www.ietdl.org

Published in IET Communications
Received on 5th May 2013
Revised on 17th November 2013
Accepted on 16th December 2013
doi: 10.1049/iet-com.2013.0379

ISSN 1751-8628

Power allocation for orthogonal frequency division multiplexing-based cognitive radio networks with cooperative relays

Sidan Du[1], Fangjiang Huang[1], Shaowei Wang[1,2]

[1]School of Electronic Science and Engineering, Nanjing University, Nanjing 210093, People's Republic of China
[2]Department of Management Science and Engineering, Stanford University, Stanford, CA 94305, USA
E-mail: wangsw@nju.edu.cn

Abstract: The power allocation problem in orthogonal frequency division multiplexing-based cognitive radio (CR) networks with cooperative relays has been investigated here, where both the interference to primary users (PUs) and the power budget of the CR network are considered. The authors try to maximise the overall throughput of the CR network within the given constraints. The coupling variables in the formulated problem make it hard to solve, so an iterative optimisation scheme to find out the optimal solution with a controllable complexity is developed. First, the original problem is decomposed into two subproblems that can be solved independently. A fast barrier method has been employed to work out the optimal solution to one of the subproblems with a complexity of $\boldsymbol{O}(L^2N)$, where L and N are the number of PUs and subcarriers, respectively. Then, an iterative procedure is developed to solve the other subproblem. Numerical results show that the proposed method can significantly increase the throughput of the CR system, comparing with other representative ones. Furthermore, the proposed algorithm gives a general power optimisation framework for CR networks with cooperative relays.

1 Introduction

Radio spectrum is a scarce resource and becomes a bottleneck to develop new wireless applications in recent years. On the other hand, it has been shown that most of the licensed spectrum is underutilised and stays idle at any given time and location [1]. Cognitive radio (CR) [2, 3] is a promising technology to improve spectrum efficiency by exploiting the spectrum portions unoccupied by licensed holders, also known as primary users (PUs). A CR user, also referred to as secondary user (SU), can access to a CR network opportunistically as long as the interference introduced to PUs is kept below a prescribed threshold [4].

Orthogonal frequency division multiplexing (OFDM), which offers a high flexibility in radio resource allocation (RA), has been widely recognised as a potential air interface for CR systems [5]. However, in an OFDM-based CR system, even with near-perfect spectrum sensing, SUs can produce significant interference to the PUs operating in the adjacent bands and vice versa, especially when the PUs do not adopt OFDM modulation. Consequently, dynamic RA is very important as the SUs have to adjust its parameters frequently to adapt to the changes of spectrum environment and try to void corrupting the normal activity of the PUs. As an attractive issue, RA in OFDM-based CR systems has been intensively studied [6–9].

One of the most challenging problems in CR networks is how to satisfy the quality-of-service (QoS) requirements of the SUs efficiently while keeping the interference to the PUs below their tolerable thresholds. However, in some cases, a reliable transmission between a certain pair of CR nodes with direct link requires too much power because of poor channel condition caused by deep fading in wireless environment, which cuts down the lifetime of the battery of the nodes and leads to heavy interference to the PUs [10]. Furthermore, even if a CR network running in a centralised manner could perform an efficient dynamic spectrum management to meet the rate requirements of the SUs, the arising high computation burden makes it difficult to apply [11]. So, conventional transmission strategy may not be suitable for CR systems. Cooperative relay is a powerful scheme to combat the effects of fading by exploiting the spatial diversity over dispersed nodes [12, 13], which is also a promising method to overcome the mentioned difficulties in CR networks [14]. In a CR network, cooperative relay can not only beat multipath fading, but also reduce the required transmission power of each CR node. Therefore, it can also mitigate the interference introduced to the PUs. The key issue is how to allocate power properly so that the throughput of the CR system can be maximised while keeping the interference to the PUs under the desired level, which is the topic of this work.

RA for CR networks with cooperative relay has attracted much attention and been studied in various perspectives [15–20]. In [15], a centralised heuristic algorithm is proposed to address the RA problem, as well as a new medium access control protocol implemented in a universal software radio peripheral-based test bed. A certain protection range for each PU is required to avoid excessive interference from the SUs. However, it is difficult to decide an accurate range in practical wireless environment because of fast-changing fading. Furthermore, it potentially decreases the spectrum efficiency. In [16], the author investigates the multi-relay cognitive transmission. A selective fusion spectrum sensing and the best relay data transmission scheme is proposed. However, the interference generated by the SUs which can deteriorate the performance of the PUs is not mentioned. A three-node CR network is developed in [17], where relay channels are divided into three categories, followed by RA approaches. However, the interference to the licensed system is ignored in the considered system model. Both power limitation and interference constraint are taken into consideration in [18], where only peak or average interference power limitations are investigated for a CR network with relays. In [19], the interference caused by each subchannel in the CR system is considered. However, it is difficult to measure the required parameters in practical wireless systems. The capacity of a CR system employing relays is studied in [21], where the sum capacity is maximised while the total transmission power is kept within a budget and the interference introduced to the PU is kept within a prescribed threshold. The optimisation problem is a mixed-integer problem which is non-polynomial (NP)-hard, and three suboptimal schemesare proposed to address the formulated problem. In [22], an alternating optimisation method is proposed to solve an optimisation task, which tries to maximise the system capacity. In [23], a general case is considered where the relay can change the subcarrier index of a two-hop system.

In this paper, we investigate the power allocation problem in a CR network with cooperative relays. We aim to maximise the end-to-end capacity of the CR system while satisfying all necessary requirements or constraints, such as the power budgets of the CR nodes and interference limitations of the licensed system. The main contributions of this work are outlined as follows:

1. *New problem formulation:* A more general power optimisation model is established for the relay-assisted CR networks with amplify-and-forward (AF) protocol. In our proposed system model, interference constraint is fully considered, in order to protect the performance of the PUs from deteriorating. The model can also be extended to other relay modes, such as decode-and-forward (DF), with few modifications.
2. *Alternating optimisation framework:* We solve the formulated problem by using an alternative optimisation method, that is, to perform power allocation at the CR source and relay iteratively, which provides a general framework to tackle the power allocation problem in cooperative relay-based CR networks.
3. *Fast barrier method with low complexity:* By exploiting the structure of the considered problem, we develop a fast barrier method for the optimal power allocation at the CR source and the relay with a low complexity.

The remainder of this paper is organised as follows: In Section 2, the system model and optimisation problem are described. In Section 3, we propose our power allocation scheme in detail. Simulation results are given in Section 4. Conclusion is drawn in Section 5.

2 System model and problem formulation

2.1 System model

Consider a three-node OFDM-based relay-enhanced CR network coexisting with a primary system which consists of L PUs as shown in Fig. 1. The CR system adopts OFDM modulation and the primary system is unnecessary to use OFDM. The primary system is a bidirectional cellular network with dynamic channel allocation and mobile receivers. A CR source ($\mathcal{S}$) transmits data to a CR destination ($\mathcal{D}$) with the help of a CR relay ($\mathcal{R}$). The relay operates in a time-division half-duplex mode using the AF protocol which can be implemented with low cost [24]. The transmission from the source to the destination is on a time-frame basis and each frame consists of two time slots with the same length. In the first time slot, the source transmits signals on all subcarriers while the relay and the destination nodes listen. In the second time slot, the source remains silent while the relay amplifies the received signals and forwards them to the destination. The signals transmitted over a subchannel in the source node are amplified and forwarded by the relay nodes over the subcarriers with the same indices in the second time slot.

Throughout this paper, we assume that the perfect knowledge of the channel state information (CSI) from the CR source network (SN) to relay network (RN) is available, as well as the perfect knowledge of the CSI between the CR RN and destination network (DN). Besides, the perfect CSI between the CR system and primary system is also supposed to be known. The former can be approximated by assuming channel reciprocity, while the latter can be estimated by listening to a beacon signal and then fed-back to the CR system. We note that our proposed solution is theoretical and the results obtained by our proposed algorithm will serve as an upper bound of the achievable capacity with channel estimation errors.

The available spectrum is equally divided into N OFDM subcarriers and the bandwidth of each subcarrier is B. The occupied spectrum bandwidth of PU l is B_l. The interference imposed on subcarrier n used by the SUs with unit transmission power can be expressed as

$$I_{l,n}^{\mathcal{SP}} = \int_{d_{n,l}-(1/2)B_l}^{d_{n,l}+(1/2)B_l} g_{l,n}^{\mathcal{SP}} \phi(f)\, \mathrm{d}f \tag{1}$$

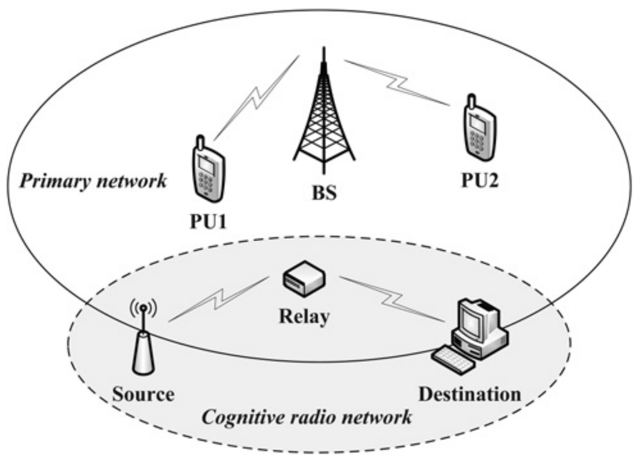

Fig. 1 *Coexistence of a relay-enhanced cognitive network with a primary network*

IET Commun., 2014, Vol. 8, Iss. 6, pp. 921–929
doi: 10.1049/iet-com.2013.0379

Similarly, the interference introduced to the PU l by the CR relay with unit transmission power on subcarrier n is

$$I_{l,n}^{\mathcal{RP}} = \int_{d_{n,l}-(1/2)B_l}^{d_{n,l}+(1/2)B_l} g_{l,n}^{\mathcal{RP}} \phi(f)\,\mathrm{d}f \tag{2}$$

where $g_{l,n}^{\mathcal{SP}}$ and $g_{l,n}^{\mathcal{RP}}$ are the channel gain of subchannel n from the CR source and relay's transmitters to the receiver of the PU l, respectively. $d_{n,l}$ is the central spectral distance between the bands of subcarrier n and PU l. $\phi(f)$ is the power spectrum density of the OFDM signal, $\phi(f) = T_s(\sin\pi f T_s/\pi f T_s)^2$, where T_s is the OFDM symbol duration.

The channel gains of subcarrier n between the CR source and relay are denoted as $h_{\mathcal{SR},n}$ and $h_{\mathcal{RD},n}$, respectively. $\sigma_{\mathcal{R}}^2$ and $\sigma_{\mathcal{D}}^2$ are assumed to be the variance of additive white Gaussian noise (AWGN) at the CR relay and destination, respectively. Moreover, we denote the interference caused by PU l to the CR relay and destination on the subcarrier n as $I_{\mathcal{PR},n}$ and $I_{\mathcal{PD},n}$ respectively, which can be measured by the CR network. The CR source sends data to the relay with power $p_{\mathcal{S},n}$ on subcarrier n in the first hop, and the relay amplifies the received signals with power

$$p_{\mathcal{R},n} = \alpha_n^2 p_{\mathcal{S},n} h_{\mathcal{SR},n} + \alpha_n^2 \sigma_{\mathcal{R}}^2 + \alpha_n^2 I_{\mathcal{PR},n}$$

on subchannel n in the second hop, where α_n is the amplification factor on subchannel n. Thus, we can rewrite the α_n into

$$\alpha_n = \sqrt{\frac{p_{\mathcal{R},n}}{p_{\mathcal{S},n} h_{\mathcal{SR},n} + \sigma_{\mathcal{R}}^2 + I_{\mathcal{PR},n}}} \tag{3}$$

Throughout this paper, we assume the destination receives the signals without diversity; that is, at the end of each transmission frame, the destination processes the signals without combination. Consequently, the received signal-to-noise ratio (SNR) on subchannel n at the CR destination is [25]

$$\begin{aligned}\Gamma_n &= \frac{p_{\mathcal{S},n} h_{\mathcal{RD},n} \alpha_n^2 h_{\mathcal{SR},n}}{\sigma_{\mathcal{D}}^2 + I_{\mathcal{PD},n} + (\sigma_{\mathcal{R}}^2 + I_{\mathcal{PR},n})\alpha_n^2 h_{\mathcal{RD},n}} \\ &= \frac{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}{1 + p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} + p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}\end{aligned} \tag{4}$$

where

$$\gamma_{\mathcal{SR},n} = \frac{h_{\mathcal{SR},n}}{\sigma_{\mathcal{R}}^2 + I_{\mathcal{PR},n}} \text{ and } \gamma_{\mathcal{RD},n} = \frac{h_{\mathcal{RD},n}}{\sigma_{\mathcal{D}}^2 + I_{\mathcal{PD},n}}$$

The transmission rate from source to destination on subchannel n can be calculated as

$$\begin{aligned}R_n &= \frac{1}{2}\log(1 + \Gamma_n) \\ &= \frac{1}{2}\log\left(1 + \frac{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}{1 + p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} + p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}\right)\end{aligned} \tag{5}$$

The factor 1/2 accounts for the two time slots in each transmission frame. Note that R_n in (5) is not jointly convex with $p_{\mathcal{S},n}$ and $p_{\mathcal{R},n}$ and the optimal solution of the resulting optimisation problem is difficult to obtain from the viewpoint of mathematical programming. To make the problem tractable, we adopt the following approximation

$$R_n \simeq \frac{1}{2}\log\left(1 + \frac{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} + p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}\right) \tag{6}$$

where Γ_n is approximated by the harmonic mean of $p_{\mathcal{S},n}$ and $p_{\mathcal{R},n}$. Such an approximation is reasonable as discussed in [26].

2.2 Problem formulation

Our objective is to maximise the total end-to-end transmission throughput of the CR system, that is, $\sum_{n=1}^{N} R_n$, while keeping the following constraints satisfied

$$\begin{aligned}&\text{Source power constraint: } \sum_{n=1}^{N} p_{\mathcal{S},n} \le P_{\mathcal{S}} \\ &\text{Relay power constraint: } \sum_{n=1}^{N} p_{\mathcal{R},n} \le P_{\mathcal{R}}\end{aligned} \tag{7}$$

where $P_{\mathcal{S}}$ and $P_{\mathcal{R}}$ are the maximum transmission powers for source and relay, respectively.

The interference happens as the primary system coexists with the CR system, and should be kept below a predefined threshold. So, the transmission power from the source to the relay in the first hop and from the relay to the destination must satisfy the following constraints

$$\begin{aligned}&\text{In the first hop: } \sum_{n=1}^{N} p_{\mathcal{S},n} I_{l,n}^{\mathcal{SP}} \le I_{\mathcal{T},l} \\ &\text{In the second hop: } \sum_{n=1}^{N} p_{\mathcal{R},n} I_{l,n}^{\mathcal{RP}} \le I_{\mathcal{T},l}\end{aligned} \tag{8}$$

where $I_{\mathcal{T},l}$ is the interference threshold of the PU l, $l = 1, 2, \ldots, L$. Then the optimisation problem can be formulated as follows

$$\begin{aligned}&\max_{p_{\mathcal{S},n}, p_{\mathcal{R},n}} \sum_{n=1}^{N} \frac{1}{2}\log\left(1 + \frac{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} + p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}\right) \\ &\text{s.t. } C_1: \sum_{n=1}^{N} p_{\mathcal{S},n} \le P_{\mathcal{S}} \\ &C_2: \sum_{n=1}^{N} p_{\mathcal{R},n} \le P_{\mathcal{R}} \\ &C_3: \sum_{n=1}^{N} p_{\mathcal{S},n} I_{l,n}^{\mathcal{SP}} \le I_{\mathcal{T},l}, \quad l = 1, 2, \ldots, L \\ &C_4: \sum_{n=1}^{N} p_{\mathcal{R},n} I_{l,n}^{\mathcal{RP}} \le I_{\mathcal{T},l}, \quad l = 1, 2, \ldots, L \\ &C_5: p_{\mathcal{S},n} \ge 0, p_{\mathcal{R},n} \ge 0, \quad n = 1, 2, \ldots, N\end{aligned} \tag{9}$$

3 Our proposed algorithm

Note that the objective function of (9) is concave and there are coupling variables, $p_{\mathcal{S},n}$ and $p_{\mathcal{R},n}$. We can divide it into two equivalently subproblems with a primary decomposition method [27]

$$\begin{aligned} &\text{SP1}: \max_{p_{\mathcal{S},n}} \frac{1}{2}\log\left(1+\frac{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n}p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n}+p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}\right) \\ &\text{s.t.}\, C_1: \sum_{n=1}^{N} p_{\mathcal{S},n} \le P_{\mathcal{S}} \\ &C_2: \sum_{n=1}^{N} p_{\mathcal{S},n} I_{l,n}^{\mathcal{SP}} \le I_{\mathcal{T},l}, \quad l=1,2,\ldots,L \\ &C_3: p_{\mathcal{S},n} \ge 0, \quad n=1,2,\ldots,N \end{aligned} \tag{10}$$

and

$$\begin{aligned} &\text{SP2}: \max_{p_{\mathcal{R},n}} \frac{1}{2}\log\left(1+\frac{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n}p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n}+p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}\right) \\ &\text{s.t.}\, C_1: \sum_{n=1}^{N} p_{\mathcal{R},n} \le P_{\mathcal{R}} \\ &C_2: \sum_{n=1}^{N} p_{\mathcal{R},n} I_{l,n}^{\mathcal{RP}} \le I_{\mathcal{T},l}, \quad l=1,2,\ldots,L \\ &C_3: p_{\mathcal{R},n} \ge 0, \quad n=1,2,\ldots,N \end{aligned} \tag{11}$$

In the first subproblem, SP1, we focus on finding the optimal values of $p_{\mathcal{S},n}$, which is a function of $p_{\mathcal{R},n}$, termed as the primal function of $p_{\mathcal{R},n}$ [27]. Whereas in the second subproblem, SP2, the optimal value of $p_{\mathcal{R},n}$ is to be achieved.

3.1 Fast barrier method for power allocation

In this subsection, we try to find the optimal solution of the subproblems mentioned above. We start from SP1. It is easy to prove that SP1 defines a convex optimisation problem with N variables and $N+L+1$ constraints. Generally, the optimal solution to the problem can be obtained with barrier method [28], which is a standard convex optimisation technique and typically has a complexity of $\boldsymbol{O}(N^3)$. However, by exploiting the structure of the problem, we can reduce the complexity to $\boldsymbol{O}(L^2N)$.

Let us denote a vector $p \in R^N$ and $p=(p_{\mathcal{S},1}, p_{\mathcal{S},2}, \ldots, p_{\mathcal{S},N})^{\mathrm{T}}$. We can write the barrier function as [28]

$$\begin{aligned} \phi(p) = &-\sum_{n=1}^{N}\log(p_{\mathcal{S},n}) - \log\left(P_{\mathcal{S}} - \sum_{n=1}^{N} p_{\mathcal{S},n}\right) \\ &- \sum_{l=1}^{L}\log\left(I_{\mathcal{T},l} - \sum_{n=1}^{N} p_{\mathcal{S},n} I_{l,n}^{\mathcal{SP}}\right) \end{aligned} \tag{12}$$

The optimal solution to (10) can be approximated by solving a sequence of unconstrained minimisation problems which have the following form

$$\min \psi_t(p) = -tf(p) + \phi(p) \tag{13}$$

where

$$f(p) = \sum_{n=1}^{N}\frac{1}{2}\log\left(1+\frac{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n}p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n}+p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}\right)$$

The barrier parameter t is positive and decides the accuracy of the approximation. The solution to (13) is denoted as $p^{\star}(t)$, which is also referred as a central point. As t increases, $p^{\star}(t)$ becomes a more accurate approximation to the optimal solution of SP1. The details of barrier method are given in Fig. 2.

Note that when we solve the unconstrained minimisation problem in (13) for each given value of t during the procedure of barrier method, Newton method is employed to compute the central point $p^{\star}(t)$. The computational load of Newton method mainly lies on the computation of Newton step Δp at p, that is, we have to solve the equation

$$\nabla^2\psi_t(p)\Delta p = -\psi_t(p) \tag{14}$$

where $\nabla^2\psi_t(p)$ is the Hessian and $\nabla\psi_t(p)$ is the gradient of $\psi_t(p)$, respectively. Generally, obtaining Δp by using matrix inversion to solve (14) has a cost of $\boldsymbol{O}(N^3)$. We now show how to reduce the complexity to $\boldsymbol{O}(L^2N)$ by exploiting the structure of the problem.

The gradient of $\psi_t(p)$ is given by

$$\begin{aligned} \frac{\partial\psi_t(p)}{\partial p_n} = &-t\frac{\gamma_{\mathcal{SR},n}M_{\mathcal{R}}^2}{(M_{\mathcal{S}}+M_{\mathcal{R}})(M_{\mathcal{S}}+M_{\mathcal{R}}+M_{\mathcal{S}}M_{\mathcal{R}})} \\ &-\frac{1}{p_{\mathcal{S},n}} + \frac{1}{P_{\mathcal{S}}-P_{\Sigma}} + \sum_{l=1}^{L}\frac{I_{l,n}^{\mathcal{SP}}}{I_l - I_{\Sigma}^l} \end{aligned} \tag{15}$$

where $M_{\mathcal{S}} = p_{\mathcal{S},n}\gamma_{\mathcal{SR},n}$, $M_{\mathcal{R}} = p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}$, $P_{\Sigma} = \sum_{n=1}^{N} P_{\mathcal{S},n}$ and $I_{\Sigma}^l = \sum_{n=1}^{N} P_{\mathcal{S},n} I_{l,n}^{\mathcal{SP}}$.

```
Give strictly feasible p, t, μ > 1 and tolerance ε_b;
while (N+L+1)/t > ε_b    /* Barrier method loop */
  Give tolerance ε_n, α ∈ (0,1/2), γ ∈ (0,1);
  while (true)    /* Newton method loop */
    Calculate Δp and λ² = −ψ_t(p)Δp;
    if λ²/2 ≤ ε_n
      break.
    endif
    Set σ := 1;
    while ψ_t(p+σΔp) > ψ_t(p) − ασλ²  /* Backtracking line search
      σ := γσ;
    endwhile
    p := p + σΔp.
  endwhile
  t := μt.
endwhile
```

Fig. 2 *Procedure of barrier method*

IET Commun., 2014, Vol. 8, Iss. 6, pp. 921–929
doi: 10.1049/iet-com.2013.0379

The Hessian of $\psi_t(p)$ is given by

$$\nabla^2 \psi_t(p) = H + \sum_{l=0}^{L} g_l g_l^{\mathrm{T}} \tag{16}$$

where

$$H_0 = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{bmatrix} \text{ and}$$

$$\lambda_n = t\frac{\gamma_{\mathcal{SR},n}^2 M_{\mathcal{R}}^2 [M_{\mathcal{R}}^2 + 2(M_{\mathcal{S}} + M_{\mathcal{R}} + M_{\mathcal{S}} M_{\mathcal{R}})]}{\left[(M_{\mathcal{S}} + M_{\mathcal{R}} + M_{\mathcal{S}} M_{\mathcal{R}})(M_{\mathcal{S}} + M_{\mathcal{R}})\right]^2} + \frac{1}{p_{\mathcal{S},n}^2}, \forall n$$

g_ls are vectors of length N.

$$g_0 = \left(\frac{1}{P_{\mathcal{S}} - P_{\Sigma}}, \frac{1}{P_{\mathcal{S}} - P_{\Sigma}}, \ldots, \frac{1}{P_{\mathcal{S}} - P_{\Sigma}}\right)^{\mathrm{T}}$$

$$\text{and } g_l = \frac{1}{I_l - I_{\Sigma}^l}\left(I_{l,1}^{\mathcal{SP}}, I_{l,2}^{\mathcal{SP}}, \ldots, I_{l,N}^{\mathcal{SP}}\right)^{\mathrm{T}}$$

Define $H_i = H + \sum_{l=0}^{i} g_l g_l^{\mathrm{T}}$, $i = 0, 1, \ldots, L$. We have the following theorems.

Theorem 1: All H_is are positive-definite.

Proof: H is diagonal and $\lambda_n > 0$, so H is positive-definite. Since for any i, we have $g_i g_i^{\mathrm{T}} > 0$, then H_i is positive-definite.

It follows that the Hessian is invertible. Since the Hessian can be treated as the sum of a diagonal matrix and $L + 1$ number of rank-one matrices, we can exploit this special structure to calculate the Newton step Δp with much lower complexity.

Theorem 2: Equation (14) can be solved with a complexity of $\boldsymbol{O}(L^2 N)$.

The proof is placed in Appendix 1. Recall that L is the number of active PUs; and generally, $L \ll N$ in CR systems, so the complexity of our proposed algorithm is almost linearly related to N.

3.2 Alternating optimisation framework

Recall that we have to solve optimisation problem (9) jointly over two variables, that is, $p_{\mathcal{S},n}$ and $p_{\mathcal{R},n}$. An alternating optimisation scheme can be applied to solve this problem due to its effectiveness and simplicity [29].

Denote $p_{\mathcal{S}} = (p_{\mathcal{S},1}, \ldots, p_{\mathcal{S},N})$ and $p_{\mathcal{R}} = (p_{\mathcal{R},1}, \ldots, p_{\mathcal{R},N})$. We can rewrite the objective function in (9) as follows

$$\max_{p_{\mathcal{S}}, p_{\mathcal{R}}} f(p_{\mathcal{S}}, p_{\mathcal{R}}) \tag{17}$$

where

$$f(p_{\mathcal{S}}, p_{\mathcal{R}}) = \sum_{n=1}^{N} \frac{1}{2}\log\left(1 + \frac{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}{p_{\mathcal{S},n}\gamma_{\mathcal{SR},n} + p_{\mathcal{R},n}\gamma_{\mathcal{RD},n}}\right)$$

Generally, maximising over both sets of variables simultaneously is not straightforward. However, minimising with respect to one variable while keeping the other one fixed is often easy and is sometimes possible analytically. In such a situation, the alternating optimisation algorithm described in Fig. 3 is well suited: start with an arbitrary feasible point $p_{\mathcal{R}}^0$; for $k \geq 1$, iteratively compute

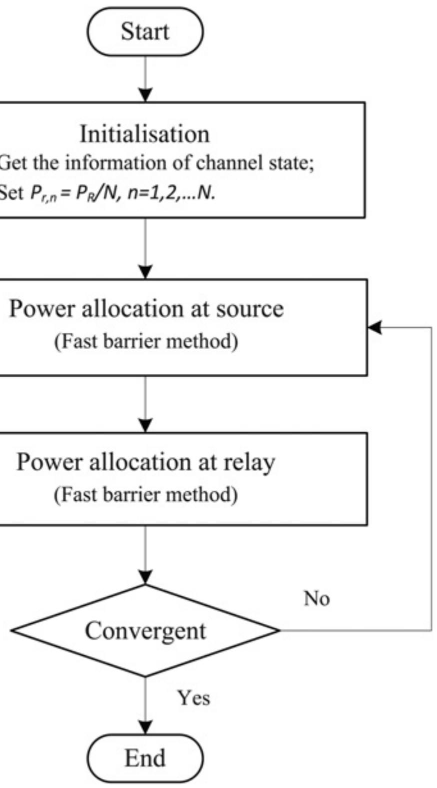

Fig. 3 *Flowchart of the algorithm*

$$\begin{aligned} p_{\mathcal{S}}^k &= \arg\max_{p_{\mathcal{S}}} f(p_{\mathcal{S}}, p_{\mathcal{R}}^{k-1}) \\ p_{\mathcal{R}}^k &= \arg\max_{p_{\mathcal{R}}} f(p_{\mathcal{S}}^k, p_{\mathcal{R}}) \end{aligned} \tag{18}$$

In other words, instead of solving the original optimisation problem over two variables, the alternating optimisation algorithm solves a sequence of optimisation problems over only one variable. The convergence of the algorithm can be proved. As (9) is concave maximisation problem, there exists a globally optimal point, which is denoted as f^*, then we have the following theorem.

Theorem 3: $\lim_{k\to\infty} f(p_{\mathcal{S}}^k, p_{\mathcal{R}}^k) = f^*$.

Proof: Let us denote $f^k = f(p_{\mathcal{S}}^k, p_{\mathcal{R}}^k)$, from (21), we have

$$f^k \geq f(p_{\mathcal{S}}^k, p_{\mathcal{R}}^{k-1}) \geq f(p_{\mathcal{S}}^{k-1}, p_{\mathcal{R}}^{k-1}) = f^{k-1} \tag{19}$$

for $k \geq 1$. Since the sequence f^k is non-decreasing, it must converge, that is, $f^k \to f^*$, because f is bounded from above. It follows that the algorithm converges, and we can take the value at the last iteration as the solution to the original problem.

Table 1 Number of iterations for convergence

number of subcarriers	16	32	64	128	256
maximal number of iteration	9	11	14	15	15
average number of iteration	2.9	3.5	4.4	5.3	6.2

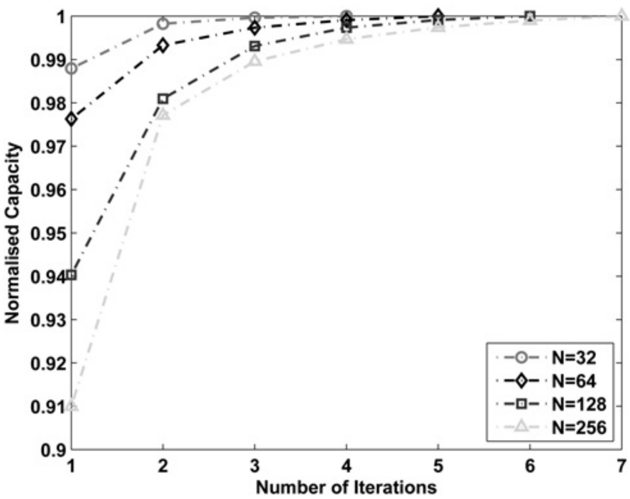

Fig. 4 *Number of iterations for alternating optimisation*

4 Simulations and discussions

Experiments are performed to evaluate the performance of our proposed method. Consider a CR system where the CR nodes and PUs are uniformly distributed within a 1 km circle area. The frequency-selective fading channel model is assumed with the path loss exponent set to 4. The variance of logarithmic normal shadow fading is 10 dB and the amplitude of multipath fading is Rayleigh. In the following simulations, the noise power is set to 10^{-13} W.

First, we investigate the convergence performance of the alternating optimisation method. For all considered scenarios, the algorithm will reach the convergence with less than 15 iterations over 1000 simulations as shown in Table 1. We also depict the normalised capacity against the iteration number of the alternating algorithm in Fig. 4,

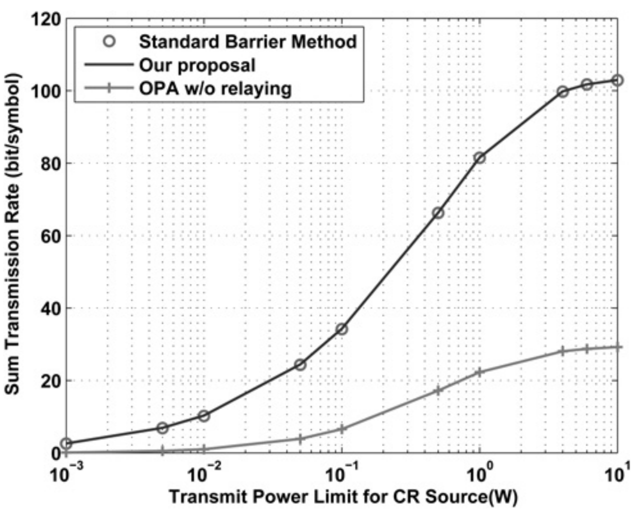

Fig. 5 *System throughput against transmission power limit of CR source*

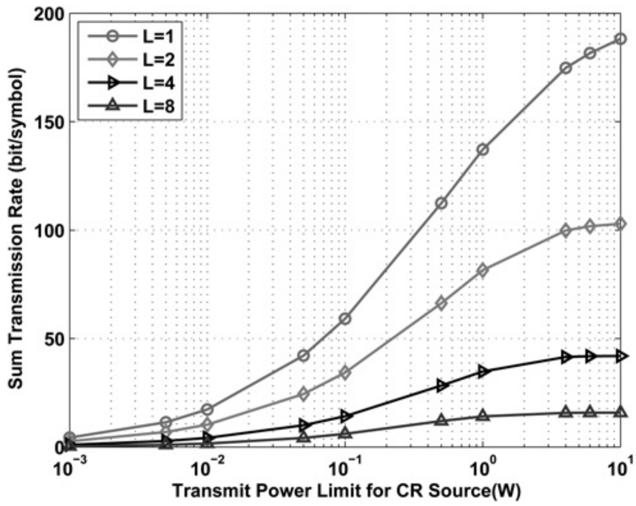

Fig. 6 *System throughput against transmission power limit of CR source in cases of different number of PUs*

where the normalised capacity represents the percentage between the achieved capacity in each iteration and the optimal solution (also referred to the maximum capacity). From Fig. 4, we can see that the method converges very fast and achieves over 98% of the maximum capacity by only three iterations. It means that the alternating optimisation method can reach the convergence quickly.

Next, the performance of our proposed power allocation scheme is illustrated in Figs. 5–8. For comparison, the performance of other two schemes is analysed:

1. *Optimal power allocation without relay (OPA w/o relay):* The help of relay is not considered and optimal power allocation on the direct link between source and destination is worked out by CVX [30].
2. *Standard barrier method:* The optimisation problem is solved by standard barrier method and optimal solution is obtained.

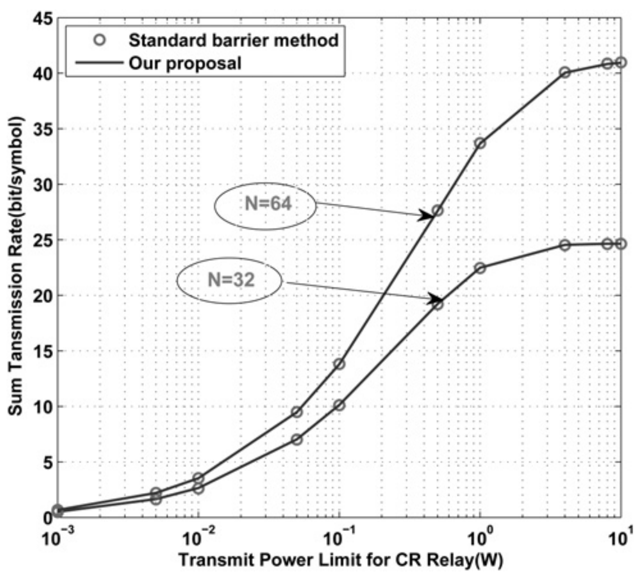

Fig. 7 *System throughput against transmission power limit of CR relay*

IET Commun., 2014, Vol. 8, Iss. 6, pp. 921–929
doi: 10.1049/iet-com.2013.0379

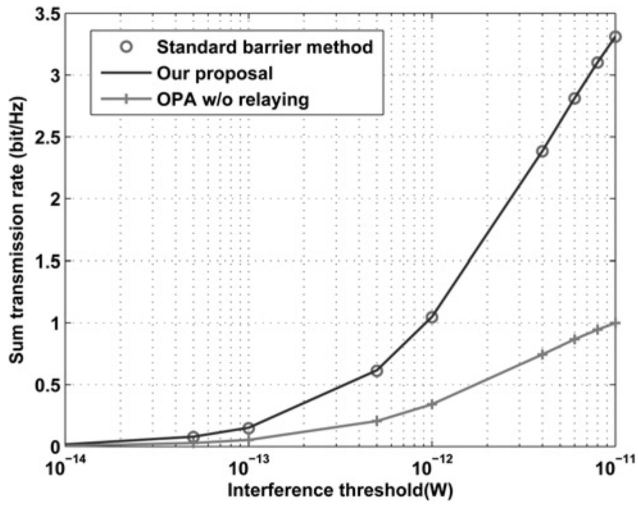

Fig. 8 *System throughput against interference threshold of PUs*

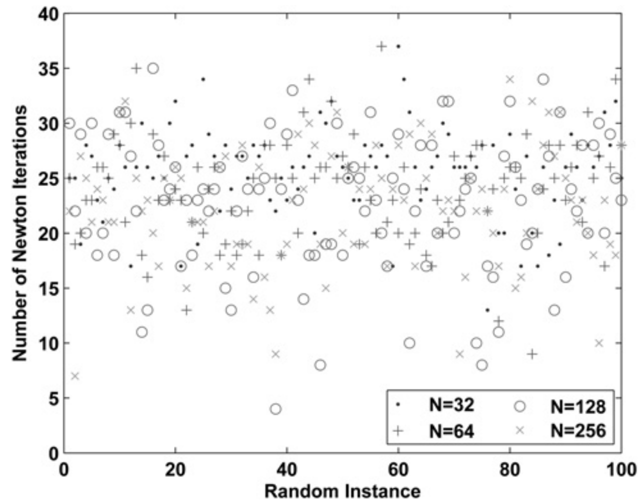

Fig. 10 *Distribution of Newton iterations*

Note that all the schemes mentioned above consider the interference constraints. Fig. 5 shows the system throughput against the overall transmission power at the CR source. In our simulation, the transmission power limit for the CR relay is $P_{\mathcal{R}} = 1$ W, and the number of PUs is two. From Fig. 5, we can see that our proposed scheme performs much better than *OPA w/o relay*. The reason is that cooperative relay can exploit spatial diversity of subcarriers. Obviously, the result of our proposed scheme is very close to the optimal solution.

Fig. 6 shows the system throughput against the overall transmission power at the CR source in cases of different number of PUs. In our simulation, the transmission power limit for the relay is $P_{\mathcal{R}} = 1$ W. The interference threshold for the primary system is 5×10^{-12} W. From the figure, we can see that for low CR source power level, the CR system is power-limited and the throughput increases with $P_{\mathcal{S}}$. When $P_{\mathcal{S}} > 1$ W, the system becomes interference-limited and the system throughput increases very little with $P_{\mathcal{S}}$. Furthermore, the system throughput decreases as the number of PUs increases, because the interference constraints are more strict.

Fig. 7 shows the throughput against the transmission power limit of the CR relay. The maximum power of the CR source is fixed at 1 W and the interference threshold is set to be 5×10^{-12} W. From Fig. 5, we can see that the throughput increases as the transmission power of the relay increases, because the increasing of the transmission power at the relay node can improve the received SNR at the destination node. Obviously, as the number of subcarriers increases, the system throughput become higher for the subcarrier diversity.

Fig. 8 shows the throughput as a function of the interference threshold of the PUs. The transmission power of the CR source and relay is 1 W. It can be seen that the throughput increases as the interference threshold of PUs increases, because larger interference thresholds lead to larger transformation power of the CR system. Obviously, our proposed method is better than *OPA w/o relay* and the solution is quite close to the optimal one.

Fig. 9 shows the program execution time as a function of number of subcarriers. The elapsed time is counted by the inbuilt function tic-toc in MATLAB. Obviously, the time complexity of our scheme is much lower than the standard convex optimisation method when the number of subcarriers is relatively large as discussed in Section 3.

Finally, we investigate the convergence of the fast barrier method. Recall that the computational load of the fast power

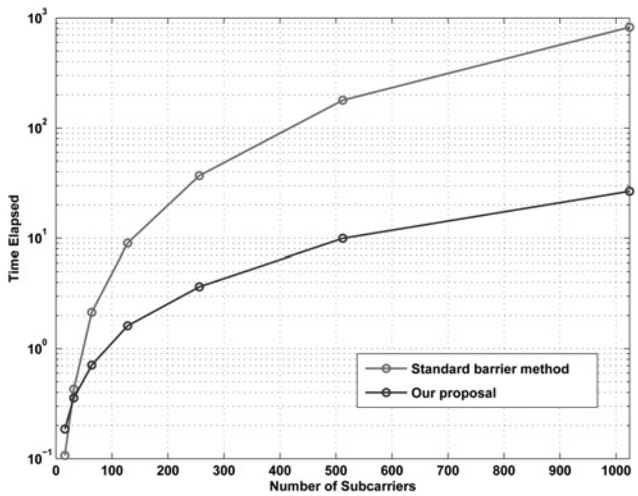

Fig. 9 *Execution time against number of subcarriers*

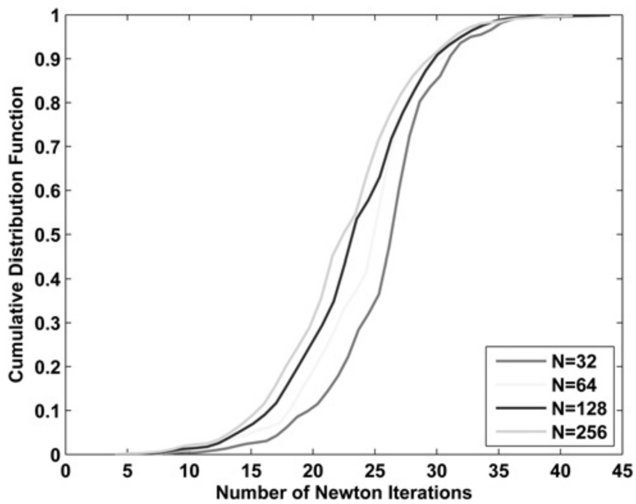

Fig. 11 *CDF of number of Newton iterations*

allocation mainly caused by Newton iterations. Fig. 10 shows the total number of Newton iterations required for a guaranteed duality gap of less than 10^{-3} for 100 channel realisations. Fig. 11 shows the cumulative distribution function (CDF) curve as the number of Newton iterations for convergence. For all cases, the number of Newton iterations is less than 30 in about 90% of the instances and varies in a narrow range, which means the fast barrier method is effective and efficient.

5 Conclusions

In this paper, we proposed an optimisation framework for cooperative CR network with cooperative relays, where the interference constraints and power limitations are fully considered. The formulated optimisation task falls into a convex problem with coupling variables. We developed a fast barrier method and an alternating optimisation algorithm to work out the optimal solution quickly. Simulation results demonstrate that our proposed scheme achieves better performance in different radio environments. Our system model is general and can be extended to other CR networks with relays intuitively. Furthermore, the alternating optimisation framework also throws some light on tackling the power allocation problem in CR network with cooperative relays.

6 Acknowledgment

This research was partially supported by the Jiangsu Natural Science Foundation (BK2011051) and the National Natural Science Foundation of China (61100111).

7 References

1 F.C. Commission: 'Spectrum policy task force report'. FCC Rep. ET Docket, November 2003, pp. 2–135
2 Mitola, J., Maguire, G.: 'Cognitive radio: making software radios more personal', *IEEE Pers. Commun.*, 1999, **6**, (4), pp. 13–18
3 Haykin, S.: 'Cognitive radio: brain-empowered wireless communications', *IEEE J. Sel. Areas Commun.*, 2005, **23**, (2), pp. 201–220
4 Khozeimeh, F., Haykin, S.: 'Dynamic spectrum management for cognitive radio: an overview', *IEEE Wirel. Commun. Mobile Comput.*, 2009, **9**, (11), pp. 1447–1459
5 Mahmoud, H.A., Yucek, T., Arslan, H.: 'OFDM for cognitive radio: merits and challenges', *IEEE Wirel. Commun.*, 2009, **16**, (2), pp. 6–15
6 Bansal, G., Hossain, M., Bhargava, V.: 'Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems', *IEEE Trans. Wirel. Commun.*, 2008, **7**, (11), pp. 4710–4718
7 Attar, A., Nakhai, M., Aghvami, A.: 'Cognitive radio game for secondary spectrum access problem', *IEEE Trans. Wirel. Commun.*, 2009, **8**, (4), pp. 2121–2131
8 Wang, S.: 'Efficient resource allocation algorithm for cognitive OFDM systems', *IEEE Commun. Lett.*, 2010, **14**, (8), pp. 725–727
9 Wang, S., Zhou, Z.-H., Ge, M., Wang, C.: 'Resource allocation for heterogeneous cognitive radio networks with imperfect spectrum sensing', *IEEE J. Sel. Areas Commun.*, 2013, **31**, (3), pp. 464–475
10 Zhang, D., Wang, Y., Lu, J.: 'QoS aware relay selection and subcarrier allocation in cooperative OFDMA systems', *IEEE Commun. Lett.*, 2010, **14**, (4), pp. 294–296
11 Khozeimeh, F., Haykin, S.: 'Dynamic spectrum management for cognitive radio: an overview', *Wirel. Commun. Mobile Comput.*, 2009, **9**, (11), pp. 1447–1459
12 Sendonaris, A., Erkip, E., Aazhang, B.: 'User cooperation diversity—Part I: System description', *IEEE Trans. Commun.*, 2003, **51**, (11), pp. 1927–1938
13 Sendonaris, A., Erkip, E., Aazhang, B.: 'User cooperation diversity—Part II: Implementation aspects and performance analysis', *IEEE Trans. Commun.*, 2003, **51**, (11), pp. 1939–1948
14 Zhang, Q., Jia, J., Zhang, J.: 'Cooperative relay to improve diversity in cognitive radio networks', *IEEE Commun. Mag.*, 2009, **47**, (2), pp. 111–117
15 Jia, J., Zhang, J., Zhang, Q.: 'Cooperative relay for cognitive radio networks'. IEEE INFOCOM 2009, April 2009, pp. 2304–2312
16 Zou, Y., Yao, Y., Zheng, B.: 'Cognitive transmissions with multiple relays in cognitive radio networks', *IEEE Trans. Wirel. Commun.*, 2011, **10**, (2), pp. 648–659
17 Zhao, G., Yang, C., Li, G., Li, D., Soong, A.: 'Power and channel allocation for cooperative relay in cognitive radio networks', *IEEE J. Sel. Top. Signal Process.*, 2011, **5**, (1), pp. 151–159
18 Musavian, L., Aissa, S., Lambotharan, S.: 'Effective capacity for interference and delay constrained cognitive radio relay channels', *IEEE Trans. Wirel. Commun.*, 2010, **9**, (5), pp. 1698–1707
19 Li, L., Zhou, X., Xu, H., Li, G., Wang, D., Soong, A.: 'Simplified relay selection and power allocation in cooperative cognitive radio systems', *IEEE Trans. Wirel. Commun.*, 2011, **10**, (1), pp. 33–36
20 Wang, S., Ge, M., Wang, C.: 'Efficient resource allocation for cognitive radio networks with cooperative relays', *IEEE J. Sel. Areas Commun.*, 2013, **31**, (11), pp. 2432–2441
21 Bharadia, D., Bansal, G., Kaligineedi, P., Bhargava, V.: 'Relay and power allocation schemes for OFDM-based cognitive radio systems', *IEEE Trans. Wirel. Commun.*, 2011, **10**, (9), pp. 2812–2817
22 Wang, S., Huang, F., Ge, M., Wang, C.: 'Optimal power allocation for OFDM-based cooperative relay cognitive radio networks'. IEEE ICC 2012, June 2012, pp. 1651–1655
23 Sidhu, G., Gao, F., Chen, W., Wang, W.: 'Joint subcarrier pairing and power loading in relay aided cognitive radio networks'. IEEE WCNC'12, April 2012, pp. 669–673
24 Laneman, J., Tse, D., Wornell, G.: 'Cooperative diversity in wireless networks: efficient protocols and outage behavior', *IEEE Trans. Inf. Theory*, 2004, **50**, (12), pp. 3062–3080
25 Li, Y., Wang, W., Kong, J., Hong, W., Zhang, X., Peng, M.: 'Power allocation and subcarrier pairing in OFDM-based relaying networks'. IEEE ICC 2008, May 2008
26 Hasna, M., Alouini, M.-S.: 'Harmonic mean and end-to-end performance of transmission systems with relays', *IEEE Trans. Commun.*, 2004, **52**, (1), pp. 130–135
27 Palomar, D., Chiang, M.: 'A tutorial on decomposition methods for network utility maximization', *IEEE J. Sel. Areas Commun.*, 2006, **24**, (8), pp. 1439–1451
28 Boyd, S., Vandenberghe, L.: 'Convex optimization' (Cambridge University Press, New York, 2004)
29 Bezdek, J.C., Hathaway, R.J.: 'Some notes on alternating optimization', *Lect. Notes Artif. Intell.*, 2002, **2275**, pp. 288–300
30 Grant, M., Boyd, S.: 'CVX: MATLAB software for disciplined convex programming [online]'. Available at: http://stanford.edu/, 2004

8 Appendix 1

8.1 Appendix: Proof of Theorem 2

Rewrite (14) as

$$H_L \Delta p_t = -\nabla \psi_t(p) \tag{20}$$

Step 1: Since $H_L = H_{L-1} + g_L g_L^{\mathrm{T}}$ and H_ls are positive defined, in particular, we have the following lemma

$$\Delta p_t = x_1^1 - \frac{g_1^{\mathrm{T}} x_1^1}{1 + g_1^{\mathrm{T}} x_2^1} x_2^1 \tag{21}$$

where the intermediate variables $x_1^1, x_2^1 \in R^N$ are the solutions of the following two sets of linear equations

$$\begin{aligned} H_{L-1} x_1^1 &= -\nabla \psi_t(p) \\ H_{L-1} x_2^1 &= g_L \end{aligned} \tag{22}$$

doi: 10.1049/iet-com.2013.0379

As mentioned above, we can obtain Δp_t if x_1^1 and x_2^1 have been worked out.

Step 2: Similarly, x_1^1 and x_2^1 can be calculated by solving the following three sets of linear equations

$$\begin{aligned} H_{L-2}x_1^2 &= -\nabla\psi_t(p) \\ H_{L-2}x_2^2 &= g_L \\ H_{L-2}x_3^2 &= g_{L-1} \end{aligned} \tag{23}$$

where $x_1^2, x_2^2, x_3^2 \in R^N$ are another intermediate variables.

Consequently, we may conclude that at Step l, $1 \le l \le L$, there are l intermediate variables which can be obtained from the intermediate variables calculated at *Step* $l+1$. Continue this process to *Step* $L+1$, $L+1$ variables are obtained by solving $L+2$ sets of linear equations

$$\begin{aligned} Hx_1^{L+1} &= -\nabla\psi_t(p) \\ Hx_2^{L+1} &= g_L \\ &\vdots \\ Hx_{L+2}^{L+1} &= g_0 \end{aligned} \tag{24}$$

Since H is diagonal, each set of equations in (22) can be solved at a cost of $\boldsymbol{O}(N)$. Then the computation cost of solving (22) is $\boldsymbol{O}(LN)$. Substituting these variables back to calculate all x_l^L, $l = 1, 2, \ldots, L+1$ with $\boldsymbol{O}(LN)$ complexity. Carry out the iteration process inversely. We can calculate all the intermediate variables $x_l^{l-1}, x_2^{l-1}, \ldots, x_l^{l-1}$ with a cost of at most $\boldsymbol{O}(LN)$ until Δp_t is worked out. The total cost is $\boldsymbol{O}(L^2N)$.