = \lambda_2$ and $(D + \lambda_1\lambda_1^T)x_3 = -\nabla\psi_t(\tilde{p}_n)$ by using Fact 1 with complexity of $O(|\mathcal{K}_n|)$. With the similar calculation, we can finally obtain the Newton step by obtaining the solution of $(D + \sum_{i=1,2}\lambda_i\lambda_i^T)\triangle\tilde{p}_n = -\nabla\psi_t(\tilde{p}_n)$. The storage complexity is bounded by $O(|\mathcal{K}_n|)$ because only six vectors and one diagonal matrix are recorded. The computational complexity of the calculation of Newton step is bounded by $O(|\mathcal{K}_n|)$, which indicates the complexity of the fast barrier method is linearly related to the number of subcarriers.

TABLE III
ALGORITHM 2: OVERALL RESOURCE ALLOCATION ALGORITHM

Step 1	1:	**for** $n \in \mathcal{N}$
	2:	Obtain R_n^* by using standard water-filling algorithm;
	3:	**end for**
Step 2	4:	**for** $m \in \mathcal{M}$, $n \in \mathcal{N}$
	5:	Obtain $R_{m,n}^*$ by using Algorithm 1;
	6:	**end for**
Step 3	7:	Obtain the optimal solution to (6);
	8:	Assign the D2D pairs by using (7).

D. Complexity Analysis

Our proposed resource allocation algorithm is presented in Table III. Specifically, we obtain R_n^* for all $\mathcal{K}_n$'s in the first step, where the complexity is $O(\mathcal{K})$. Then, we work out $R_{m,n}^*$'s by using Algorithm 1. As mentioned before, the complexity of our proposed fast barrier method is $O(|\mathcal{K}_n|)$. Thus the complexity of calculating all $R_{m,n}^*$'s is $O(MK)$. Finally, the original problem is reduced to (5), and we assign D2D pairs by exploiting the optimal solution to the relaxation (6).

IV. SIMULATION RESULTS

We conduct a series of experiments to evaluate the performance of our proposed algorithm. The CUs and the D2D-Txes are uniformly distributed in a circle within 500 m from the BS. Each D2D-Rx is uniformly distributed in a circle within 50 m from the pairing D2D-Tx. The number of CUs is same to the number of D2D pairs. The CUs have the same number of subcarriers. The path loss model (in dB) is $15.3 + 37.6 \log_{10} d$ for distance d (in m). The variance of shadowing is 10 dB and the amplitude of multipath fading is Rayleigh. The noise power on each subcarrier is -100 dBm. The tolerance ϵ is set to 10^{-3}. The parameters of barrier method are as follows: $t^{(0)} = 0.1, \epsilon_b = \epsilon_n = 10^{-3}, \mu = 10, \alpha = 0.01, \beta = 0.1$.

First, we investigate the sum rate of the system with different number of CUs and different number of subcarriers occupied by each CU, as illustrated in Fig. 1. The maximum transmission power of each transmitter is 20 dBm, i.e. $P_m^D = P_n^C = 20$ dBm, $\forall m \in \mathcal{M}, n \in \mathcal{N}$. We compare our proposed algorithm with an upper bound and Hungarian method. The upper bound is the optimal value to (6) and Hungarian method can always work out the optimal solution to (5) with the complexity of $O(N^3)$. Note that the globally optimal solution to (8) is hard to obtain since it requires to generate all local optima. Thus, $R_{m,n}^*$ is calculated by using Algorithm 1 in all three schemes. It can be observed from Fig. 1 that the sum rate of our proposed algorithm is close to the upper bound. The gap is always less than 1% for different scenarios. Nevertheless, our proposed algorithm can always find the optimal solution to (6), which confirms the effectiveness of the proposed D2D pair assignment scheme.

Then, we compare our proposed resource allocation scheme with equal power allocation (EPA) scheme that equally distributes power over all subcarriers. The sum rate with different transmission power limits is presented in Fig. 2, where the maximum transmission power varies from -10 dBm to 30 dBm and the number of the subcarriers is 100. We can see from Fig. 2 that our proposed scheme outperforms the EPA scheme. It can always achieve more than at least 50% of the EPA scheme when the maximum transmission power is larger than 10 dBm. We can conclude that our proposal can enhance the system capacity effectively.

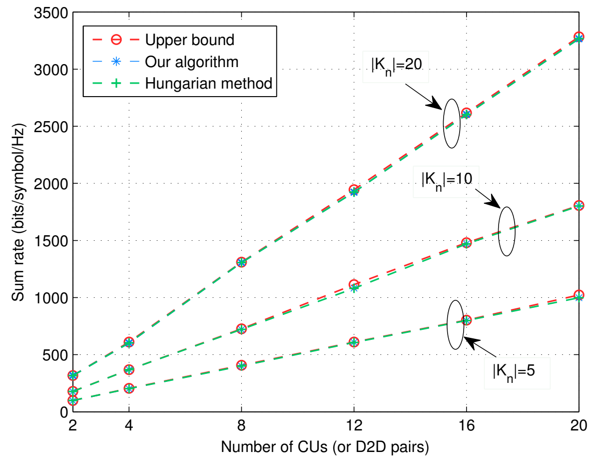

Fig. 1. Sum rate as a function of number of CUs with different number of subcarriers. $P_m^D = P_n^C = 20$ dBm, $\forall m \in \mathcal{M}, n \in \mathcal{N}$.

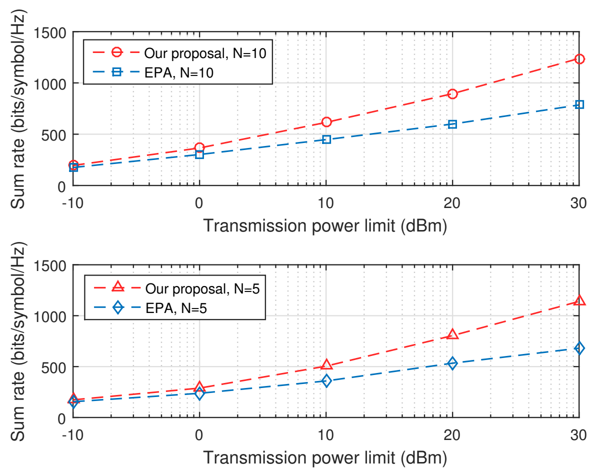

Fig. 2. Sum rate as a function of transmission power limit with different number of CUs. $K = 100$.

The convergence of the proposed power allocation algorithm is illustrated in Fig. 3. The average number of iterations is less than 5 as can be seen from Fig. 3. Moreover, the number of iterations varies in a narrow range. Only 4% of them are larger than 10. Furthermore, the proposed algorithm is robust for different number of subcarriers.

Finally, we give the cumulative distribution function (CDF) of the number of Newton iterations over 1000 Monte Carlo simulations, as shown in Fig. 4. It can be seen from Fig. 4, for all cases, 95% of Newton iterations are less than 100. The number of Newton iterations increases slightly with increase

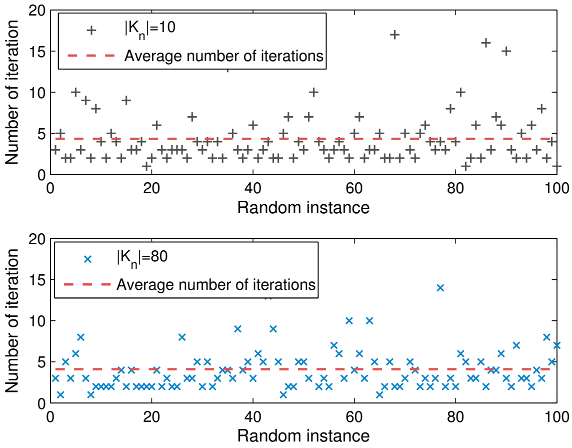

Fig. 3. Number of iterations required for convergence of Algorithm 1 over 100 channel realizations. The tolerance ϵ is 10^{-3}.

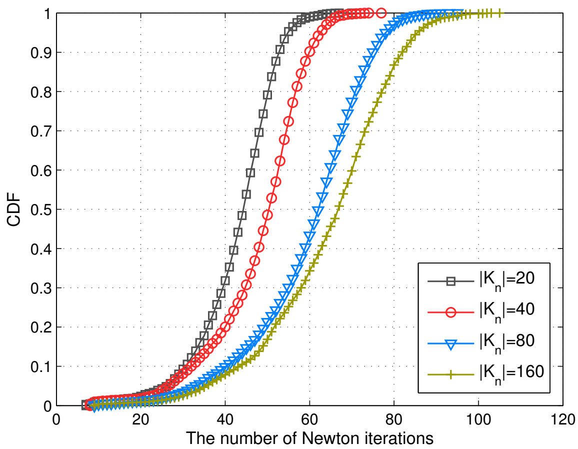

Fig. 4. CDF of the number of Newton iterations required for convergence over 1000 channel realizations. $t^{(0)} = 0.1, \epsilon_b = \epsilon_n = 10^{-3}, \mu = 10, \alpha = 0.01, \beta = 0.1$.

of the number of subcarriers. We can conclude that the fast barrier method works effectively and efficiently.

V. Conclusion

In this work, we investigated the resource allocation for OFDMA-based D2D communication underlaying cellular networks. We try to maximize the sum rate of the system. The resource allocation problem is decoupled into two procedures: D2D pair assignment and power distribution. First, we convert the formulated problem into an equivalent form to make the original one more trackable. Then, a D2D pair assignment scheme was proposed to allocate the D2D pairs efficiently. Finally, we developed a low complexity iterative algorithm to work out the locally optimal power allocation. Both theoretical analysis and numerical results confirm the efficiency and effectiveness of our proposed algorithm.

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