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Joint subchannel and power allocation in multiuser OFDM systems with minimal rate constraints

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SUMMARY

In this paper, we study the adaptive resource allocation in multiuser orthogonal frequency division multiplexing (OFDM) systems. We try to maximize the sum capacity of an OFDM system with given transmission power budget, while meeting users' minimal rate requirements. Unlike other resource allocation schemes, which generally separate subchannel allocation and power distribution into independent procedures, our proposed algorithm implements joint subchannel and power allocation. Given a set of subchannels, the required power to satisfy a user's minimal rate constraint is calculated by water-filling policy. Then, the user who requires the maximum power to meet the rate requirement has a priority to obtain an additional subchannel. The procedure continues until all subchannels are consumed, by which time the consumed power to meet all users' rate requirements is also worked out. Finally, the margin power is allocated among all subchannels in an optimal manner to maximize the sum capacity of the OFDM system. Simulation results show that our proposed algorithm performs better than other existing ones. The solution produced by our proposed algorithm is close to the upper bound, while its complexity is relatively lower compared with other methods, which makes it attractive for applications. Copyright © 2012 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising technique for wireless systems because of its high spectrum efficiency and flexibility in allocating radio resource. An OFDM system divides broad spectrum into a set of orthogonal narrowband subchannels to combat frequency-selective fading in wireless environment and provides a high performance physical layer. Adaptive resource allocation (RA) is an important issue in OFDM systems, attracting much attention in the past decade. Generally, the RA problem in OFDM systems consists of uplink and downlink scenarios. In this paper, we focus on the downlink scenarios, which are mainly fallen into two classes: margin adaptive and rate adaptive. The former is to minimize transmission power while satisfying users' rate requirements [1–6]. The optimization objective of the latter is usually to maximize the sum capacity with a given transmission power budget [7–18], which is also the topic of this work.

In [7], it is shown that the sum capacity can be maximized when each subchannel is assigned to the user with the largest channel signal-to-noise ratio (SNR), and the transmission power is distributed over all subchannels by water-filling policy. The algorithm proposed in [8] separates subchannel and power allocation into independent procedure to reduce complexity. A suboptimal subchannels allocation scheme is proposed, as well as an optimal power allocation. The proposed method exhibits a good tradeoff between sum capacity and fairness. In [9], a joint subchannel and power allocation

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algorithm is developed. Both subchannel allocation and power distribution are suboptimal. In [10], a three-step suboptimal resource allocation scheme is proposed, which exhibits a good performance as shown in simulation results. In [11], a user selection strategy is developed considering fairness. The objective of [7–11] is to concentrate on the system capacity. However, the users' rate requirements are not carefully considered, limiting their applications.

In [12–18], the optimization objective is to maximize the sum capacity while keeping some users' rate requirements satisfied. In [12], resource allocation procedure is decoupled into two steps. The proposed Hungarian algorithm has complexity of $O(N^4)$, which is too high for practical wireless systems. In [13], a two-round procedure RA algorithm is developed. First, each subchannel is assigned to the user with the largest SNR without considering the rate constraint. Then, subchannels are reassigned to the users whose rate requirements are not satisfied in the first round. A cost function for reallocating subchannel is proposed to obtain the least reduction of the overall capacity. A similar algorithm to [13] is presented in [14], except for a different cost function of reallocating subchannels. The disadvantage of the algorithms proposed in [13, 14] is that more subchannels are consumed by the user with lower channel gain because there is an assumption that the total transmission power is equally distributed over all subchannels. Besides, when some of the minimal rate requirements are high, the solution of the proposed algorithms is far from the optimal. In [15], an efficient subchannel reallocation algorithm is proposed. Comparing with [13, 14], the algorithm proposed in [15] has lower complexity while obtaining almost the same capacity.

In [16], an increment update strategy is developed to meet users' minimal rate requirements. When allocating a subchannel to a user, the required power over this subchannel is incremented by a constant. Then, the achievable rate of this user is updated. When all users' rate requirements are satisfied, each of the remaining subchannels is assigned to the user with the largest SNR over this subchannel. In [17], fairness is taken into consideration and a multiuser bargaining algorithm is proposed, which is based on the optimal coalition pairs among users. In [18], both delay-constrained and non-delay-constrained traffic scenarios are discussed. To make the formulated optimization task tractable, the problem is transformed into a convex programming form by time-sharing strategy. A suboptimal algorithm with low computational complexity is also proposed.

In this paper, we consider both fairness and sum capacity of a multiuser OFDM system, that is, we try to maximize the sum capacity of the system while satisfying users' minimal rate requirements. The user, who needs the maximum power to satisfy the rate requirement, is given a priority to get a new subchannel. When all users' rate requirements are satisfied, the required power to obtain target rates of all users is also worked out. Then, the margin power is distributed among all subchannels in a water-filling manner to maximize the sum capacity. Our proposed algorithm can produce solutions close to the optimal. Moreover, its complexity is lower than the others that have comparable sum capacity.

The remainder of this paper is organized as follows. In Section 2, system model is described, as well as the formulated problem. In Section 3, joint subchannel and power allocation schemes are presented in detail. Simulation results and discussions are presented in Section 4. Conclusion is drawn in Section 5.

2. SYSTEM MODEL AND PROBLEM FORMULATION

2.1. System model

Consider the downlink of a multiuser OFDM system with K users, in which base station can acquire channel state information through feedback channel at the beginning of each OFDM symbol. The channel between transceivers of base station and users undergoes independent frequency selective fading. The sets of users and the required rates are denoted as $\mathcal{K} = \{1, 2, \ldots, K\}$ and $\{R_{1,min}, R_{2,min}, \ldots, R_{K,min}\}$, respectively. The transmission power of the base station is limited to P_T. The total available bandwidth is B, which is divided into N subchannels denoted as $\mathcal{N} = \{1, 2, \ldots, N\}$. Each subchannel is exclusively assigned to one user for eliminating the intra-cell interference [7]. $p_{k,n}$ is the power allocated to user k on subchannel n. $h_{k,n}$ is the corresponding channel gain. Then, the achievable rate $r_{k,n}$ of user k on subchannel n can be calculated as

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$$r_{k,n} = \log_2\left(1 + \frac{p_{k,n} h_{k,n}^2}{\Gamma N_0 B/N}\right), \tag{1}$$

where Γ is the SNR gap. Generally, Γ is a constant that is related to a given bit-error-rate (BER) for a specific modulation/demodulation scheme, for example, $\Gamma = -\ln(5\text{BER})/1.6$ for an uncoded multilevel quadrature amplitude modulation system [10]. N_0 is the power spectral density of additive white Gaussian noise. Denote Ω_k as the allocated subchannel set of user k, the sum rate of user k is

$$R_k = \sum_{n\in\Omega_k} r_{k,n} = \sum_{n\in\Omega_k} \log_2\left(1 + p_{k,n} H_{k,n}\right), \tag{2}$$

where $H_{k,n} = \frac{h_{k,n}^2}{\Gamma N_0 B/N}$.

2.2. Problem formulation

Our goal is to find the best assignment of Ω_k and $p_{k,n}$ to maximize the overall system rate under the maximal power and the users' minimal rate requirements. Mathematically, the optimization task can be formulated as follows:

$$\begin{aligned} &\max_{\{\Omega_k, p_{k,n}\}} \sum_{k=1}^{K} R_k \\ s.t.\quad & C_1: R_k - R_{k,min} \geqslant 0, \forall k \in \mathcal{K} \\ & C_2: p_{k,n} \geqslant 0, \forall k \in \mathcal{K}, n \in \Omega_k \\ & C_3: P_T - \sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n} \geqslant 0 \\ & C_4: \Omega_1 \cup \Omega_2 \cup \cdots \cup \Omega_K \subseteq \mathcal{N} \\ & C_5: \Omega_k \cap \Omega_k' = \emptyset, \forall k \neq k'. \end{aligned} \tag{3}$$

C_1 is the minimal rate requirement. C_2 is intuitive because power of a subchannel is nonnegative. C_3 is the total power limitation. C_4 and C_5 indicate that each subchannel is allocated to one and only one user.

3. JOINT SUBCHANNEL AND POWER ALLOCATION

The equation (3) is generally very hard to solve because it is a mixed binary integer programming problem. There are K^N combinations of subchannel allocation assignment, which makes the optimal solution impossible to obtain even though the problem scale is moderate. The relaxed form of (3) can be obtained by introducing sharing factor $\rho_{k,n}$s, which will be shown in Section 3.1 to be a convex optimization problem, and the optimal solution can be worked out by standard convex optimization techniques. However, the solution of the relaxed problem is not feasible in practice because $\rho_{k,n}$s are continuous variables, violating the integer constraints in (3). We take the solution of the relaxed problem as an upper bound and design efficient algorithm to approach the bound.

3.1. The relaxation of the optimization problem

As mentioned earlier, an alternative approach to solve (3) is to relax constraints C_4 and C_5 to continuous variables [1, 18, 19]. A sharing factor $\rho_{k,n}$ is introduced to convert (3) into a convex optimization problem, as illustrated in the following:

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$$\begin{aligned}
&\max_{\{\rho_{k,n},p_{k,n}\}} \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{k,n}\log_2\left(1+\frac{p_{k,n}H_{k,n}}{\rho_{k,n}}\right)\\
s.t.\quad &C_1: \sum_{n=1}^{N}\rho_{k,n}\log_2\left(1+\frac{p_{k,n}H_{k,n}}{\rho_{k,n}}\right)-R_{k,min}\geqslant 0, \forall k\in\mathcal{K}\\
&C_2: p_{k,n}\geqslant 0, \forall k\in\mathcal{K}, n\in\mathcal{N}\\
&C_3: -\sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n}+P_T\geqslant 0\\
&C_4: \sum_{k=1}^{K}\rho_{k,n}=1, \forall n\in\mathcal{N}\\
&C_5: 1\geqslant\rho_{k,n}>0, \forall k\in\mathcal{K}, n\in\mathcal{N}.
\end{aligned}\tag{4}$$

It is easy to prove that (4) defines a convex problem because the objective function and all the constraints are convex. So it can be solved by standard convex optimization techniques [20]. The complexity of these algorithms is generally $O((KN)^3)$, which is too high to apply in practical wireless systems. More importantly, the solution of (4) is infeasible because nonzero sharing factor $\rho_{k,n}$ means that a subchannel can be used by multiple users, which is not the case for our considered problem. However, we can take the solution of (4) as the upper bound to measure the performance of other algorithms.

3.2. *Joint subchannel and power allocation*

Our intuition for resource allocation is following. Given a subchannel assignment, if each user achieves the required rate by consuming power as little as possible, there will be more power left to maximize the sum capacity of the OFDM system. So, the key idea of our proposed RA algorithm is: A user that costs much power to obtain the minimal rate has a priority to get an available subchannel. By doing so, we can reduce the total power consumed for satisfying all users' rate requirements.

At the beginning of our RA process, we have no information on how much power a user needs to meet the rate requirement. So we should assign at least one subchannel to the user with rate requirement $R_{k,min}$. Furthermore, we should determine a profitable selection order among K users to make the power cost as small as possible. Denote A as the set of the available subchannels and B as the set of the users with rate requirements, respectively. SNR_k^{max} is the maximum SNR over N subchannels for the k-th user, and

$$k^* = \arg\min_{k\in B} SNR_k^{max},$$

we allocate user k^* the subchannel n^*, which has the maximum SNR for this user first. Then, we set $A = A - \{n^*\}$ and $B = B - \{k^*\}$. This procedure continues until all users obtain a subchannel. Denote P_k as the required power of user k to achieve the minimal rate $R_{k,min}$, when all users have been allocated a subchannel, we have

$$P_k = \frac{2^{R_{k,min}}-1}{H_{k,n}},$$

where $H_{k,n}$ indicates that subchannel n is allocated to user k. The total consumed power is $P_{total} = \sum_k P_k$.

In the following, we assign the rest of the subchannels to minimize total power cost. We should solve such an optimization problem: Given a set Ω_k, representing the subchannels allocated to user k, how to minimize the power cost if satisfying the k-th user's rate requirement. The optimization problem can be described as

$$\begin{aligned}
&\min_{\{s_{k,n}\}} \sum_{n\in\Omega_k} s_{k,n}\\
s.t.\quad &C_1: \sum_{n\in\Omega_k}\log_2(1+s_{k,n}H_{k,n})\geqslant R_{k,min}\\
&C_2: s_{k,n}\geqslant 0, \forall n\in\Omega_k,
\end{aligned}\tag{5}$$

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where $s_{k,n}$ is the allocated power of user k on subchannel n. The equation (5) is also a convex problem and has a unique optimal solution. To work out the optimal solution, we apply the Karush–Kuhn–Tucker (KKT) conditions of (5) to find the primal and dual optimal points [20]. The Lagrangian of (5) is

$$J = \sum_{n\in\Omega_k} s_{k,n} - \eta_k \left(\sum_{n\in\Omega_k} \log_2(1 + s_{k,n}H_{k,n}) - R_{k,min} \right) - \xi_{k,n}s_{k,n}, \tag{6}$$

where η_k and $\xi_{k,n}$ are the Lagrange multipliers. Let $s^*_{k,n}$, η^*_k, and $\xi^*_{k,n}$ be the primal and dual optimal points with zero duality gap [20]; based on the KKT conditions, we have the following equations:

$$1 - \frac{\eta^*_k}{\ln 2}\frac{H_{k,n}}{1 + s^*_{k,n}H_{k,n}} - \xi^*_{k,n} = 0. \tag{7}$$

$$\eta^*_k \left(\sum_{n\in\Omega_k} \log_2(1 + s^*_{k,n}H_{k,n}) - R_{k,min} \right) = 0 \quad \eta^*_k \geqslant 0. \tag{8}$$

$$\xi^*_{k,n}s^*_{k,n} = 0 \quad \xi^*_{k,n} \geqslant 0. \tag{9}$$

According to (7) and (9), we have $\xi^*_{k,n} = 0$ and $s^*_{k,n} = \eta^*_k/\ln 2 - 1/H_{k,n}$, for the case of $s^*_{k,n} > 0$. Otherwise, $s^*_{k,n} = 0$, which means that no power is distributed to subchannel n. So, the optimal solution of (5) is given by

$$s^*_{k,n} = \left(L_k - \frac{1}{H_{k,n}} \right)^+ \quad \texttt{for} \;\; n \in \Omega_k, \tag{10}$$

where $(x)^+ = max(x, 0)$, $L_k = \eta^*_k/\ln 2$. Furthermore, the numerical value of η^*_k must be greater than zero if $s^*_{k,n}$ is greater than zero. According to (8), $s^*_{k,n}$ can be worked out by the following equation:

$$\sum_{n\in\Omega_k} \log_2(1 + s^*_{k,n}H_{k,n}) = R_{k,min}. \tag{11}$$

The user with the largest power cost has the priority to get an additional subchannel. We allocate user k the subchannel that has the largest SNR for user k only if $L_k > 1/H_{k,n}$. And L_k and P_k are updated simultaneously. Otherwise, we assign no subchannel to user k. Thus, the distributed power to user k over the selected subchannel n is $s^*_{k,n} = L_k - 1/H_{k,n}$. Substituting the $s^*_{k,n}$ into (11), we have

$$\sum_{n\in\Omega_k} \log_2(1 + s^*_{k,n}H_{k,n}) = \sum_{n\in\Omega_k} \log_2(L_kH_{k,n}) = \log_2(L_k^{N_k} \prod_{n\in\Omega_k} H_{k,n}) = R_{k,min}. \tag{12}$$

So, the closed-form expression of L_k is

$$L_k = \left[\frac{2^{R_{k,min}}}{\prod_{n\in\Omega_k} H_{k,n}} \right]^{1/N_k}, \tag{13}$$

where N_k is the number of subchannels allocated to user k. Then, P_k can be calculated as

$$P_k = \sum_{n\in\Omega_k} s^*_{k,n} = N_kL_k - \sum_{n\in\Omega_k} \frac{1}{H_{k,n}}. \tag{14}$$

If some users' rate requirements are not satisfied when all subchannels are consumed and the required power is larger than P_T, an outage event occurs, which means that there is no feasible

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solution. When all users' rate requirements are satisfied and the required power is less than P_T, the optimal solution exists. We allocate each of the remaining subchannels to the user who has the largest SNR over this subchannel to maximize the system sum capacity.

The outline of our joint subchannel and power allocation algorithm is described in Table I. At the first step, we assign user $k \in B$ one subchannel to initialize P_k, as well as water-filling level L_k.

Table I. Jointly Subchannel and Power Allocation

Initialization:
$\Omega_k = \emptyset$ for all k, $A = \mathcal{N}$ and B is the set of the users with $R_{k,min} > 0, k \in B$.
Step 1:
$B^* = B$;
while $B^* \neq \emptyset$
    $k^* = \arg\min_{k \in B^*} SNR_k^{max}$;
    $n^* = \arg\max_{n \in A} H_{k^*,n}$;
    $A = A - \{n^*\}$; $B^* = B^* - \{k^*\}$; $\Omega_{k^*} = \Omega_{k^*} \cup \{n^*\}$;
    $L_{k^*} = 2^{R_{k^*,min}}/H_{k^*,n^*}$; $P_{k^*} = L_{k^*} - 1/H_{k^*,n^*}$;
endwhile
$P_{total} = \sum_{k \in B} P_k$;
Step 2:
$B^* = B$;
while $P_{total} > P_T$
    if $A = \emptyset$ or $B^* = \emptyset$
        the algorithm terminates and an outage event occurs;
    endif
    $k^* = \arg\max_{k \in B^*} P_k$;
    $n^* = \arg\max_{n \in A} H_{k^*,n}$;
    if $L_{k^*} - 1/(H_{k^*,n^*}) > 0$
        $A = A - \{n^*\}$; $\Omega_{k^*} = \Omega_{k^*} \cup \{n^*\}$;
        update L_{k^*} using (13);
        update P_{k^*} using (14);
        $P_{total} = \sum_{k \in B} P_k$;
    else
        $B^* = B^* - \{k^*\}$;
    endif
endwhile
Step 3:
while $A \neq \emptyset$
    $\forall n^* \in A$; $k^* = \arg\max_{k=1,\ldots,K} H_{k,n^*}$;
    $A = A - \{n^*\}$; $\Omega_{k^*} = \Omega_{k^*} \cup \{n^*\}$;
    if $L_{k^*} - 1/(H_{k^*,n^*}) > 0$ and $R_{k^*,min} > 0$
        update L_{k^*} using (13);
        update P_{k^*} using (14);
        $P_{total} = \sum_{k \in B} P_k$;
    endif
endwhile

When all users obtain at least one subchannel, the consumed sum power P_{total} is initialized. At the second step, when $P_{total} > P_T$, we allocate the user with the largest power cost an additional subchannel, which has the largest SNR for this user. If some users' rate requirements are not satisfied when all subchannels have been allocated and P_{total} cannot decrease, an outage event occurs. Otherwise, when all users' rate requirements are satisfied and $P_{total} \leqslant P_T$, we assign each of the remaining subchannel to the user who has the largest SNR over this subchannel and update P_{total}, P_k, and L_k if necessary, as shown in the third step of Table I.

3.3. Distribute margin power allocation: an optimal strategy

When joint subchannel and power allocation procedure completes, all subchannels have been allocated and the consumed sum power is also determined if feasible solution exists . There is margin power that can be used to maximize the sum capacity of the system. In this subsection, we propose an optimal power distribution algorithm. Given a subchannel assignment, (3) is converted into a continuous optimization problem as follows:

$$\begin{aligned} &\max_{\{p_{k,n}\}} \sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2(1 + p_{k,n} H_{k,n}) \\ s.t. \quad &C_1 : \sum_{n \in \Omega_k} \log_2(1 + p_{k,n} H_{k,n}) - R_{k,min} \geqslant 0, \forall k \in \mathcal{K} \\ &C_2 : p_{k,n} \geqslant 0, \forall k \in \mathcal{K}, n \in \Omega_k \\ &C_3 : P_T - \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} \geqslant 0. \end{aligned} \tag{15}$$

Again, we can see that (15) is a convex problem [20] with a global optimal solution.

The Lagrangian of (15) is

$$\begin{aligned} J = &\sum_{k=1}^{K} \sum_{n \in \Omega_k} \log_2(1 + p_{k,n} H_{k,n}) \\ &+ \sum_{k=1}^{K} \lambda_k \left(\sum_{n \in \Omega_k} \log_2(1 + p_{k,n} H_{k,n}) - R_{k,min} \right) \\ &+ \sum_{k=1}^{K} \sum_{n \in \Omega_k} \nu_{k,n} p_{k,n} - \mu \left(\sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n} - P_T \right), \end{aligned} \tag{16}$$

where $\lambda_k, \nu_{k,n}, \mu$ are Lagrange multipliers for the constraints C_1, C_2, and C_3 of (15), respectively. The necessary conditions for optimality are given by KKT conditions,

$$\frac{1 + \lambda_k^*}{\ln 2} \frac{H_{k,n}}{1 + p_{k,n}^* H_n} + \nu_{k,n}^* - \mu^* = 0, \tag{17}$$

$$\lambda_k^* \left(\sum_{n \in \Omega_k} \log_2(1 + p_{k,n}^* H_{k,n}) - R_{k,min} \right) = 0 \quad \lambda_k^* \geqslant 0, \tag{18}$$

$$\nu_{k,n}^* p_{k,n}^* = 0 \quad \nu_{k,n}^* \geqslant 0, \tag{19}$$

$$\mu^* \left(P_T - \sum_{k=1}^{K} \sum_{n \in \Omega_k} p_{k,n}^* \right) = 0 \quad \mu^* \geqslant 0, \tag{20}$$

where $p_{k,n}^*$, λ_k^*,$\nu_{k,n}^*$ and μ^* are the primal and dual optimal point with zero dual gap.

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According to (19), we have $\nu^*_{k,n} = 0$, if $p^*_{k,n} > 0$. So, $p^*_{k,n}$ can be worked out by substituting $\nu^*_{k,n} = 0$ into (17). Otherwise, $p^*_{k,n} = 0$. That is,

$$p^*_{k,n} = \left(\frac{1}{\mu^* \ln 2} + \frac{\lambda^*_k}{\mu^* \ln 2} - \frac{1}{H_{k,n}} \right)^+, \quad \texttt{for} \quad n \in \Omega_k. \tag{21}$$

The optimal power allocation follows a multilevel water-filling principle [16]. More power is allocated to the subchannel with higher SNR. It is worth noting that the water levels of the users can be different. When

$$\sum_{n \in \Omega_k} \log_2(1 + p^*_{k,n} H_{k,n}) - R_{k,min} > 0. \tag{22}$$

where $L = \frac{1}{\mu^* \ln 2}$, we have $\lambda^*_k = 0$ and the water-filling level of user k is L. And when

$$\sum_{n \in \Omega_k} \log_2(1 + p^*_{k,n} H_{k,n}) - R_{k,min} = 0, \tag{23}$$

we have $\lambda^*_k \geqslant 0$.

Notice that if (23) is satisfied, the optimal power distribution for user k involves the same structure as (5). Hence, the water-filling level of user k is L_k, which has been obtained in the joint subchannel and power allocation procedure. The power allocated to subchannel n can be calculated as

$$p^*_{k,n} = \left(max(L, L_k) - \frac{1}{H_{k,n}} \right)^+ = max(L - \frac{1}{H_{k,n}}, s^*_{k,n}), n \in \Omega_k. \tag{24}$$

When L and L_k (or $s^*_{k,n}$) are known, we can obtain optimal power distribution over subchannels directly by (24). Given a subchannel assignment, the water level L_k of each user (or $s^*_{k,n}$) is confirmed. So, the optimal power allocation can be achieved by using standard water-filling algorithm to find the L,

$$\sum_{k=1}^{K} \sum_{n \in \Omega_k} \left(L - max(L_k, \frac{1}{H_{k,n}}) \right)^+ = P_T - P_{total}, \tag{25}$$

where P_{total} has been worked out by the joint subchannel and power allocation scheme.

3.4. Complexity analysis

The computational complexity of our proposed algorithm can be analyzed roughly as follows. We only consider float operations. When assigning a subchannel to a user at the first step, the complexity is at most $\boldsymbol{O}(N)$ because the number of subchannels is always larger than that of the users. When updating P_{total}, P_k, and L_k of this user at the second step, we can record the values of $V_k = 2^{R_{k,min}} / \prod_{n \in \Omega_k} H_{k,n}$ and $U_k = \sum_{n \in \Omega_k} 1/H_{k,n}$, and store them. The complexity of updating L_k, P_k, and P_{total} is $\boldsymbol{O}(1)$. Then, the total complexity of calculating L_k, P_k and P_{total} is $\boldsymbol{O}(N)$ at the second step. The third step has also a complexity of $\boldsymbol{O}(N)$. For margin power allocation, the worst scenario is that all users have no rate constraints, for which case power allocation has a complexity of $\boldsymbol{O}(N \log_2 N)$. Thus, the overall complexity of our proposed algorithm is $\boldsymbol{O}(3N + N \log_2 N)$, bounded by $\boldsymbol{O}(N \log_2 N)$. The overall storage complexity is obviously bounded by $\boldsymbol{O}(K + N)$.

4. SIMULATIONS AND DISCUSSIONS

Experiments were performed to test the performance of our proposed algorithm. Consider a multiuser OFDM system with $N = 64$ subchannels. Each user locates randomly within a 3×3-km area. The path loss exponent is 4, the variance of logarithmic normal shadow is 10 dB, and the

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amplitude of multipath fading is Rayleigh. The noise power is 10^{-13}W and Γ is set to 1. All results are averaged by 10000 Monte Carlo simulations.

First, we investigate the service outage probability. There are $K = 4$ users. Each user has a fixed minimal rate requirement of $R_{min} = 10$,bits/symbol. Figure 1 illustrates the service outage probability as a function of transmission power budget. For comparison, the service outage probabilities of the relaxation method proposed in Section3.1, the schemes proposed in [16] and [18], which have the same complexity as ours, are also shown in Figure 1. We observe that the performance of our algorithm is close to the upper bound, both of which outperform the schemes proposed in [16] and [18]. The outage probability decreases as the power budget increases. We also give the curve of sum rate as a function transmission power budget in Figure 2 under the same simulation scenarios. From Figure 2, we can see that our proposed algorithm can approach the upper bound. The gap is less than 5% for different power budgets.

Then, we investigate the achievable sum rate as a function of minimal rate constraints R_{min} s for an OFDM system with $K = 4$ users. The power limitation is $P_T = 0.2$ W. Consider two cases, one of which is shown in Figure 3, where all users have the same rate requirement R_{min} that varies from 2–20 bits/symbol. The other is shown in Figure 4, where the rate requirements are $0.5R_{min}$, R_{min}, $0.5R_{min}$, and $10R_{min}$, respectively, and R_{min} varies from 1–10 bits/symbol. From Figures 3 and 4, we can observe that the sum rate gap between our proposed algorithm and the upper

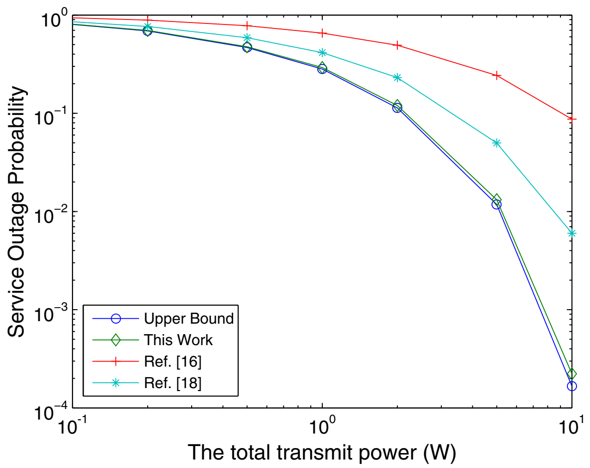

Figure 1. Service outage probability as a function of transmission power. $K = 4$ and $R_{min} = 10$ bits/symbol.

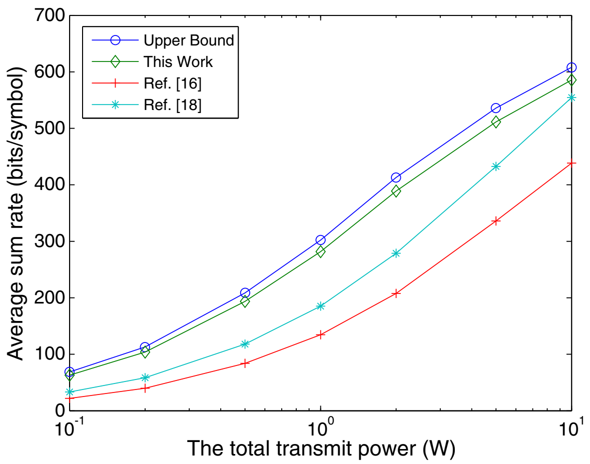

Figure 2. Sum rate as a function of transmission power. $K = 4$ and $R_{min} = 10$ bits/symbol.

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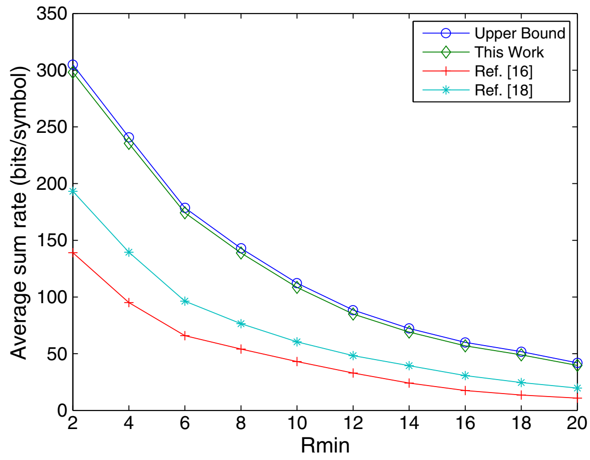

Figure 3. Sum rate as a function of minimal rate R_{min}. $K = 4$ users and $P_T = 0.2$W. All users have the same minimal rate requirements.

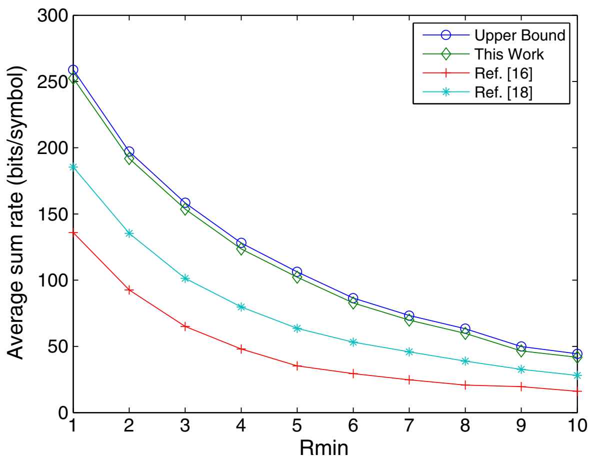

Figure 4. Sum rate as a function of minimal rate R_{min}. $K = 4$ users and $P_T = 0.2$W. The users have the minimal rate requirements of $0.5R_{min}$, $2R_{min}$, $0.5R_{min}$ and $10R_{min}$, respectively.

bound is always smaller than 5%. The methods proposed in [16] and [18] produce large capacity gap as seen in Figures 3 and 4. Especially, the gap between [16] and the upper bound is larger than 40%. We also observe that the sum rate decreases as the minimal rate requirements increases. It is because many subchannels are allocated to the users with low channel gains to satisfy their rate requirements, resulting that the system benefits little from multiuser diversity.

Finally, we investigate the sum capacity as a function of the number of users. The transmission power is $P_T = 2$ W. Each user has a minimal rate requirement of $R_{min} = 10$. From Figure 5, we can see when the number of users is not more than five, the sum rate of the OFDM system increases as the number of users increases. This phenomenon can also be explained that the OFDM system benefits from multiuser diversity [1], which means that a subchannel is more possible to have better channel gain as the increasing of the number of users. On the other hand, as seen in Figure 5, when the number of users is more than five, the sum rate decreases as the number of users increases. It is because the users' minimal rate requirements result that many subchannels are allocated to the users with lower channel gain to meet the rate requirements, which weakens the multiuser diversity effect. Again, our proposed algorithm can achieve the upper bound while other algorithms perform worse than ours.

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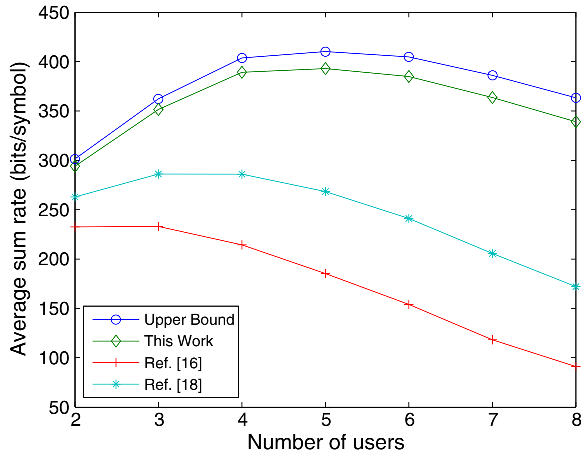

Figure 5. Sum rate as a function of the number of users. $K = 4$, $P_T = 2$W and each user has the same minimal rate requirement $R_{min} = 10$ bits/symbol.

5. CONCLUSIONS

We studied the resource allocation problem in multiuser OFDM systems, where the optimization objective is to maximize the sum capacity under transmission power and minimal rate constraints. Subchannels and power are jointly allocated. The user who needs the largest power to achieve its minimal rate requirement has the priority to obtain an additional subchannel. When all subchannels are allocated to the users, the required power of each user is also worked out. Then, margin power is distributed among all subchannels in a water-filling manner to maximize the sum capacity of the system. Simulation results show that the performance of our proposed algorithm is better than other ones and close to the optimal, while the computational complexity is low, making it promising for applications.

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