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Energy-efficient power allocation for cognitive radio networks with minimal rate requirements

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SUMMARY

Because the energy consumption is growing rapidly, green radio, which lays emphasis on the energy efficiency (EE) in wireless networks, is becoming increasingly important. As potential paradigms for future wireless network design, cognitive radio (CR) is a promising technology to solve the spectrum shortage and inefficiency issues, while cooperative relay is capable of improving spectral efficiency by combating severe fading in wireless environment. In this paper, we investigate the energy-efficient power allocation problem in orthogonal frequency division multiplexing (OFDM)-based relaying CR networks. We develop a general framework to maximize the overall EE of the CR system, under the constraints of transmission power budget, traffic demands, and the interference constraints of the primary users. Our problem formulation is a nonconvex optimization task, and it is hard to obtain the optimal solution. We first convert our formulated problem into a convex optimization problem via its hypograph form, which can be solved by the barrier method. Then we further speed up the computation of Newton step during the barrier method, significantly reducing the complexity of the algorithm by exploiting its special structure. Numerical results validate that our method can exploit the overall EE of CR systems, while the algorithm converges efficiently and stably.

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KEY WORDS: energy efficiency; cognitive radio; OFDM; relay; resource allocation; optimization

1. INTRODUCTION

Cognitive radio (CR) is an attractive paradigm to alleviate the scarcity of spectrum resource [1]. CR users, which are also referred as secondary users (SUs), can access the licensed spectrum, as long as the interference to the primary users (PUs) is kept below a preset threshold. As to meet the opportunistic access requirements, the physical layer of a CR network should be very flexible, which necessitates multicarrier technologies in CR networks. Orthogonal frequency division multiplexing (OFDM) is adopted as a promising air interface of CR networks, which can offer an impressive flexibility in radio resource allocation (RA) [2, 3].

However, CR networks also introduce many challenging problems; one of them is the transmission opportunity exploitation of the SUs. Specifically, the SUs must exploit the transmission opportunity to satisfy their quality-of-service (QoS) requirements, while not causing much unacceptable service degradation of the PUs. In some cases, large power may be used to guarantee the transmission between some CR nodes, which probably causes severe interference to the PUs. Hence, traditional transmission scheme is no longer suitable for CR networks in this case. Instead, as a key spatial diversity technique, relay technology is effective to weaken the effect of fading by means of generating an alternative path to support the communication between two CR nodes. Therefore, relay scheme has the ability to boost the overall performance of wireless

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systems, especially for CR networks [4]. Besides the advantage of improving QoS of the CR networks, relay scheme can also protect the PUs due to lower transmit powers of the source node (SN) and relay node (RN) compared with the power of the direct link, which generates less interference to the PUs.

For the current CR system, adaptive RA has attracted much attention in recent years. It can be found that much effort has been made to enhance the throughput of networks for RA over the past decade. In [5], an improved water-filling power allocation scheme is proposed, based on which a simple subcarrier allocation algorithm is adopted. In [6], RA problem for the multiuser OFDM-based CR systems under the constraints of proportional rate is discussed, for both capacity and fairness considerations. RA in relaying CR system has also been studied. In [7], the authors propose a simplified algorithm to select relay and allocate power to maximize the capacity under the interference constraint. Biyanwilage *et al.* [8] aim to maximize the instant capacity of the CR system with optimal power distribution schemes, while Nouri and Noori [9] propose power allocation to maximize usage efficiency of the secondary network in which SUs transmit data through the same spectral band that has been assigned to the PUs. Biyanwilage *et al.* [10] study the optimal power allocation schemes to maximize the capacity of the CR system, where the average interference constraints should be reached for the PUs, and the CR transmitters have maximum transmit power constraints.

As the high-data-rate applications grow, the energy consumption is also growing rapidly. Meanwhile, many greenhouse gases and high-operation expenditures are caused by the increase of energy consumption in wireless networks [11]. Green radio, which focuses on energy efficiency (EE) in wireless networks, has attracted many research and standard development activities [12]. Several research projects and organizations dedicated to develop more energy-efficient architectures and techniques, are being carried out recently. Different from the two classes of conventional dynamic RA schemes, which are rate adaptation and margin adaptation [13], energy-efficient resource management is a special case that is generally to maximize the EE of wireless networks.

Recently, a growing number of researches have investigated the EE in wireless communications. A survey depicts the technology of some international projects and discusses the state-of-the-art research that can be found in [14]. Xiong *et al.* [15] mainly address the fundamental tradeoff between EE and spectral efficiency (SE) in downlink OFDMA networks. A power allocation scheme with low complexity is developed by considering time-averaged EE metrics for an uplink OFDMA system in [16]. Ng *et al.* [17] transform the formulated problem in fractional form into a problem in subtractive form, then an RA scheme is proposed to maximize the EE. In [18], the considered non-convex optimization problem is converted to a convex form by using time-sharing, which can be solved by an iterative algorithm.

Relaying CR system can also help improve EE. The potential advantage of RN in developing EE comes from the fact that the distance between SN and RN, as well as the distance between RN and destination node (DN), is smaller than the direct link between SN and DN; thus, the radio links between SN and RN, and the links between RN and DN suffer lower path losses as compared with the direct links between SN and DN. As a result, the relay technology can provide required service with higher EE. However, it is noteworthy that there is few work on the energy-efficient RA of the relaying CR systems. Actually, dynamic energy-efficient RA is extremely important for CR networks, because it is a prerequisite for fully exploiting the overall EE of CR systems, and achieving the highly utilization of the limited transmission power. Moreover, most existing algorithms cannot be used for this case.

In this paper, energy-efficient power allocation issue is investigated for an OFDM-based CR system, enhanced by an RN. Here, we formulate an optimization problem to maximize overall EE of the CR relaying system under the individual power constraint of SN and RN while satisfying the

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minimal capacity requirement. On the other hand, the interference to the PUs must be strictly kept in a tolerable range. The formulated optimization problem is a nonlinear fractional programming problem, which can be equivalently transformed into a convex optimization problem via its hypograph form. By intensively analyzing the equivalent convex problem, an efficient barrier method is adopted to calculate the optimal solution.

The rest of this paper is organized as follows. In Section 2, system model and optimization problem are given. In Section 3, an efficient algorithm based on the barrier method is developed. Simulation results and discussions are shown in Section 4. Finally, conclusion are given in Section 5.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a three-node relay-enhanced CR network, which consists of an SN, an RN, and a DN, coexisting with a licensed system with L PUs. The relay operates in a time-division half-duplex mode using the amplify-and-forward (AF) protocol. As we know, RNs can operate with different kinds of modes. AF relays can be used as gap fillers; however, it is known that AF relays amplify the interference and noise as well when they amplify the desired signal. Decode-and-forward (DF) relays have the ability to detect the desired signal and then encode and forward it. As a result, DF relays can be applied in interference limited environment. However, DF relays need many standardization efforts, and the system will be very complex. As a consequence, RN operates with AF protocol in this paper because of its simplicity. The operation of the relaying CR network is as follows: In the first time slot, the SN transmits, while the RN and DN listen; then the relay amplifies the signal received slot and transmits them to the DN in the second time slot. We assume the direct link between the SN and DN experiences deep fading, which is completely weak. Although the system model is simple, our power allocation scheme can be easily applied to other complex communication systems such as multiuser scenarios, combined with subchannel assignments.

Throughout this paper, destination is assumed to receive the signals from the source without diversity. Besides, we assume the perfect channel state information is available at both the CR source and relay. Under this circumstance, the results obtained by our method are an upper bound on the achievable EE with channel estimation errors.

The whole available bandwidth is divided into N subchannels and denoted as $\mathcal{N} = \{1, 2, \ldots, N\}$. The spectrum of the nth subchannel is $f_0 + (n-1)B$ to $f_0 + nB$, where f_0 is the starting frequency and B is the bandwidth of each subchannel. Assume the lth PU's band spans from f_l to $f_l + B_l$, where f_l and B_l are the starting frequency and bandwidth, respectively. Thus, the interference introduced to the lth PU in the first slot on the nth subchannel with unit transmission power is [19]

$$I_{n,l}^{SP} = \int_{f_l - f_0 - (n-1/2)B}^{f_l + B_l - f_0 - (n-1/2)B} g_{n,l}^{SP} \phi(f) df, \tag{1}$$

where $g_{n,l}^{SP}$ is the power gain from the CR SN's transmitter to the receiver of the lth PU on the nth subchannel. $\phi(f)$ is the baseband power spectral density (PSD) of OFDM signal with $\phi(f) = T\left(\frac{\sin \pi f T}{\pi f T}\right)^2$, where T is OFDM symbol duration.

Similarly, interference caused by the CR relay to the lth PU in the second slot with unit transmission power on the nth subchannel is given by

$$I_{n,l}^{RP} = \int_{f_l - f_0 - (n-1/2)B}^{f_l + B_l - f_0 - (n-1/2)B} g_{n,l}^{RP} \phi(f) df, \tag{2}$$

where $g_{n,l}^{RP}$ is the power gain from the CR RN to the lth PU's receiver on the nth subchannel.

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The interference introduced into the CR RN and DN by the lth PU on the nth subchannel can be calculated as follows:

$$
\begin{aligned}
I_{n,l}^{PR} &= \int_{f_0+(n-1)B-f_l-B_l/2}^{f_0+nB-f_l-B_l/2} g_{n,l}^{PR}\phi_l(f)df, \\
I_{n,l}^{PD} &= \int_{f_0+(n-1)B-f_l-B_l/2}^{f_0+nB-f_l-B_l/2} g_{n,l}^{PD}\phi_l(f)df,
\end{aligned} \tag{3}
$$

where $g_{n,l}^{PR}$ and $g_{n,l}^{PD}$ are the power gain from the lth PU's receiver to the RN and DN on the nth subchannel, respectively. $\phi_l(f)$ is the PSD of the lth PU's signal.

Denote $p_{s,n}$ is the transmission power on the nth subchannel in the first hop, while the relay amplifies the signal using power $p_{r,n}$ on the nth subchannel in the second hop, the amplification factor β_n on the nth subchannel is

$$
\beta_n = \sqrt{\frac{p_{r,n}}{p_{s,n}|h_{sr,n}|^2 + N_r B + \Sigma_{l=1}^{L} I_{n,l}^{PR}}}, \tag{4}
$$

where $h_{sr,n}$ is the channel gain from the SN to RN and N_r is the PSD of additive white Gaussian noise (AWGN) at the CR relay. Consequently, the received signal-to-noise ratio (SNR) on the nth subchannel is

$$
\begin{aligned}
\gamma_n &= \frac{p_{s,n}|h_{rd,n}|^2\beta_n^2|h_{sr,n}|^2}{N_d B + \sum\limits_{l=1}^{L} I_{n,l}^{PD} + \left(N_r B + \sum\limits_{l=1}^{L} I_{n,l}^{PR}\right)\beta_n^2|h_{rd,n}|^2} \\
&= \frac{p_{s,n}a_n p_{r,n}b_n}{1 + p_{s,n}a_n + p_{r,n}b_n},
\end{aligned} \tag{5}
$$

where $h_{rd,n}$ is the channel gain between the CR relay and destination on nth subchannel, and N_d the PSD of AWGN at the destination. For notation brevity, let $a_n = |h_{sr,n}|^2/\left(N_r B + \Sigma_{l=1}^{L} I_{n,l}^{PR}\right), b_n = |h_{rd,n}|^2/\left(N_d B + \Sigma_{l=1}^{L} I_{n,l}^{PD}\right)$.

Because we assume the DN receives the signals without diversity and the signal from the direct path is not taken into account, the transmission rate on the nth subchannel from source to the destination can be calculated as [20]

$$
R_n = \frac{1}{2}\log(1 + \gamma_n), \tag{6}
$$

where the rate is scaled by 1/2 because the transmission takes two time slots.

Here, the EE is defined as the system throughput for unit energy consumption in $bits/Joule$. Besides the transmission power to guarantee the reliable data transmission, the energy consumption also includes circuit consumed power in the active mode, which is caused by signal processing, active circuit blocks, and so on. The associated circuit energy consumption is generally modeled as a constant, denoted by P_c [21, 22]. Thus, the EE in bits/Joule of the CR relaying system is given by

$$
\eta_{EE} = \frac{\sum_{n=1}^{N} R_n}{\sum_{n=1}^{N} p_{s,n} + \sum_{n=1}^{N} p_{r,n} + P_c}. \tag{7}
$$

Throughout this paper, we aim to maximize the EE of the CR system with cooperative relay, under the individual transmission power budget of the CR SN and RN. Besides, the total throughput should meet the minimal requirement $R_{\min}$, while the interference to the PUs should be kept below a preset threshold. Thus, we can formulate the energy-efficient optimization problem as follows:

$$\begin{aligned}
\max_{p_{s,n},p_{r,n}} \quad & \eta_{EE} = \frac{\sum_{n=1}^{N} R_n}{\sum_{n=1}^{N} p_{s,n} + \sum_{n=1}^{N} p_{r,n} + P_c} \\
s.t.\ C1 \quad & \sum_{n=1}^{N} R_n \geqslant R_{\min} \\
C2 \quad & \sum_{n=1}^{N} p_{s,n} \leqslant P_S \\
C3 \quad & \sum_{n=1}^{N} p_{r,n} \leqslant P_R \\
C4 \quad & \sum_{n=1}^{N} p_{s,n} I_{n,l}^{SP} \leqslant I_l^{th}, l = 1, \ldots, L \\
C5 \quad & \sum_{n=1}^{N} p_{r,n} I_{n,l}^{RP} \leqslant I_l^{th}, l = 1, \ldots, L \\
C6 \quad & p_{s,n} \geqslant 0,\ p_{r,n} \geqslant 0, \forall n,
\end{aligned} \tag{8}$$

where P_S and P_R are the total transmission power limit of the SN and RN, respectively. I_l^{th} is the preset interference threshold of the lth PU. C1 guarantees the minimal rate requirement of the relaying CR system. C2 and C3 represent the constraints on transmission power budget of CR source and relay individually, while C4 and C5 keep the interference to PUs under the tolerable range.

3. THE OPTIMAL POWER ALLOCATION ALGORITHM

Obviously, Equation (8) defines a nonlinear fractional programming problem, which is generally challenging to deal with. Furthermore, the transmission rate R_n is a nonconvex function associated with power allocation variables $p_{s,n}$ and $p_{r,n}$, because of the existence of numeric constants in Equation (5), which makes the problem even difficult. To make the problem tractable and reduce the computation complexity, we adopt the following approximation

$$R_n \approx \frac{1}{2} \log\left(1 + \frac{p_{s,n} a_n p_{r,n} b_n}{p_{s,n} a_n + p_{r,n} b_n}\right), \tag{9}$$

where γ_n in Equation (5) is approximated by the harmonic mean of $p_{s,n}$ and $p_{r,n}$. The reasonability of this approximation is shown in [23, 24]; the gap between the optimal result and the proposed one by using such a approximation is proven to be tight. Note that R_n in Equation (9) is jointly convex with $\boldsymbol{p}_s$ and $\boldsymbol{p}_r$, where we stack all variables $\{p_{s,n}\}_{n=1}^N$ and $\{p_{r,n}\}_{n=1}^N$ into the vectors $\boldsymbol{p}_s = \{p_{s,1}, \ldots, p_{s,N}\}$ and $\boldsymbol{p}_r = \{p_{r,1}, \ldots, p_{r,N}\}$, respectively.

3.1. *The equivalent hypograph problem form*

Note that Equation (8) is still a nonconvex optimization problem, even with the approximation of R_n in (8). Nevertheless, an equivalent transformation of Equation (8) can lead to a convex optimization problem via its hypograph form [25], which has also been used in [26]. The hypograph of the objective function η_{EE} is defined as

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$$\texttt{hypo}\ \eta_{EE} = \{(\boldsymbol{p}_s, \boldsymbol{p}_r, y) | y \leqslant \eta_{EE}(\boldsymbol{p}_s, \boldsymbol{p}_s)\}, \tag{10}$$

The hypograph form of Equation (8) is given by

$$\begin{aligned} \max_{\boldsymbol{p}_s, \boldsymbol{p}_r, y} \quad & y \\ s.t. \quad & \eta_{EE}(\boldsymbol{p}_s, \boldsymbol{p}_r) \geqslant y \\ & C1 \sim C6 \text{ in Equation(8)} \\ & y \geqslant 0, \end{aligned} \tag{11}$$

where the domain $y \geqslant 0$ is determined by the inequality $\eta_{EE} \geqslant 0$. Obviously, such transformation guarantees the equivalence between problems (8) and (11). The hypograph form problem (13) can be analyzed geometrically as an optimization problem in the 'graph space' of $(\boldsymbol{p}_s, \boldsymbol{p}_r, y)$; that is, we maximize y over the hypograph of η_{EE}, subject to the constraints in Equation (8), which is equivalent to solve Equation (8) directly.

We can take the equivalent inequalities $\varphi(\boldsymbol{p}_s, \boldsymbol{p}_r, y) \geqslant 0$ to substitute for $\eta_{EE}(\boldsymbol{p}_s, \boldsymbol{p}_r) \geqslant y$ in Equation (11), that is,

$$\eta_{EE}(\boldsymbol{p}_s, \boldsymbol{p}_r) \geqslant y \Longleftrightarrow \varphi(\boldsymbol{p}_s, \boldsymbol{p}_r, y) \geqslant 0, \tag{12}$$

where we have $\varphi(\boldsymbol{p}_s, \boldsymbol{p}_r, y) = \sum_{n=1}^{N} R_n - y \left(\sum_{n=1}^{N} p_{s,n} + \sum_{n=1}^{N} p_{r,n} + P_c \right)$.

Thus, the equivalent hypograph problem form of Equation (8) is established as follows:

$$\begin{aligned} \max_{\boldsymbol{p}_s, \boldsymbol{p}_r, y} \quad & y \\ s.t. \quad & \varphi(\boldsymbol{p}_s, \boldsymbol{p}_r, y) \geqslant 0 \\ & C1 \sim C6 \text{ in Equation(8)} \\ & y \geqslant 0. \end{aligned} \tag{13}$$

It is easy to prove Equation (13) is a convex optimization problem, because the objective function and the inequality constraints functions are all convex [25]. Equivalent to solve the problem (8) or (11), we now turn to work out the optimal solution of Equation (13).

3.2. *Fast barrier method*

Barrier method has been recognized as a standard technique to solve convex optimization problems, which can make all inequality constraints implicit in the optimization objective. The original problem can be converted into a sequence of minimization problems, by introducing a logarithmic barrier function with a parameter t. The solution to each converted minimization problem is called a central point in the central path related to the original problem. The central point will be closer to the optimal solution as the parameter t increases. Newton method is generally employed for searching the center point with a given value of t.

In brief, the barrier method is always carried out via two essential steps, namely centering step and Newton step. The former is the outer iteration, executed to calculate the central point starting from the previously computed one. And the latter is the inner iteration implemented during each centering step. The computational complexity of the barrier method mainly lies in the computation of Newton step that needs matrix inversion with complexity of $O(N^3)$ for our considered problem.

To reduce the computational cost, we introduce a fast algorithm by studying the special structure of this problem [27–29]. First, a preparatory procedure is necessary for the barrier method, by transforming the objective y in Equation (13) into a twice differentiable function $U(y)$. Here, we give the complete form of the optimization problem with the transformation as follows:

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$$
\begin{aligned}
\max_{\boldsymbol{p}_s, \boldsymbol{p}_r, y} \quad & U(y) \\
s.t.\ C1 \quad & \varphi(\boldsymbol{p}_s, \boldsymbol{p}_r, y) \geqslant 0 \\
C2 \quad & \sum_{n=1}^{N} R_n \geqslant R_{\min} \\
C3 \quad & \sum_{n=1}^{N} p_{s,n} \leqslant P_S \\
C4 \quad & \sum_{n=1}^{N} p_{r,n} \leqslant P_R \\
C5 \quad & \sum_{n=1}^{N} p_{s,n} I_{n,l}^{SP} \leqslant I_l^{th}, l = 1, \ldots, L \\
C6 \quad & \sum_{n=1}^{N} p_{r,n} I_{n,l}^{RP} \leqslant I_l^{th}, l = 1, \ldots, L \\
C7 \quad & p_{s,n} \geqslant 0,\ p_{r,n} \geqslant 0, \forall n \\
C8 \quad & y \geqslant 0,
\end{aligned}
\tag{14}
$$

where the function $U(y)$ is monotone increasing in y in order to preserve the convexity of the problem. Evidently the associated problem (14) and the original form problem (13) are equivalent; indeed, the feasible sets are identical, as well as the optimal points [25]. In this paper, we take $U(y) = \log(1 + y)$ to guarantee the equivalence between the two problems, which is obviously not unique.

Then, inequality constraints are converted into a logarithmic barrier function $\phi(\boldsymbol{x})$,

$$
\begin{aligned}
\phi(\boldsymbol{x}) = & -\log \varphi(\boldsymbol{p}_s, \boldsymbol{p}_r, y) - \log y - \log\left(\sum_{n=1}^{N} R_n - R_{\min}\right) \\
& - \log\left(P_S - \sum_{n=1}^{N} p_{s,n}\right) - \log\left(P_R - \sum_{n=1}^{N} p_{r,n}\right) \\
& - \sum_{l=1}^{L} \log\left(I_l^{th} - \sum_{n=1}^{N} p_{s,n} I_{n,l}^{SP}\right) - \sum_{n=1}^{N} \log p_{s,n} \\
& - \sum_{l=1}^{L} \log\left(I_l^{th} - \sum_{n=1}^{N} p_{r,n} I_{n,l}^{RP}\right) - \sum_{n=1}^{N} \log p_{r,n},
\end{aligned}
\tag{15}
$$

where all variables $\{\boldsymbol{p}_s, \boldsymbol{p}_r, y\}$ are collected into one vector $\boldsymbol{x}$, that is, $\boldsymbol{x} = \{p_{s,1}, p_{r,1}, \ldots, p_{s,N}, p_{r,N}, y\}$.

Thus, the optimal solution to the problem (13) can be approximated by solving the following problem:

$$
\min \quad \psi_t(\boldsymbol{x}) = -t \log(1 + y) + \phi(\boldsymbol{x}). \tag{16}
$$

Because the optimal solution to Equation (16) is an approximation of that to the original problem (13), such approximation will become more and more close to the optimal solution as t increases.

During the centering step of barrier method, Newton method is executed to compute the central point. With a given parameter t, Newton step $\Delta \boldsymbol{x}$ can be given by the following Karush–Kuhn–Tucker system,

$$
\bigtriangledown^2 \psi_t(\boldsymbol{x}) \Delta \boldsymbol{x}_{nt} = - \bigtriangledown \psi_t(\boldsymbol{x}), \tag{17}
$$

where $\Delta \boldsymbol{x}_{nt} \in \Re^{2N+1}$. $\bigtriangledown^2 \psi_t(\boldsymbol{x})$ and $\bigtriangledown \psi_t(\boldsymbol{x})$ are the Hessian and the gradient of $\psi_t(\boldsymbol{x})$, respectively.

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The outline of the barrier method is summarized in Table I. ϵ and ϵ_n are the tolerances of the barrier method and the Newton step, respectively. α and β are two constants utilized in backtracking line search with $\alpha \in (0, 0.5)$ and $\beta \in (0, 1)$. The step size of the backtracking line search is s with $s > 0$. t and μ are parameters that associated with a tradeoff between outer iterations and inner iterations

If calculating the Newton step in (17) by matrix inversion directly, it will generate a complexity of $O(N^3)$, which is too high for application. We analyze the problem (16) and develop a fast computation of the Newton step by exploiting its special structure.

For simplicity, denote

$$\begin{aligned} f_s &= P_S - \sum_{n=1}^{N} p_{s,n}, \quad f_r = P_R - \sum_{n=1}^{N} p_{r,n}, \\ f_{s,l} &= I_l^{th} - \sum_{n=1}^{N} p_{s,n} I_{n,l}^{SP}, l = 1, \ldots, L \\ f_{r,l} &= I_l^{th} - \sum_{n=1}^{N} p_{r,n} I_{n,l}^{RP}, l = 1, \ldots, L \\ g_r &= \sum_{n=1}^{N} R_n - R_{\min}. \end{aligned} \tag{18}$$

The Hessian of $\psi_t(\boldsymbol{x})$ is given by

$$\begin{aligned} \bigtriangledown^2 \psi_t(\boldsymbol{x}) = D &+ \frac{\bigtriangledown f_s \bigtriangledown f_s^T}{f_s^2} + \sum_{l=1}^{L} \frac{\bigtriangledown f_{s,l} \bigtriangledown f_{s,l}^T}{f_{s,l}^2} \\ &+ \frac{\bigtriangledown f_r \bigtriangledown f_r^T}{f_r^2} + \sum_{l=1}^{L} \frac{\bigtriangledown f_{r,l} \bigtriangledown f_{r,l}^T}{f_{r,l}^2} \\ &+ \frac{\nabla \varphi \bigtriangledown \varphi^T}{\varphi^2} + \frac{\nabla g_r \bigtriangledown g_r^T}{g_r^2} \\ &= D + \sum_{i=1}^{2L+4} G_i G_i^T, \end{aligned} \tag{19}$$

Table I. The barrier method.

Algorithm 2

1.**Initialization for the Barrier method**
2. A feasible point $\boldsymbol{x}, t > 0$, tolerance $\epsilon > 0, \mu > 1$
3.**Outer Loop for Barrier method**
4. Centering step: Compute $\boldsymbol{x}^*(t)$ derived by problem (16)
5. **Initialization for Newton method**
6. Starting point $\boldsymbol{x}$, tolerance $\epsilon_n > 0, \alpha \in (0, 1/2), \beta \in (0, 1)$
7. **Inner Loop for Newton method**
8. Compute $\Delta \boldsymbol{x}_{nt}$ and $\lambda = - \bigtriangledown \psi_t(\boldsymbol{x}) \Delta \boldsymbol{x}_{nt}$;
9. Quit if $\lambda^2/2 \leqslant \epsilon_n$
10. Backtracking line search on $\psi_t(\boldsymbol{x}), s := 1$
11. ***while*** $\psi_t(\boldsymbol{x} + s\Delta \boldsymbol{x}) > \psi_t(\boldsymbol{x}) - \alpha s \lambda^2$
12. $s := \beta s$
13. ***endwhile***
14. Update:$\boldsymbol{x} = \boldsymbol{x} + s\Delta \boldsymbol{x}$,
15. Update:$\boldsymbol{x}^*(t) = \boldsymbol{x}$.
16. Stopping criterion:$(2N + 2L + 4)/t < \epsilon$.
17. Increase:$t := \mu t$.

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where
$D = \text{diag}(D_1, \ldots, D_N, Y) \in \Re^{2N+1}$ with

$$D_n = \begin{bmatrix} \frac{1}{p_{s,n}^2} & 0 \\ 0 & \frac{1}{p_{r,n}^2} \end{bmatrix} - \left(\frac{1}{g_0} + \frac{1}{g_r}\right) \begin{bmatrix} \frac{\partial^2 R_n}{\partial p_{s,n}^2} & \frac{\partial^2 R_n}{\partial p_{s,n} \partial p_{r,n}} \\ \frac{\partial^2 R_n}{\partial p_{r,n} \partial p_{s,n}} & \frac{\partial^2 R_n}{\partial p_{r,n}^2} \end{bmatrix}, \quad (20)$$
$$Y = t/(1+y)^2 + 1/y^2.$$

And $G_i \in \Re^{2N+1}$ with

$$G_i = \begin{cases} \nabla f_s / f_s, & i = 1 \\ \nabla f_{s,l} / f_{s,l}, & i = l+1, l = 1, \ldots, L \\ \nabla f_r / f_r, & i = L+2 \\ \nabla f_{r,l} / f_{r,l}, & i = L+l+2, l = 1, \ldots, L \\ \nabla \varphi / \varphi, & i = 2L+3 \\ \nabla g_r / g_r, & i = 2L+4 \end{cases}. \quad (21)$$

The Hessian is positive definite because the diagonal matrix D is positive definite matrixes and all $G_i G_i^T \geqslant 0$. Denote $G_0 = -\nabla \psi_t(\boldsymbol{x})$, $H_0 = \nabla^2 \psi_t(\boldsymbol{x})$ and $M = 2L + 4$.

Theorem 1
Suppose $D \in \Re^{n \times n}$ is a nonsingular matrix, $u, v \in \Re$ with $1 + v^T D u \neq 0$. For the following two equation

$$Dx = b, \quad (D + uv^T)\tilde{x} = b,$$

$\tilde{x}$ is called a rank-one update of x, and we have

$$\tilde{x} = x - \frac{v^T x}{1 + v^T w} w,$$

where $w = D^{-1} u$.

Proof
For the second equation, we can derive

$$\begin{aligned} \tilde{x} &= (D + uv^T)^{-}1b \\ &= \left(D^{-1} - \frac{1}{1 + v^T D u} D^{-1} u v^T D^{-1}\right) b. \end{aligned}$$

Substituting $Dx = b$ into the previous equation with $w = D^{-1} u$, it follows

$$\begin{aligned} \tilde{x} &= D^{-1} b + \frac{v^T x}{1 + v^T D u} D^{-1} u \\ &= x - \frac{v^T x}{1 + v^T w} w. \end{aligned}$$

□

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Instead of computing $\Delta \boldsymbol{x}_{nt}$ via matrix inversion, we propose a fast algorithm by exploiting the structure of the problem as analyzed earlier based in Theorem 1, to speedup the Newton step with an M-step procedure as follows [30]:

Step 1 Let $H_0 = H_1 + G_1 G_1^T$.
Then we have $\Delta \boldsymbol{x}_{nt} = \upsilon_1^1 - \frac{G_1 \upsilon_1^1}{1 + G_1 \upsilon_2^1} \upsilon_2^1$,
Where $H_1 \upsilon_1^1 = G_0$ and $H_1 \upsilon_2^1 = G_1$.

Step 2 Further decompose Λ_1 with $H_1 = H_2 + G_2 G_2^T$,
Similarly, the two variables in Step 1 can be updated by $\upsilon_i^1 = \upsilon_i^2 - \frac{G_2 \upsilon_i^2}{1 + G_2 \upsilon_3^2} \upsilon_3^2, i = 1, 2$,
where $H_2 \upsilon_i^2 = G_{i-1}, i = 1, 2, 3$.

$\vdots$

Without loss generality, Consider the step m

Step m Let $H_{m-1} = H_m + G_m G_m^T$.
Update the m variables in Step $m-1$ by
$\upsilon_i^{m-1} = \upsilon_i^m - \frac{G_m^T \upsilon_i^m}{1 + G_m \upsilon_{m+1}^m} \upsilon_{m+1}^m, i = 1, \ldots, m$,
where $H_m \upsilon_i^m = G_{i-1}, i = 1, \ldots, m+1$ with $H_i = D + \sum_{j=i+1}^M G_j G_j^T$.

Through M steps as discussed earlier, there produce $M+1$ matrix systems $\Lambda_M \upsilon_i^M = G_{i-1}, i = 1, \ldots, M+1$. During the derivation, we can find that the m variables $\upsilon_i^{m-1}, i = 1, \ldots, m$ in Step $m-1$ can be obtained by the $m+1$ variables $\upsilon_i^m, i = 1, \ldots, m+1$ in Step m. Hence, if we can figure out the $M+1$ variables $\upsilon_i^M, i = 1, \ldots, M+1$, the Newton step and the associated dual variable in (17) will be indirectly obtained. Obviously, a reverse derivation of the M steps' decomposition discussed earlier is necessary to be executed, after we solve the $M+1$ matrix system in the Step M.

Then we demonstrate the exhaustive process to solve the matrix system $H_M \upsilon_i^M = G_{i-1}$ as follows. Without loss of generality, we convert the equations into a unified form $Du = G$ because we have $\Lambda_M = D$, where $u, G \in \Re^{2N+1}$. It follows

$$\begin{bmatrix} u_{2n-1} \\ u_{2n} \end{bmatrix} = D_n^{-1} \begin{bmatrix} G_{2n-1} \\ G_{2n} \end{bmatrix}, n = 1, \ldots, N, \quad u_{2N+1} = G_{2N+1}/Y. \tag{22}$$

Thus, the $M+1$ matrix systems in Step M can be solved in the same way.

3.3. Warm start procedure for barrier method

For the initialization of the barrier method, a strictly feasible starting point is required. Thus, a preparatory procedure is necessary to obtain feasible points or prove its inexistence. We execute the warm start procedure in two step. First, we try to find a feasible point $\left(\boldsymbol{p}_s^0, \boldsymbol{p}_r^0\right)$, satisfying the constraints C3∼C7 in Equation (13). Then we can take any value in the interval $\left(0, \eta_{EE}\left(\boldsymbol{p}_s^0, \boldsymbol{p}_r^0\right)\right)$ as a feasible y, denoted as y^0.

During the first step, finding a feasible solution is equivalent to solve a minimization problem by introducing a crucial indicator parameter z as discussed in [25]. The optimization problem for the warm start procedure can be formulated as

$$\begin{aligned} \min_{p_{s,n}, p_{r,n}, z} \quad & z \\ s.t.\ C1 \quad & \sum_{n=1}^N R_n \geqslant R_{\min} - z \\ & C3 \sim C7 \text{ in Equation}(13), \end{aligned} \tag{23}$$

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where z can be interpreted as a bound on the maximum infeasibility of the inequality C1 and our goal is to drive it below zero. Because it is easy to find $\boldsymbol{p}_s$, $\boldsymbol{p}_r$ to satisfy C3$\sim$C7 in Equation (13), we can choose a feasible z to satisfy C1. So the feasible solution to Equation (23) always exists.

Note that Equation (23) also defines a convex problem whose structure is similar to Equation (13). Because of its special structure, we can also apply the fast barrier method developed in section III-B to solve the problem (23). By solving Equation (23), a strictly feasible point $\left(\boldsymbol{p}_s^0, \boldsymbol{p}_r^0, y^0\right)$ may be computed, or it proves no feasible point exists. If the optimal solution to Equation (23) satisfies $z \leqslant 0$, the associated solution of $\boldsymbol{p}_s$ and $\boldsymbol{p}_r$ can be used as the starting point of the barrier method to solve Equation (13). Otherwise, no feasible point exists for the Equation (13), and we regard such a case as system outage.

3.4. *On the complexity*

The computational complexity can be counted roughly as follows. The fast barrier algorithm of solving Equation (13) consumes M decomposition, while each decomposition yields an additional equation. First, we need to solve $M + 1$ matrix system according to (22) with the computational complexity $O(N)$ for each one. Then, a reverse substitution procedure is carried out, and the total computational cost for the fast barrier method is $O(M^2N)$, after M reverse substitution steps. Thus, we can conclude the complexity for solving the optimal solution to Equation (13) is measured by $O(M^2N)$. If we take the standard barrier method and compute the Newton step directly by matrix inversion, the total complexity is $O(N^3)$.

Because we can also apply the proposed fast algorithm to solve the warm start problem, the complexity is roughly equal to that of solving Equation (13), because of the similar structure. Therefore, we conclude the complexity of the optimal power allocation is $O(M^2N)$. Notice that the number of PUs is always much smaller than that of the subchannels in wireless systems, that is, $M \leqslant N$; the complexity is reduced dramatically.

4. SIMULATION RESULTS

Experiments are performed to evaluate the performance of our proposed method. Consider a CR system where the SN and PUs are uniformly distributed within a 1-km circle area. The frequency-selective fading is assumed, and the path loss exponent is set to 4. The variance of logarithmic normal shadow fading is 10 dB, and the amplitude of multipath fading is Rayleigh. We assume that each PU's bandwidth is randomly generated by uniform distribution and the maximum value is $2W/3L$. The noise power is 10^{-13} W. To emphasize the advantages of energy-efficient power loading scheme for green communication, we compare the EE of our proposed optimal energy-efficient power allocation algorithm with that of rate adaptive power allocation algorithm, which always maximizes the throughput of the relaying CR system.

First, we illustrate the EE of CR system versus the transmission power limit at the SN and RN in Figures 1 and 2, respectively. Two cases of different numbers of subchannels, which are $N = 32$ and $N = 64$, are considered. There are two PUs, and the interference threshold of each PU is uniformly set to 5×10^{-12} W. The static circuit power is fixed to 0.5 W, with $P_R = 1$ W in Figure 1 and $P_S = 1$ W in Figure 2. Besides, the minimal rate requirement is 20 bits/symbol. For both cases of $N = 32$ and $N = 64$, the EE of energy-efficient power allocation algorithm grows with the increase of P_S (P_R), until the power is sufficient to satisfy the rate requirements. Because more power budget will lower the probability of system outage and enhance the EE of the CR system.

For rate adaptive power allocation algorithm, the curve of EE first increases with the growth of P_s (P_r), and the loss of EE occurs at a cut-off of the transmission power limit, where a decrease of EE can be found when P_S (P_R) becomes larger. This phenomenon can be explained intuitively. When the power limit is relatively small, P_c occupied the main part of the total power consumption, and larger P_S or P_R can achieve more capacity. However, when P_S or P_R gets larger enough to ignore the static circuit power, the EE will decrease because the logarithmic growth of the capacity exhausted the total power budget.

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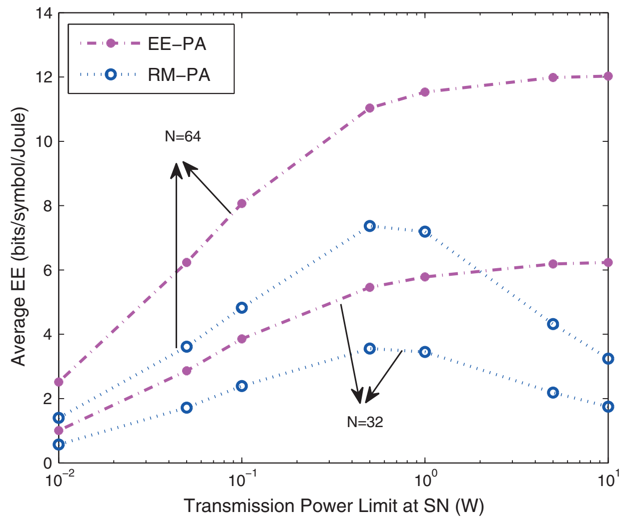

Figure 1. The energy efficiency (EE) of cognitive radio system as a function of transmission power limit at source node (SN). $L = 2$, $P_R = 1$ W, $R_{\min} = 20$ bits/symbol. EE-PA, energy-efficient power allocation algorithm; RM-PA, rate adaptive power allocation algorithm.

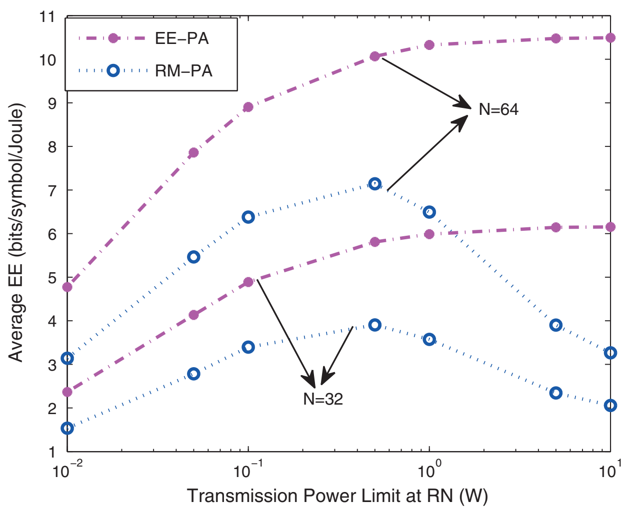

Figure 2. The energy efficiency (EE) of cognitive radio system as a function of transmission power limit at relay node (RN). $L = 2$, $P_S = 1$ W, $R_{\min} = 20$ bits/symbol. EE-PA, energy-efficient power allocation algorithm; RM-PA, rate adaptive power allocation algorithm.

Figure 3 shows the EE of the CR system as a function of the minimal rate requirements for different numbers of PUs. There are 64 subchannels with the transmission power budgets $P_S = 10$ W and $P_R = 1$ W, while the static circuit power $P_c = 0.5$ W. The interference threshold of each PU is 5×10^{-12} W. We can observe that the EE of CR system decreases with the growth of the rate requirements for both $L = 2$ and $L = 4$, because the growth of rate requirements will not only result in exponentially increase of power consumption but also more frequently system outage. Comparing the curves of two cases, we find more PUs will lower the total EE, because more subchannels will be interference limited and fail to maintain the rate requirements.

Finally, we investigate the convergence of our proposed algorithm in Figure 4. As discussed in Section III-B, the computational load of the algorithm mainly lies in the computation of Newton step. If the number of Newton iterations is large or varies in a wide range, the algorithm would be difficult to be applied in practical wireless systems. Figures 4 and 5 demonstrate that it is not the case

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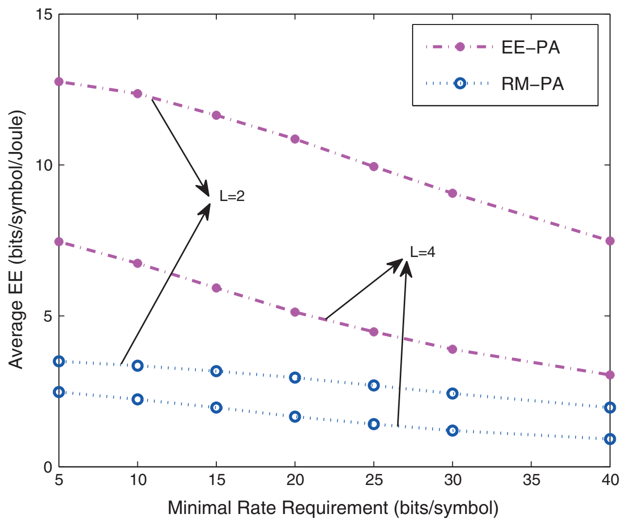

Figure 3. The energy efficiency (EE) of cognitive radio system as a function of minimal rate requirement. $L = 2$, $P_S = 10$ W, $R_R = 1$ W. EE-PA, energy-efficient power allocation algorithm; RM-PA, rate adaptive power allocation algorithm.

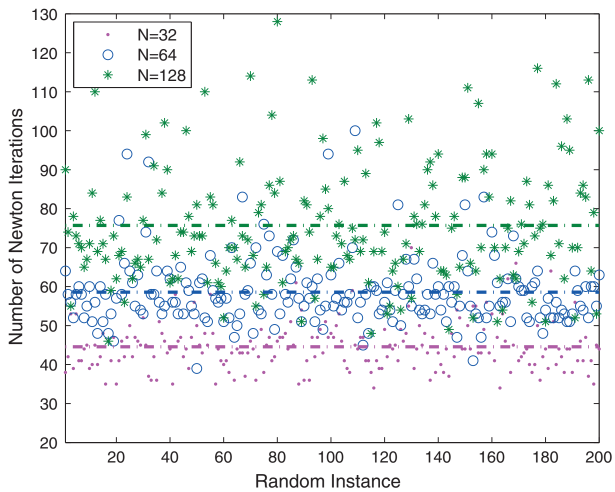

Figure 4. Number of Newton iterations required for convergence.

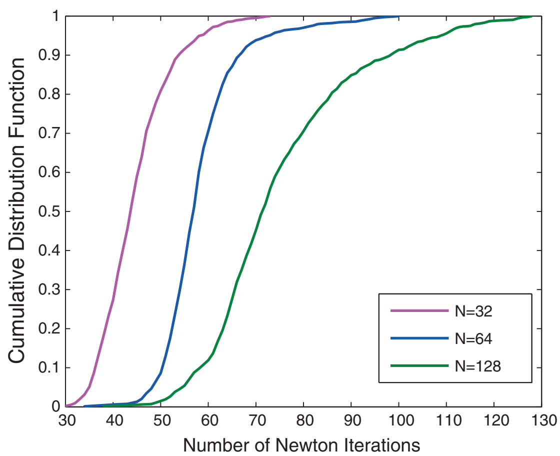

Figure 5. CDF of the number of Newton iterations required for convergence for 1000 channel realizations.

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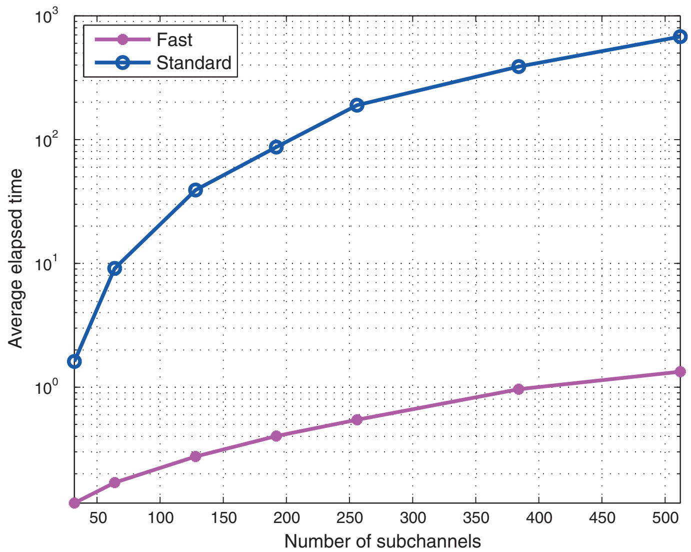

Figure 6. Average time cost as a function of the number of subchannels.

for our proposed algorithm. Figure 4 shows the number of Newton iterations for the barrier method to converge in 200 random instances, while Figure 5 gives the cumulative distribution function (CDF) of the number of Newton iterations for our proposal with different settings of N. Besides, the average number of Newton iterations is shown in Figure 4 by the corresponding dashed line for a certain number of subchannels. It can be observed that the number of Newton iterations always varies in a narrow range with a given N, which validates that our proposed algorithm is effective and efficient.

We also compare the time cost of our proposed algorithm (Fast) exploiting the fast barrier method in section III-B with the standard one (Standard), which computes Newton step by matrix inversion. Figure 6 shows the average time cost as a function of number of subchannels over 1000 instances. The elapsed time is counted by in-built *tic-tac* function in *Matlab*. From Figure 6, we can see the time cost of our proposed algorithm is much less than the standard technique.

5. CONCLUSION

In this paper, we studied the energy-efficient power allocation in the relaying CR networks. Because the formulated problem is nonconvex and completely difficult to tackle, the problem is first converted into a convex optimization problem in its hypograph form. By extensively analyzing the equivalent problem, we developed a fast barrier algorithm to work out the optimal solution quickly, which always updates the Newton step in an ingenious way by exploiting its special structure. Numerical simulations verified the effectiveness and efficiency of our proposed method.

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