Clustering-based Interference Management in Densely Deployed Femtocell Networks

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***Abstract*—Deploying femtocells underlaying macrocells is a promising way to improve the capacity and enhance the coverage of a cellular system. However, densely deployed femtocells in urban area also give rise to intra-tier and cross-tier interference issue that should be addressed properly to acquire the expected performance gain. In this paper, we propose an interference management scheme based on joint clustering and resource allocation for the two-tier Orthogonal Frequency Division Multiplexing (OFDM)-based femtocell networks. We formulate an optimization task to maximize the sum throughput of the femtocell users (FUs) under the consideration of intra-tier interference mitigation, while controlling the interference to the macrocell user (MU) under its bearable threshold. The formulate problem is addressed by a two-stage procedure: femtocells clustering and resource allocation. First, disjoint femtocell clusters with dynamic sizes and numbers are generated to minimize intra-tier interference. Then each cluster is taken as a resource allocation unit to share all subchannels, followed by a fast algorithm to distribute power among these subchanels. Simulation results show that our proposal can improve the throughput of the FUs with reasonable complexity.**

I. Introduction

Wireless data traffic has been increasing dramatically, requiring more efficient use of the scarce radio spectrum. Heterogeneous network, which consists of macrocells and the overlaying femtocells, is an economical and effective way to improve system capacity and coverage [1]. However, such a heterogeneous infrastructure also gives rise to nonnegligible interference issue, which may seriously degrade the performance of the cellular networks. In a two-tier cellular network, there are two kinds of interference [2]: cross-tier interference, that is, the aggressor (e.g., a femtocell user (FU)) and the victim of interference (e.g., a macrocell user (MU)) belong to different tiers; intra-tier interference, which means that the aggressor and the victim belong to the same tier. Dense femtocell deployment is expected in the future [3], where the femtocells suffer from severe intra-tier interference due to dense deployment in a small area. Therefore, there are many new challenges that should be carefully addressed for the high density of femtocells scenario, such as resource allocation (RA) and interference management.

Previous researches have provided an overview on interference avoidance mechanisms in a two-layer network, e.g., power control, multiple antennas, adaptive femtocell access point (FAP) access scheme, and spectrum allocation. These studies mainly focus on cross-tier interference mitigation. However, considering the fact that the number of FAPs is very large, many proposed intra-tier interference mitigation schemes are not scalable because they often yield a non-linear non-convex problem. Clustering can be used as a technique to reduce intra-tier interference by coordinating the transmissions of FAPs in a dense deployment scenario, which generally divides the RA task into a series of subproblems that are not difficult to deal with. The femtocells can be divided into disjoint clusters, where the entire set of subchannels is available for each cluster, however, no two femtocells in the same cluster are allowed to transmit on the same subchannel. Hence, clustering-based interference mitigation schemes have been researched in the literature [4, 5]. In [4], a clustering algorithm based on semi-definite programming is proposed to manage the intra-tier interference with a lower complexity. In [5], an efficient clustering algorithm is proposed to solve the interference management problem. However, it ignores the FU's QoS requirement.

An important issue that follows is how to effectively assign orthogonal radio resources between macrocell and femtocells after dividing the femtocells into clusters meanwhile considering the cross-tier interference. In [6], the authors proposed a dynamic clustering-based subband allocation scheme in a dense femtocell environment. [7] proposed a joint power control and resource allocation algorithm in an orthogonal frequency division multiplexing (OFDM) femtocell network, where femtceolls are grouped into disjoint clusters.

In this paper, we try to maximize the sum throughput of all FUs and control the interference to the MU under its bearable threshold. Our general formulation leads to a computationally intractable problem, which is NP-hard. Therefore, it is divided into two procudures, the clustering and resource allocation. In the clustering, two femtocells which have strong interference with each other are grouped into clusters. And the femtocells in the same cluster use different subchannels to mitigate intra-tier interference. Then in each cluster, one femtocell is selected as the cluster center (CC) to perform subchannel and power allocation in this cluster. We propose a two-step method to address the resource allocation problem: subchannel allocation and power distribution. The subchannel allocation procedure can roughly satisfy the rate requirements of all FUs and the power allocation algorithm can achieve a near optimal solution. Numerical results validate the effectiveness and efficiency of our proposal.

The rest of this paper is organized as follows. In Section II, we illustrate system model and formulate an optimization task. Section III discuss the clustering subproblem, together with the proposed low-complexity algorithm to obtain the best cluster configuration. In Section IV, we propose a suboptimal subchannel allocation algorithm and achieve an optimal power allocation scheme by developing an efficient fast method. Numerical results are given in Section V with discussions. Conclusion and future work are presented in Section VI.

II. System Model and Problem Formulation

Consider a two-tier heterogeneous network with densely deployed femtocells operating within a macrocell. For simplification, we only consider the case of each femtocell with one FU and one macrocell with an MU. All users, i.e., the FUs and the MU exist indoor. Denote the set of femtocells by $\mathcal{F}$ with $F = |\mathcal{F}|$. We define that an FU belongs to femtocell f is k_f and the MU belongs to the macrocell is k_0. The bandwidth is divided into N OFDM subchannels in the cellular network. Denote $g^n_{k_i,j}$ by the channel gain between FU k_i and FAP j on the nth subchannel and we assume perfect channel state information (CSI) is available at the transceivers of the MUs and the FUs.

In the indoor area where femtocells are densely deployed, the FU k_i and its serving FAP i are very close, so the channel gain between femtocell j and FU k_i is approximated to the channel gain between the two femtocells, i.e., $g^n_{k_i,j} \approx g^n_{i,j}$. The k_fth FU has a minimal rate requirement of $R_{k_f,min}$. The total available bandwidth of the system is W. The interference to the MU introduced by FAP f on the nth subchannel with unit transmission power is I^f_n.

Define the signal-to-interference plus noise power ratio (SINR) of the k_fth FU in a macrocell on the nth subchannel is

$$H_{k_f,n} = \frac{|c_{k_f,n}|^2}{\Gamma(N_0 W/N + I_{k_f,n})}, \tag{1}$$

where $c_{k_f,n}$ is the channel gain of the k_fth FU over subchannel n, N_0 is the PSD of additive white Gaussian noise, Γ is the SNR gap and can be represented as $\Gamma = -\frac{\ln(5BER)}{1.5}$ for an uncoded multiple quadrature amplitude modulation (MQAM) with a specified bit error rate (BER). The interference caused by the MU's signal is $I_{k_f,n}$, which can be regarded as noise. And the transmission rate of the k_fth FU on the nth subchannel is

$$r_{k_f,n} = \log_2(1 + p_{k_f,n} H_{k_f,n}), \tag{2}$$

where $p_{k_f,n}$ is the k_fth FU's transmission power on the nth subchannel.

To reduce intra-tier interference, the femtocells can be divided into disjunct clusters. Denote the set of clusters as $\mathcal{C}$. A femtocell cluster $c_m \subseteq \mathcal{F}, \forall m \in 1, 2, ..., |\mathcal{C}|$, $\cup^{|\mathcal{C}|}_{m=1} c_m = \mathcal{F}$, and $\cap^{|\mathcal{C}|}_{m=1} c_m = \emptyset$. Attention that every cluster can use the entire set of subchannels $\mathcal{N}$ and no two femtocells in the same cluster transmit on the same subchannel in the meantime. In other words, there is no intra-tier interference within a cluster. As femtocells which have low interference with each other are grouped into different clusters, they can use the same subchannel for transmission.

Our target is to maximize the sum rate of the FUs under the transmit power limitation and the MU's interference constraint while reducing the intra-tier interference, which leads to the following optimization problem:

$$\begin{aligned}
&\max_{c_m, p_{k_f,n}, \rho_{k_f,n}} \sum_{m=1}^{|C|} \sum_{f\in c_m} \sum_{n=1}^{N} \rho_{k_f,n} r_{k_f,n} \\
s.t.\quad & \mathrm{C1}: \textstyle\sum_{n=1}^{N} \rho_{k_f,n} r_{k_f,n} \geq R_{k_f,min}, \forall k_f, \\
& \mathrm{C2}: \textstyle\sum_{n=1}^{N} \rho_{k_f,n} p_{k_f,n} \leq P_t, \forall f, \\
& \mathrm{C3}: \textstyle\sum_{f\in c_m} \sum_{n=1}^{N} \rho_{k_f,n} p_{k_f,n} I^f_n \leq I_{th}, \forall m, \\
& \mathrm{C4}: \textstyle\sum_{f\in c_m} \rho_{k_f,n} = 1, \forall n, m, \\
& \mathrm{C5}: \cup^{|\mathcal{C}|}_{m=1} c_m = \mathcal{F}, \mathrm{C6}: \cap^{|\mathcal{C}|}_{m=1} c_m = \emptyset, \\
& \mathrm{C7}: |c_m| \leq S, \forall m, \\
& \mathrm{C8}: p_{k_f,n} \geq 0, \forall k_f, n, \\
& \mathrm{C9}: \rho_{k_f,n} \in \{0, 1\}, \forall k_f, n,
\end{aligned} \tag{3}$$

where $R_{k_f,min}$ is the minimal rate requirement of the k_fth FU. $\rho_{k_f,n}$ can only be either 1 or 0, indicating whether the nth subchannel is used by the k_fth FU or not, P_t is the power limit of each femtocell and I_{th} is the interference power threshold of the MU. C1 is the throughput requirements of the FUs. C2 is the power limitation and C3 is the interference constraint, which enforces that the sum interference power at the MU in every cluster stays below I_{th}. C4 is the exclusion constraint that in cluster c_m, subchannel n can only be occupied by one femtocell. C5 and C6 indicate that the entire set of clusters $\mathcal{C}$ form the femtocell set $\mathcal{F}$ and the set of clusters are disjoint. C7 limits the maximum cluster size to S. C8 and C9 are intuitive.

III. Efficient Clustering Algorithm

Note that (3) defines a computationally intractable problem that involves variables c_m's, binary variables $\rho_{k_f,n}$'s and real variables $p_{k_f,n}$'s, which is NP-hard. We address it by dividing the original optimization task into two procedures: clustering, and subchannel and power allocation.

First, we propose an efficient clustering scheme to group the femtocells into clusters based on interference degree. Femtocells which have high interference degree with each other are grouped into the same cluster and in each cluster, no two femtocells transmit on the same subchannel. As femtocells which have low interference with each other are grouped into different clusters, they can use the same subchannel for transmission. In practice, femtocell density changes all times, so some clustering algorithms based on a given number of clusters are impractical. Our proposed scheme can change the cluster size and cluster number as the femtocell density differs, which is of practical merit.

To acquire the clustering formation, we give an undirected graph $G = (V, E)$, where V is the set of vertices which represents femtocells and $(i, j) \in E$ is the set of edges between two vertices. Every edge (i, j) is given a non-negative weight $w_{i,j}$, which represents the interference degree between femtocell i and femtocell j. In the scene of femtocell networks, femtocell i and femtocell j have high $w_{i,j}$ if they have strong interference with each other. In fact, the two femtocells which have high channel gain $g_{i,j}^n$ between them will severely interfer with each other. Then, the weight $w_{i,j}$ is made in directly proportion to the channel gain between the two femtocell i, j by setting $w_{i,j} = g_{i,j}^n$.

The procedure is described in details. The procedure initializes by setting up the femtocell interfering graph. Based on this graph, the femtocell gateway firstly selects arbitrary initial CCs, $z_1, z_2, ..., z_{N_c}$, where N_c is the number of clusters. After all CCs are determined, the rest femtocells are then attached to the nearest CC and act as cluster members (CMs). A femtocell x belongs to the ith CC when $w_{x,i} > w_{x,j}, \forall j \neq i$, where $w_{x,i}$ is the interference degree between femtocell x and CC i while $w_{x,j}$ is the interference degree between femtocell x and CC j. We define $\mathcal{W}_f$ as the position of femtocell f in interference graph. When all femtocells are classified into clusters, we update the CCs by

$$z_j = \frac{1}{|c_j|} \sum_{f \in c_j} \mathcal{W}_f, j = 1, 2, ..., N_c. \tag{4}$$

Then the average interference degree $\overline{d_{z_j}}$ between CMs and CC in cluster $\mathcal{C}_j$ is $\frac{1}{|\mathcal{C}_j|} \sum_{f \in \mathcal{C}_j} w_{f,z_j}$ and the average interference degree of the femtocell network $\overline{D}$ is calculated by $\frac{1}{N_c} \sum_{j=1}^{N_c} \overline{d_{z_j}}$. If the number of femtocells in a cluster is less than our expected minimal cluster number, our clustering algorithm finds cluster with largest variance in which the interference level differs roughly and splits the cluster into two clusters. The number of clusters increases by one. And the variance in a cluster is calculated by $\sigma_j = \frac{1}{|c_j|} \sum_{f \in c_j} (w_f - \overline{w_{z_j}})^2$. The splitting process continues until minimal cluster number satisfied. Nevertheless, if the interference level between two CCs exceeds minimal interference level, which means the interference between the two clusters is large, then this two clusters should merge together. This process repeats until the stopping criteria is met.

Our clustering algorithm is described in details in Table I. Denote E as the expected minimal cluster number, θ_N as the minimal cluster size and θ_c as the allowed maximal interference degree between two CCs. L is the maximal iterations. If $\Delta \overline{D} < \theta_D$, the clustering algorithm converges and the clustering formation is done.

TABLE I: Efficient Clustering Algorithm

Algorithm: Clustering Algorithm for Femtocells

1: **Input**: $\mathcal{W} = [\mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_F], z1, ..., z_{N_c}, E, L, \theta_N, \theta_c, \theta_D, l$
2: **While** $l < L$ and $\Delta \overline{D} > \theta_D$
3: Map femtocells into clusters
4: If $|c_m| < \theta_N, \forall m$, cancel this cluster, Nc=Nc-1, go to step 3
5: Update new CCs according to (4)
6: For each c_m, update average interference degree $\overline{d_{z_m}}$
7: Update average interference level for femtocell network $\overline{D}$
8: **While** $N_c < E/2$
9: Calculate variance of each cluster $\sigma_m, \forall m$
10: Find σ_m^* satisfies $\sigma_m^* > \sigma_m, \forall m$
11: c_m splits into two clusters with CCs z_m^+ and z_m^-
12: **End while**
13: If there exists z_i and z_j, $i \neq j$ that $w_{z_i,z_j} > \theta_c$, combine cluster c_i and cluster c_j
14: $l = l + 1$
15: **End while**
16: **Return**: Femtocell clusters $c_1, c_2, ..., c_{Nc}$

IV. Subchannel and Power Allocation

After getting the cluster configuration, the femtocell gateway sends these configurations in sequence to the femtocells through the wired backhaul. In each cluster, the CC will take charge with the subchannel and power allocation for all CMs in this cluster [4, 5]. We try to maximize the sum capacity of all femtocells within each cluster, under minimal rate requirements for all FUs and the interference constraint for the MU. Therefore, we can formulate the RA problem in the cluster m for CC to solve:

$$\begin{aligned}
&\max_{p_{k_f,n}, \rho_{k_f,n}} \sum_{f \in c_m} \sum_{n=1}^{N} \rho_{k_f,n} r_{k_f,n} \\
s.t. \quad & \mathrm{C1}: \sum_{n=1}^{N} \rho_{k_f,n} r_{k_f,n} \geq R_{k_f,min}, k_f = 1, ..., |c_m|, \\
& \mathrm{C2}: \sum_{n=1}^{N} \rho_{k_f,n} p_{k_f,n} \leq P_t, \forall f, \\
& \mathrm{C3}: \sum_{f \in c_m} \sum_{n=1}^{N} \rho_{k_f,n} p_{k_f,n} I_n^f \leq I_{th}, \\
& \mathrm{C4}: \sum_{k_f=1}^{|c_m|} \rho_{k_f,n} = 1, \forall n, \\
& \mathrm{C5}: p_{k_f,n} \geq 0, \forall k_f, n, \\
& \mathrm{C6}: \rho_{k_f,n} \in \{0, 1\}, \forall k_f, n.
\end{aligned} \tag{5}$$

A. Subchannel Allocation

We propose a suboptimal approach to allocate subchannels to the FUs. In a femtocell network, the subchannel with high SNR for an FU may also bring more interference to the MU that uses this subchannel. In other words, the traditional water-filling-like method [8] is not appropriate because interference constraint also lays an upper bound of transmit power for each subchannel. That is to say, the interference introduced to the MU and the SINR of a subchannel should be jointly considered to calculate the rate of the subchannel. Our method measures the achievable rate of the nth subchannel used by the k_fth FU as follows,

$$r_{k_f,n}^{max} = \log_2(1 + p_{k_f,n}^{max} H_{k_f,n}), \tag{6}$$

where $p_{k_f,n}^{max}$ is the maximum achievable power for the k_fth FU on the nth subchannel,

$$p_{k_f,n}^{max} = \min(P_t, I_{th}/I_n^f). \tag{7}$$

Denote Ω_{k_f} as the subchannel set occupied by the k_fth FU. We allocate the FUs subchannels to meet their minimal

TABLE II: Subchannel Allocation

Algorithm: Subchannel Allocation Algorithm for the Cluster m
1: **Initialization**:
2: $\mathcal{N}_t = \mathcal{N}, \Omega_{k_f} = \emptyset, \forall k_f$
3: Set the FMS's rates to zero: $R_{k_f} = 0$ for any $1 \leq k_f \leq \lvert c_m \rvert$
4: **For FUs**:
5: While $\mathcal{N}_t \neq \emptyset$ and $R_{k_f} < R_{k_f,min}$ for any $1 \leq k_f \leq \lvert c_m \rvert$
6: Find k_f^* satisfies $R_{k_f^*} - R_{k_f^*,min} \leq R_{k_f} - R_{k_f,min}$
7: For k_f^*, find n^* satisfies $r_{k_f^*,n^*} \geq r_{k_f^*,n}, \forall n$
8: Update $R_{k_f^*} = R_{k_f^*} + \log_2(1 + p_{k_f^*,n^*} H_{k_f^*,n^*})$
9: Update $\Omega_{k_f^*} = \Omega_{k_f^*} \bigcup n^*, \mathcal{N}_t = \mathcal{N}_t \setminus n^*$
10: **endwhile**

TABLE III: Simulation Parameters

System Parameters	Radius of Macro-network	500 m (LTE-A)
	Radius of the femtocell	20 m
	Carrier Frequency	2 GHz
	Total Bandwidth	10 MHz
	Thermal Noise PSD	-174 dBm/Hz
Shadowing	Shadow Fading	Log-normal
Macrocell Parameters	Transmit Power	46 dBm
	Antenna Gain	14 dBi
	Noise Figure	7 dB
Femtocell Parameters	Transmit Power	20 dBm
	Noise Figure	7 dB
M(F)U Parameters	Anternna Gain	0 dBi
	Noise Figure	7dB

rate requirements. The principle of our subchannel allocation algorithm for the FUs is that the FU whose current rate is the farthest away from the target one has the priority to get a subchannel among the available ones. The operational procedure of the proposed algorithm for the cluster m is described in Table II.

B. Fast Barrier Method for Power Allocation

After subchannel allocation, the power allocation problem in the cluster m can be rewritten as

$$\begin{aligned} &\max_{p_{k_f,n}} \textstyle\sum_{f\in c_m} \sum_{n\in\Omega_{k_f}} r_{k_f,n} \\ s.t.\quad & \mathrm{C1}: \textstyle\sum_{n\in\Omega_{k_f}} r_{k_f,n} \geq R_{k_f,min}, k_f = 1,...,\lvert c_m\rvert, \\ & \mathrm{C2}: \textstyle\sum_{n\in\Omega_{k_f}} p_{k_f,n} \leq P_t, \forall f, \\ & \mathrm{C3}: \textstyle\sum_{f\in c_m}\sum_{n=1}^{N} \rho_{k_f,n} p_{k_f,n} I_n^f \leq I_{th}, \\ & \mathrm{C4}: p_{k_f,n} \geq 0, \forall k_f, n. \end{aligned} \tag{8}$$

(8) defines a convex optimization problem and can be solved by barrier method [9]. Collect all $p_{k_f,n}$'s into one vector $\mathbf{x}$, the logarithmic barrier function is

$$\begin{aligned} \phi(\mathbf{x}) = \; & -\textstyle\sum_{k_f=1}^{\lvert c_m\rvert} \ln(\sum_{n\in\Omega_{k_f}} r_{k_f,n} - R_{k_f,min}) \\ & -\textstyle\sum_{f\in c_m} \ln(P_t - \sum_{n\in\Omega_{k_f}} p_{k_f,n}) \\ & -\ln(I_{th} - \textstyle\sum_{f\in c_m}\sum_{n=1}^{N} \rho_{k_f,n} p_{k_f,n} I_n^f) \\ & -\textstyle\sum_{k_f=1}^{\lvert c_m\rvert} \sum_{n\in\Omega_{k_f}} \ln p_{k_f,n}. \end{aligned} \tag{9}$$

Note that the subscript k_f can be omitted as it has been determined by subchannel allocation. Denote

$$f(\mathbf{x}) = \textstyle\sum_{k_f=1}^{\lvert c_m\rvert} R_{k_f}, \tag{10}$$

where $R_{k_f} = \sum_{n\in\Omega_{k_f}} r_{k_f,n}$, the optimal solution to (8) can be approximated by solving the following unconstrained minimization problem

$$\min \psi_t(\mathbf{x}) = -tf(\mathbf{x}) + \phi(\mathbf{x}), \tag{11}$$

where $t \geq 0$ is a parameter to control the accuracy of solution. Newton method can efficiently solve this unconstrained minimization problem [9]. Newton step at $\mathbf{x}$, denoted by $\Delta\mathbf{x}_{nt}$, is given by

$$\nabla^2\psi_t(\mathbf{x})\Delta\mathbf{x}_{nt} = -\nabla\psi_t(\mathbf{x}), \tag{12}$$

where $\nabla\psi_t(\mathbf{x})$ and $\nabla^2\psi_t(\mathbf{x})$ are the gradient and the Hessian of $\psi_t(\mathbf{x})$, respectively. The computational complexity of the barrier method mainly lies in the computation of Newton step that needs matrix inversion. In order to reduce the computational cost, we exploit the structure of (8) and develop a fast algorithm to calculate the Newton step with lower complexity. Denote

$$\begin{aligned} & s_f = P_t - \textstyle\sum_{n\in\Omega_{k_f}} p_{k_f,n}, f = 1,...,\lvert c_m\rvert, \\ & f_{k_f} = \textstyle\sum_{n\in\Omega_{k_f}} r_{k_f,n} - R_{k_f,min}, k_f = 1,...,\lvert c_m\rvert, \\ & g_0 = I_{th} - \textstyle\sum_{f\in c_m}\sum_{n=1}^{N} \rho_{k_f,n} p_{k_f,n} I_n^f. \end{aligned} \tag{13}$$

The Hessian of $\psi_t(\mathbf{x})$ is

$$\begin{aligned} \nabla^2\psi_t(\mathbf{x}) &= \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \ddots & \\ & & & D_N \end{bmatrix} + \frac{\nabla g_0 \nabla g_0^T}{g_0^2} \\ & + \textstyle\sum_{f=1}^{\lvert c_m\rvert} \frac{\nabla s_f \nabla s_f^T}{s_f^2} + \sum_{k_f=1}^{\lvert c_m\rvert} \frac{\nabla f_{k_f} \nabla f_{k_f}^T}{f_{k_f}^2} \\ &= D + \sum_{i=1}^{M} F_i F_i^T. \end{aligned} \tag{14}$$

where $D = \mathrm{diag}(D_1, D_2, \cdots, D_N)$ and $M = 2\cdot\lvert c_m\rvert + 1$ with

$$D_n = (t + \frac{1}{f_{k_f}})\frac{H_{k_f,n}^2}{(1 + p_{k_f,n}H_{k_f,n})^2} + \frac{1}{p_{k_f,n}^2}. \tag{15}$$

F_i are all vectors with N elements,

$$F_i = \begin{cases} \frac{\nabla s_f}{s_f}, & f = 1,\cdots,\lvert c_m\rvert, i = f, \\ \frac{\nabla f_{k_f}}{f_{k_f}}, & k_f = 1,\cdots,\lvert c_m\rvert, i = k_f + \lvert c_m\rvert, \\ \frac{\nabla g_0}{g_0}, & i = 2\cdot\lvert c_m\rvert + 1. \end{cases} \tag{16}$$

Theorem 1: The problem defined in (8) can be solved with the complexity of $O(M^2N)$. We give the proof in detail in Appendix. If we solve (8) via standard convex optimization technique, it has a complexity of $O(N^3)$. In practical wireless systems, $M \ll N$ and our proposed algorithm has a significant advantage to solve the RA problem that can be tackled in an online manner.

V. Numerical Results and Discussions

Consider an LTE-advanced network where a macrocell is in the center of a circle with radius of 500 m. Each FU is uniformly distributed within a circle with radius of 20 m from the pairing FAP. We consider an indoor area with densely deployed femtocells within the coverage of the macrocell. A dual-stripe building model, which was initially proposed in [10], is adopted to evaluate the performance of our algorithm. The simulation parameters are listed in Table III.

The distance dependent path loss attenuation varies according to the characteristics of the evaluated link. We give a summary of the different situations in our simulations.

- Macrocell to MU

$$PL(d) = 15.3 + 37.6\log_{10}(d) + L_{ow},$$

where d (in m) is the distance between the macrocell to the indoor MU/FU and L_{ow} is the penetration loss in the external walls of the building.

- Femtocell to FU

$$\begin{aligned} PL(d) = \; & 38.46 + 20\log_{10}(d) + 0.7d_{2D} \\ & + qL_{iw} + 18.3n^{(\frac{n+2}{n+1}-0.46)}, \end{aligned} \tag{17}$$

where d is the distance between the femtocell to the FU, d_{2D} is the indoor distance of the link, L_{iw} is the penetration loss in the internal walls of the building, $q(n)$ denotes the number of penetrated walls (floors).

Shadow fading is modeled as a log-normal random variable, whose standard deviation is 4 dB and 8 dB for the MU and the FUs respectively. The parameters of the clustering algorithm are as follows: The maximal iterations L for clustering is set by 20 and the maximal interference degree between two CCs is 10^{-10} W.

Fig. 1 shows the average capacity of femtocells as a function of power limit achieved by our proposed algorithm with other two algorithms: equal power allocation (EPA) algorithm and IFPA [11] based on the same clustering and subchannel allocation methods proposed in above. EPA assumes that power is equally allocated among all subchannels and IFPA allocates power inversely proportional to the interference level. From Fig. 1 we can see that the average capacity of the FUs grows with the increase of the power budget. Our proposed algorithm performs better than the other algorithms. When power budget grows larger, our algorithm performs much better than the EPA and IFPA.

We also study the average capacity of femtocell networks in various femtocell densities in Fig. 2. We compare the performance of our proposed clustering algorithm with K-means algorithm. K-means algorithm is introduced in [12], which executes clustering based on a given cluster size and cluster number. Both two algorithms have a complexity of $O(K_f)$, which K_f is the number of all FUs. Both algorithms decrease as the femtocells density increases. However, the capacity in our proposed algorithm is higher than the K-means algorithm. This is mainly because that in the K-means algorithm the cluster size and the number of clusters are predefined, which is not fit for different femtocell intensively. Our algorithm dynamically change the cluster size and cluster number as the femtocell density change.

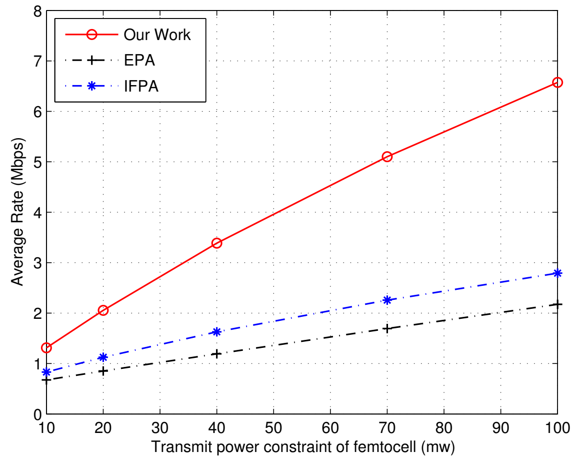

Fig. 1: Average capacity as a function of transmit power limit.

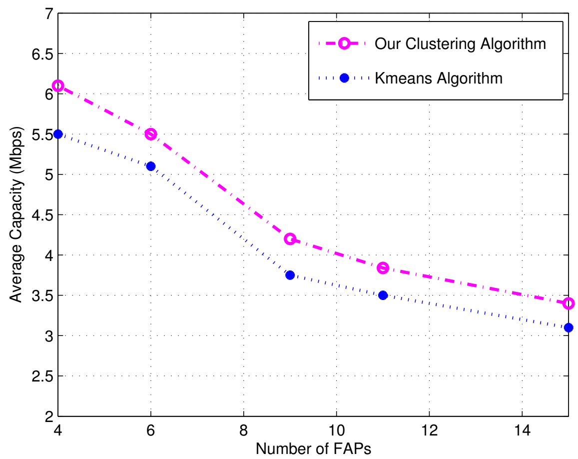

Fig. 2: Average capacity vs the number of FAPs.

Finally, we investigate the convergence of our proposed fast algorithm. As discussed above, the computational load of the proposed algorithm mainly lies in the computation of Newton step. Fig. 3 gives the cumulative distribution function (CDF) of the number of Newton iterations for solving the optimal power allocation with different number of N. As seen in Fig. 3, the number of Newton iterations is not large and varies in a narrow range, indicating our proposed algorithm is efficient.

VI. Conclusion

In this paper, we studied the RA and interference management problem in dense OFDM femtocell networks. Our general formulation leads to a mixed integer programming problem which is computationally complex. We divided the problem into two tractable subproblems: clustering, subchannel and power allocation, which are solved by efficient algorithms. Numerical simulations validate the effectiveness and efficiency of our proposal.

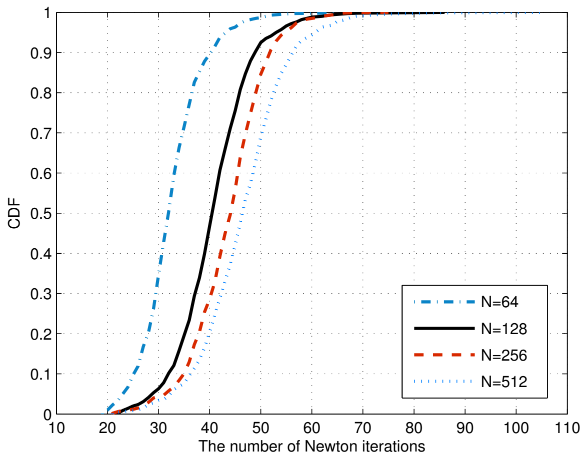

Fig. 3: The Empirical CDF of number of Newton iterations.

APPENDIX

A. Proof of Theorem 1

Rewrite the KKT system (12) as follows,

$$\Lambda_0 \Delta \mathbf{x} = F_0, \quad (18)$$

where $\Lambda_0 = \nabla^2 \psi_t$ and $F_0 = -\nabla \psi_t$. According to the (14), Λ_0 can be written as

$$\Lambda_0 = D + \sum_{i=1}^{M} F_i F_i^T, \quad (19)$$

which can be decomposed into M equations,

$$\Lambda_i = \Lambda_{i+1} + F_{i+1} F_{i+1}^T, i = 0, 1, \cdots, M-1. \quad (20)$$

By exploiting the structure of Λ_i's, we give an M-step procedure to compute Newton step efficiently.

First, use (20) to decompose Λ_0, that is, $\Lambda_0 = \Lambda_1 + F_1 F_1^T$. Denote two intermediate variables as the solutions of the following linear equations: $\Lambda_1 v_1^1 = F_0$ and $\Lambda_1 v_2^1 = F_1$. Then $\Delta \mathbf{x}$ can be obtained by $\Delta \mathbf{x} = v_1^1 - \frac{F_1 v_1^1}{1+F_1 v_2^1} v_2^1$. And we can figure out $\Delta \mathbf{x}$ if obtaining the two new variables v_1^1 and v_2^1.

Continue the procedure, decompose Λ_1 with $\Lambda_1 = \Lambda_2 + F_2 F_2^T$. Then the two variables introduced in step 1 can be updated by solving the following three sets of linear equations, $\Lambda_2 v_i^2 = F_{i-1}$,$i = 1, 2, 3$, where v_1^2, v_2^2 and v_3^2 are three new intermediate variables.

For the mth step, decompose Λ_{m-1} with $\Lambda_m = \Lambda_m + F_m F_m^T$. We can update the m variables introduced in step $m-1$ by $v_i^{m-1} = v_i^m - \frac{F_m^T v_i^m}{1+F_m v_{m+1}^m} v_{m+1}^m$, $i = 1, 2, \cdots, m$, which is obtained by solving the following $m+1$ sets of linear equations, $\Lambda_m v_i^m = F_{i-1}$, $i = 1, 2, \cdots, m+1$.

Continue the procedure to the Mth step, it yields $M+1$ matrix systems $\Lambda_M v_i^M = F_{i-1}$, $i = 1, 2, \cdots, M+1$. From the derivation process, we can find that the m variables v_i^{m-1}, $i = 1, 2, \cdots, m$ in the $(m-1)$th step can be obtained by the $m+1$ variables v_i^m, $i = 1, 2, \cdots, m+1$ in the mth step. Thus, if we figure out the $M+1$ variables v_i^M, $i = 1, 2, \cdots, M+1$, $\Delta \mathbf{x}$ will be indirectly obtained.

Equation $\Lambda_M v_i^M = F_{i-1}$ can be solved as follows: According to the analysis given in Section IV, we have $\Lambda_M = D$. Unify these equations into

$$\begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \ddots & \\ & & & D_N \end{bmatrix} v = g. \quad (21)$$

Since D is a diagonal matrix, we can easily obtained

$$v_i = D_i^{-1} g_i, i = 1, \ldots, N. \quad (22)$$

Thus the computational complexity of solving the $M+1$ matrix systems is $O(MN)$. We also need an M-step reverse iteration to figure out $\Delta \mathbf{x}$. The total computation cost of the proposed method is $O(M^2 N)$.

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