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Cooperative Resource Allocation in OFDM-Based Multicell Cognitive Radio Systems

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***Abstract*—In this paper, we investigate the resource allocation problem for an orthogonal frequency division multiplexing (OFDM) based multicell cognitive radio (CR) system. Secondary users (SUs) served by the CR system distribute randomly in multiple cells and share radio spectrum with primary users (PUs) in a licensed system, where the interference introduced to the PUs must be kept below their tolerable thresholds. In our system model, SUs in different CR cells can transmit signals with the same OFDM subchannel, so cochannel interference among the SUs should also be considered. We propose an efficient algorithm, named as multi-level waterfilling (MLWF), to allocate power among OFDM subchannels for all CR cells by jointly considering transmission power and interference constraints. The MLWF always allocates much power to a subchannel which generates less interference to the PUs. Simulation results show that our proposed algorithm provides better performance than other existing ones. Moreover, the complexity of the MLWF is much lower than other representative algorithms.**

***Index Terms*—Cognitive radio, Cooperative resource allocation, OFDM, Optimization**

I. Introduction

Spectrum scarcity becomes a severe problem as the rapid development of wireless service demand. However, in despite of the looming spectrum shortage crisis, investigations show that radio spectrum is far away from fully utilized [1]. Although radio spectrum has already been assigned to licensed primary users (PUs), part of them is usually unused at a certain time or location. That is to say, spectrum holes exist in both time and location. In order to fulfill the requirements of spectrum-hungry applications, cognitive radio (CR) is brought up [2, 3] and attracts more and more attention in the past ten years [4]. On the other hand, to prohibit the performance degeneration of the PUs, the interference generated by the SUs must be regularly controlled. Hence, the physical layer of CR systems should be very flexible to meet these requirements.

Orthogonal frequency division multiplexing (OFDM) has been considered as an appropriate modulation scheme for CR systems [5], owing to its high flexibility in dynamic resource allocation. A heuristic algorithm called Max-Min is proposed in [6]. Simulation results show its performance is close to the optimal, but the computational cost is relatively high. A simple but fast efficient algorithm is implemented in [7] by introducing a normalized index to measure the ability of a subchannel to carry bits. It achieves performance close to the optimal with a very low computational complexity. However, both of the algorithms in [6] and [7] are only suitable for single cell scenario.

For the single cell scenario, SUs are generally assumed to transmit signals on different subchannels simultaneously. Nevertheless, for multicell case, it is more reasonable to allow that SUs in different cells use the same subchannel simultaneously [8–10]. In [8], a distributed algorithm is proposed to maximize the throughput of the considered CR system while guaranteeing the rates of nominal users. In [9], a fully distributed subchannel selection and power allocation algorithm is proposed by combining an unconstrained optimization method with a constrained partitioning procedure. However, interference introduced to PUs is not considered in [8, 9]. In [10], a greedy-like heuristic method, referred to as multicell Max-Min algorithm, is proposed to solve the resource allocation problem in multicell CR networks. The computational complexity of the proposed algorithm is very high because it has to solve a set of nonlinear equations during each iteration.

In this paper, we focus on the resource allocation in the downlink of a multicell OFDM-based CR system and try to maximize the overall data rate of the SUs. Firstly, we give a greedy-like water-filling procedure to load bits for subchannels. Each CR user greedily loads bits on a subchannel based on its signal to interference plus noise ratio (SINR). The MLWF applies a simple cooperative adjustment procedure after the greedy waterfilling algorithm, which can reduce both cochannel interference and mutual interference. By cooperatively moving bits from the subchannels which generates high interference to the subchannels with lower interference, the MLWF makes an effort to reduce the capacity loss caused by the greedy-like algorithms. Simulation results show our proposed algorithm has a better performance with remarkable lower complexity compared to other existing methods.

The rest of this paper is organized as follows. Section II describes system model and formulates the optimization problem. In III, proposed algorithm is shown in detail. Simulation results and analysis are presented in Section IV. Conclusion is drawn in Section V.

II. System Model and Problem Formulation

A. System Model

Consider that multiple OFDM-based CR cells share spectrum with a licensed system. Each CR cell locates a base

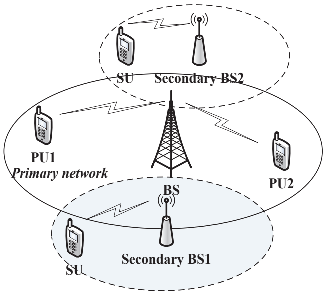

Fig. 1. System model: Coexistence of primary and secondary cells

station (BS) and serves an SU. Fig.1 shows the system model considered in this work. We consider the downlink of the CR system in this work and try to maximize the sum throughput of the SUs served by CR cells. We assume that perfect channel-state information (CSI) is available by channel estimation at the receiver side, and the CSI is also available at each CR cell's transmission side by feedback. Perfect CSI between the secondary BSs and a specified PU is also available by searching the database or sensing the channel.

The primary BS transmits to L PUs. The whole available spectrum is divided into N subchannels with equal bandwidth B and the starting frequency is f_s. There are M CR cells and each cell is permitted to employ the N subchannels. Let $\mathcal{N} := \{1, 2, \ldots, N\}$, $\mathcal{L} := \{1, 2, \ldots, L\}$, $\mathcal{M} := \{1, 2, \ldots, M\}$ denote the set of subchannels, PUs and CR cells, respectively. The nominal spectrum of subchannel n, $\forall n \in \mathcal{N}$, ranges from $f_s+(n-1)B$ to f_s+nB. The PU l's nominal band is supposed to span from f_l^{PU} to $f_l^{PU} + B_l$, where f_l^{PU} is the starting frequency and B_l is the occupied bandwidth of PU l, $\forall l \in \mathcal{L}$.

The power spectrum density (PSD) of the OFDM subchannel n used by an SU can be expressed as

$$\phi_n^{SU}(f) = T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2, \tag{1}$$

where T_s is the symbol duration. The interference introduced to the PU l from CR cell m's BS on subchannel n with unit transmit power is

$$I_{m,l,n}^{SP} = \int_{f_l^{PU}-f_s-(n-\frac{1}{2})B}^{f_l^{PU}-f_s-(n-\frac{1}{2})B+B_l} g_{m,l,n}^{SP} \phi_n^{SU}(f) df, \tag{2}$$

where $g_{m,l,n}^{SP}$ is the power gain from the CR cell m's BS to the PU l's receiver on subchannel n.

On the other hand, assume the primary BS casts unit power on subchannel n, the interference introduced to the SU in CR cell m on subchannel n is

$$I_{l,m,n}^{PS} = \int_{f_s+(n-1)B-f_l^{PU}-\frac{1}{2}B_l}^{f_s+nB-f_l^{PU}-\frac{1}{2}B_l} g_{n,m}^{PS} \phi_l^{PU}(f) df, \tag{3}$$

where $g_{n,m}^{PS}$ is the power gain on subchannel n from primary BS to the SU served in CR cell m. $\phi_l^{PU}(f)$ is the PSD of PU l's signal.

Since the primary network and CR network coexist in system, there are two kinds of interference introduced to the SUs. First, cochannel interference between CR cells arises when multiple SU in different CR cells use the same subchannel. That is to say, the signal of one CR cell is treated as interference by other CR cells transmitting on the same subchannel. On the other hand, the PUs also generate interference to the subchannels. Let $I_{m,n}^{SS}$ denote the interference to the SU of the mth CR cell on the nth subchannel introduced by the other CR cells,

$$I_{m,n}^{SS} = \sum_{j \neq m, \forall j \in \mathcal{M}} p_n^j g_{j,m,n}^{SS}, \tag{4}$$

where p_n^j is the CR cell j's transmit power on subchannel n and $g_{j,m,n}^{SS}$ is power gain from CR cell j's BS to CR cell m's user.

The SINR of SU served in the CR cell m on subchannel n can be defined as

$$H_{m,n} = g_{m,m,n}^{SS} / (\sigma^2 + I_{m,n}^{PS} + I_{m,n}^{SS}), \tag{5}$$

where σ^2 denotes the additive white gaussian noise variance. And $I_{m,n}^{PS}$ is the total interference from PUs to the subchannel n of SU served in CR cell m. From (3), it can be calculated by

$$I_{m,n}^{PS} = \sum_{l=1}^{L} p_n^l I_{l,m,n}^{PS}, \tag{6}$$

where p_n^l is the PU l's signal power on subchannel n.

The achievable transmission rate on the nth subchannel of the mth CR cell is

$$r_{m,n} = B \log_2 \left(1 + (p_{m,n} H_{m,n})/\Gamma\right), \tag{7}$$

where Γ is the SINR gap.

B. Problem Formulation

The optimization objective is to maximize the sum capacity of the CR cells which operate in a power-limited situation, while keeping the interference to the PUs not exceeding specified thresholds $\left\{I_l^U, l = 1, 2, \ldots, L\right\}$. Thus, the constrained optimization problem can be formulated as follows,

$$\begin{aligned} & \max \textstyle\sum_{m=1}^{M} \sum_{n=1}^{N} r_{m,n} \\ s.t. \quad & C_1 : \textstyle\sum_{n=1}^{N} p_{m,n} \leq P_m, p_{m,n} \geq 0, \forall m \in \mathcal{M} \\ & C_2 : \textstyle\sum_{m=1}^{M} \sum_{n=1}^{N} p_{m,n} I_{m,l,n}^{SP} \leq I_l^U, \forall l \in \mathcal{L}. \end{aligned} \tag{8}$$

The C_1 is the power constraint of the mth CR cell. P_m is the maximum transmit power. The C_2 are the interference constraints, where the interference threshold of PU l is I_l^U, $\forall l \in \mathcal{L}$.

III. The Proposed Algorithm

The optimization problem formulated in (8) is a nonlinear programming coupling in the constraint C_2 which poses a prohibitive computational burden to the system and is generally very hard to solve. In [11], an iterative waterfilling (IWF) algorithm yields a nash equilibrium if we regard the resource allocation problem as a non-cooperative game. Although the complexity of the IWF algorithm is much less than that of the exhaustive search algorithm and converges fast in various network scenarios, it has sub-optimal performance with greedy criterion.

To address limitations of IWF, many other methods have been proposed. In [10], a heuristic algorithm called multicell Max-Min is proposed to solve the problem formulated in (8). The complexity of the multicell Max-Min algorithm is $\mathcal{O}\left(RM^4N\right)$, where R is the total number of bits allocated to all SUs. Obviously, the computational complexity of the algorithm is too high to be applied in practical CR systems.

We consider the problem formulated in (8) as a non-cooperative game in our previous work [12] and propose an iterative algorithm which decomposes (8) into m independent sub-problem. Using iterative method to update interference level of each CR system makes the interference constrains to be satisfied while maximizing the sum capacity of the CR system. Moreover, in this work, we develop a more efficient algorithm with better performance and lower complexity.

A. MLWF Optimization Model

From (8), we can see that the constraint C_1 is independent for each CR cell while the constraints C_2 is not. Without loss of generality, suppose each SU m on subchannel n has a virtual power spectral mask (PSM) $p^U_{m,n}$ which can keep the the constraint C_2 satisfied in (8). We rewrite the optimization problem as

$$\begin{array}{ll} & \max \sum_{m=1}^{M} R_m \\ s.t. & C_1 : \sum_{n=1}^{N} p_{m,n} \leq P_m, \forall m \in \mathcal{M} \\ & C_2 : 0 \leq p_{m,n} \leq p^U_{m,n}, \forall m \in \mathcal{M}, \forall n \in \mathcal{N} \end{array} \tag{9}$$

where $R_m = \sum_{n=1}^{N} r_{m,n}$ is the sum rate of cell m. To meet the interference constraint C_2 in (8), we have

$$\sum_{m=1}^{M}\sum_{n=1}^{N} p^U_{m,n} I^{SP}_{m,l,n} \leq I^U_l, \forall l \in \mathcal{L} \tag{10}$$

We firstly solve the optimization problem (9) and then use MLWF algorithm to ensure (10) by a power (bits) adjustment procedure. We propose to use Lagrangian multipliers to model the problem (9).

Define the Lagrangian

$$\begin{aligned} L\left(\{p_{m,n}\}\right) &= \sum_{m=1}^{M}\left\{\sum_{n=1}^{N} r_{m,n} - \lambda_m\left(\sum_{n=1}^{N} p_{m,n} - P_m\right)\right. \\ &\left. - \sum_{n=1}^{N}\mu_{m,n}(p_{m,n} - p^U_{m,n}) + \sum_{n=1}^{N}\nu_{m,n} p_{m,n}\right\} \end{aligned} \tag{11}$$

where $\{\lambda_m\}_{m=1,\ldots,M}$,and $\{\mu_{m,n},\nu_{m,n}\}_{m=1,\ldots,M,n=1,\ldots,N}$ are nonnegative Lagrangian multipliers. Based on Karush-Kuhn-Tucker (KKT) conditions, we obtain the following results.

The transmit power allocated to subchannel n for user m is given by

$$p_{m,n} = \left(1/(\lambda_m + \mu_{m,n}) - 1/(\gamma_{m,n})\right)^+, \tag{12}$$

where $(*)^+$ denotes $\max\{*,0\}$, and $\gamma_{m,n} = \Gamma_m/H_{m,n}$. Here, dual variables λ_m and $\mu_{m,n}$ control the waterfilling level of SU m on subchannel n. Since $\mu_{m,n}$ changes with n, each SU m has many WF levels.

Generally, to account for the transmit power and the PSM constraints C_2 in (9), following iterations based on sub-gradient search may be implemented

$$\lambda^{k+1}_m = \lambda^k_m - \beta_m(P_m - \sum_{n=1}^{N} p_{m,n}) \tag{13}$$

$$\mu^{k+1}_{m,n} = \mu^k_{m,n} - \beta_m\left(p^U_{m,n} - p_{m,n}\right) \tag{14}$$

where $0 \leq \beta_m \leq 1$ is a gradient search step size, and λ^k_m and $\mu^k_{m,n}$ denote the values of λ_m and $\mu_{m,n}$ at the kth step, respectively. Moreover, this approach converges in theory when the dual update stepsize β_m is small enough [13]. However, gradient-based search method has a very slow convergence due to the large number of dual variables involved. To overcome this problem, we design a multicell cooperative MLWF algorithm which can achieve a fast and stable convergence.

B. Multicell MLWF Algorithm

We adopt a cooperative manner to allocate the power among all subchannels. First, we use the waterfilling algorithm to allocate the total power P_m in each cell while keeping $p_{m,n} \leq p^U_{m,n}$. If the solution already satisfies the constraints (10), we can directly obtain the power allocation for problem (8). Otherwise, we cooperatively adjust the power (bits) to mitigate the interference introduced by SUs to PUs.

Similar to the Levin-Campello loading [14], the MLWF algorithm also maintains an incremental energy table for discrete power (bits) adjustment procedure. With η as the granularity of the discrete bits ($\eta = 1$ for integer bits or $\eta = 0.5$ for a complex channel quadrature amplitude modulation signal). Assume user m's current bit distribution is $r_{m,n}$. The energy required to maintain the bit distribution on subchannel n of user m is calculated as [15]

$$\varepsilon^n_m(r_{m,n}) = 2(\Gamma_m/H_{m,n})(2^{r_{m,n}} - 1). \tag{15}$$

The incremental energy e^n_m to load an additional η bit on subchannel n of user m is thus

$$e^n_m(r_{m,n}) = (\Gamma_m/H_{m,n})2^{r_{m,n}+1}(2^\eta - 1). \tag{16}$$

The pseudo-code of the multicell cooperative MLWF algorithm is in Table I. The third step of the algorithm is the fixed-margin version of the greedy bit-loading procedures by exploiting waterfilling. The fifth step performs the multicell cooperative power (bits) adjustment procedure to reduce the mutual interference from SUs to PUs. The key idea of this step is to move the power (bits) for one subchannel with high interference to PUs to the other one which generates lower interference to PUs until the power allocation scheme satisfied the interference constraint (10). Obviously, the main computational loads lie on the greedy power allocation procedure and multi-level cooperative power (bits) adjustment procedure.

TABLE I
THE PSEUDO-CODE OF MLWF ALGORITHM

Algorithm: Multicell Cooperative MLWF Algorithm

Step 1:Initialize

Set $r_{m,n} \leftarrow 0$, $\varepsilon_m^n(r_{m,n}) \leftarrow 0$, $p_{m,n} \leftarrow 0$, $\forall m \in \mathcal{M}$, $\forall n \in \mathcal{N}$. Given P_m.

Calculate the $I_{m,l,n}^{SP}$,$I_{l,m,n}^{PS}$ and $I_{m,n}^{PS}$ respectively by (2), (3),(4). And compute $e_m^n(r_{m,n}+1)$, $\forall m \in \mathcal{M}$,$\forall n \in \mathcal{N}$.

flag=1;

while(flag==1)

step 2:Calculate the SINR

Calculate the SINR by (5);

step 3:Greedy power allocation procedure

for $m = 1$ to M

  Perform the waterfilling algorithm with P_m constraints and obtain the $p_{m,n}$ and $r_{m,n}$;

end for

step 4:Test the interference constraint

if the constraint (10) is met

  flag=0, break;

end if

step 5: Multi-level Cooperative Power (bits) Adjustment

for $n = 1$ to N

  $g \leftarrow argmax_{(m,p_{m,n} \leq P_m)} \{p_{m,n} I_{m,l,n}\}$, $\forall m \in \mathcal{M}$;

  $h \leftarrow argmin_{(m,p_{m,n} \leq P_m)} \{p_{m,n} I_{m,l,n}\}$, $\forall m \in \mathcal{M}$;

  if $(\varepsilon_h + e_g^n(r_{g,n}) - e_h^n(r_{h,n})) \leq P_h$ &&

    $(\varepsilon_g + e_h^n(r_{h,n}) - e_g^n(r_{g,n})) \leq P_g$

    $p_{g,n} \leftarrow p_{g,n} + e_h^n(r_{h,n}) - e_g^n(r_{g,n})$;

    $p_{h,n} \leftarrow p_{h,n} + e_g^n(r_{h,n}) - e_h^n(r_{h,n})$;

    $r_{g,n} \leftarrow r_{g,n} - 1$, $r_{h,n} \leftarrow r_{h,n} + 1$;

  end if

end for

end while

TABLE II
COMPUTATIONAL COMPLEXITY COMPARISON

Algorithm	Complexity
Exhaustive Search	$\mathcal{O}(e^{MN})$
Multicell Max-Min	$\mathcal{O}(RM^4N)$
Iterative algorithm	$\mathcal{O}(M^2N\log^2 N)$
MLWF	$\mathcal{O}(MNlog^2N)$

C. On the Complexity and Convergence

The MLWF greedy power allocation procedure and the multi-level cooperative power (bits) adjustment, the dominated parts of the whole algorithm, have the same complexity which is equal to $\mathcal{O}(N \log N)$ where N is the number of subchannels. MLWF contains a simple line-search with bisection or diminishing step size and thus adds $\log N$ search steps to each bit adjustment procedure. For M CR cells, the MLWF adds a linear complexity with regard to the M coordinates to be searched. Therefore the total complexity of the MLWF algorithm is $\mathcal{O}(MNlog^2N)$. Table II compares the complexity of the MLWF algorithm with other resource allocation algorithms. R is the total bits allocated to all SUs. From Table II, we can observe the complexity of the MLWF algorithm is the lowest compared to the others.

We analyze the convergence of the MLWF briefly. In theory, if we take an iterative method using the (13) and (14), the algorithm converges slowly. In practical, the MLWF can approach to solution more closely after each adjustment. Since the solution can be always improved to maintain the interference constraints by reducing the mutual interference in each adjustment procedure, the solution can be found after certain number of adjustments.

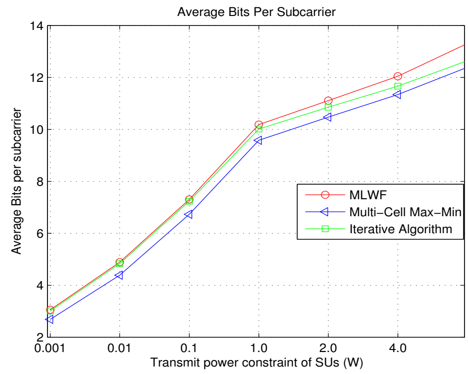

Fig. 2. Average bits per subcarrier as a function of power limit

IV. SIMULATION AND DISCUSSION

We conduct a series of experiments to assess the performance of our proposed multicell multi-level waterfilling algorithm. Consider an OFDM-based multicell CR system where a primary cell with two PUs and three CR cells coexist. All users are located within a 3×3 km areas. The primary base station is in the middle of the area and the secondary base stations are randomly distributed. Each PU or SU is uniformly distributed within a 500-meter circle of its corresponding base station. The path loss exponent is 4, the variance of the shadowing effect is 10 dB, and the multipath fading is assumed to be Rayleigh [16]. There are 16 subchannels. The noise power of each subchannel is set to 10^{-13}W. The frequency bands occupied by PUs are generated randomly with the maximum number of OFDM subchannels is $\frac{2N}{3}$. The transmission power of a PU is equal to the number of OFDM subchannel within the PU's band and the interference threshold of all PUs are set to 5×10^{-13} W. The results presented in this section is obtained from over 1000 Monte Carlo simulations.

To evaluate the performance of the MLWF, we compare the average bits per subchannel of the MLWF with other two schemes: Multicell Max-Min [10] and iterative algorithm [12]. Fig.2 illustrates the average bits per subchannel as a function of the transmit power limit. It can be seen that the capacity of all algorithm increase as the transmit power limit increases. The solution obtained by the MLWF is better than the others, suggesting that cooperative power allocation can achieve more capacity than the greedy-like algorithms.

Fig.3 shows the time complexity of the three algorithms mentioned above. The elapsed time is counted by the inbuilt function tic-toc in *Matlab*. We can see the complexity of the MLWF is much lower than the other algorithms just as the analysis in section III. Moreover, when the transmit power constraint of the SU (PSU) is small, greedy power allocation procedure (step 3 in algorithm) dominates the whole elapsed time because it needs more time to find the water

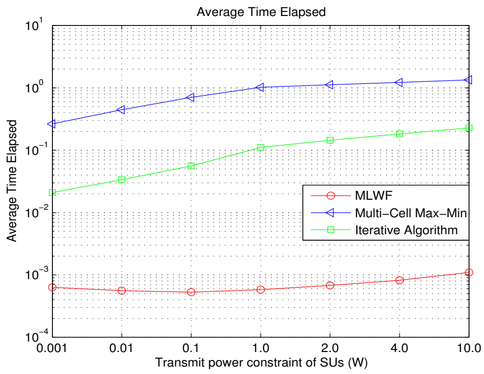

Fig. 3. Average time elapsed as a function of power limit

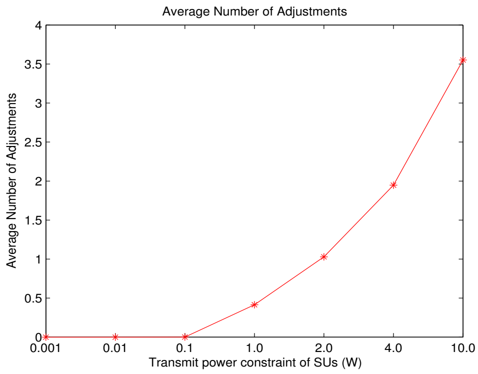

Fig. 4. Average number of adjustments as a function of power limit

level while no power (bits) adjustment is required [see Fig.4]. However, when PSU is large enough, cooperative power (bits) adjustment (step 5 in algorithm) need to be performed many times to meet the interference-temperature constraint resulting more time consumption [see Fig.4 and Fig.5(b)]. Naturally, if the PSU is appropriate, there is no need to require the adjustment procedure [see Fig.4 and Fig.5(a)] and greedy power allocation procedure does less iterations. As analysis above, it is reasonable to see the elapsed time of MLWF is lower at the point of PSU$=0.1$.

Finally, we investigate the convergence of the MLWF. As discussed in section III, one of the main computational loads of the MLWF lies in the number of the adjustment procedure. From Fig.4 and Fig.5, we observe the average number of the adjustment procedure lies in a narrow range $[0, 4]$ and the random instance total adjustment lies in [0,30]. Conservatively, we conclude the MLWF method is effective and efficient according to the complexity and convergence analysis in section III.

V. Conclusion

In this paper we studied the resource allocation problem in a multicell OFDM-based CR system. We try to maximize the sum capacity of the system under transmission power and interference constraints. An efficient and effective multilevel waterfilling (MLWF) algorithm is developed by jointly considering the co-channel and mutual interference. The proposed algorithm achieves better capacity performance with a lower complexity, comparing to other existing algorithms. Besides,

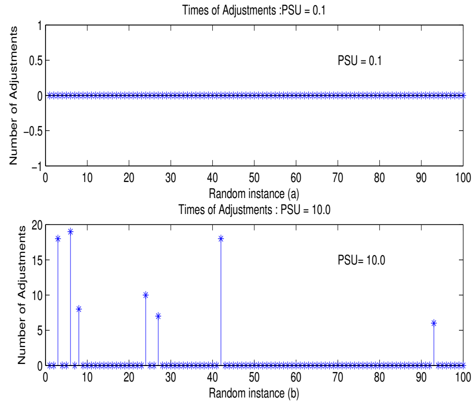

Fig. 5. Adjustments as a function of power limit

our proposed MLWF algorithm always converges fast and stably, which makes it promising for practical applications.

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