

# Cell Planning for Heterogeneous Networks: An Approximation Algorithm

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**Abstract**—Low-power access points, such as pico base stations (BSs), femto BSs, and relays are introduced to the next generation cellular systems to enhance coverage and improve system capacity. Deploying low-power access points to offload the conventional macro BSs is deemed as a spectrum- and cost-efficient way to meet the sharp increase of traffic requirements of cellular networks. However, it also leads to heterogeneous network framework and raises new challenges for cell planning. In this paper, we study the minimum cost cell planning problem in such a heterogeneous network. Our optimization task is to select a subset of candidate sites to lay BSs, including macro BSs, pico BSs and relays, to minimize the total deployment cost while satisfying the rate requirements of the demand nodes (DNs) served by the cellular network. We prove that the general case of the formulated problem is APX-hard, where a DN is constrained to be associated with only one BS. However, if each DN can be served by multiple BSs, which is a reasonable case for practical cellular systems, we show it is not APX-hard and develop an approximation algorithm to work out promising solutions. Our proposed algorithm guarantees an approximation ratio of  $O(\log R)$  to the global optimum, where  $R$  is the maximum achievable capacity of the BSs. Numerical results indicate that our proposal can significantly reduce the deployment cost of the cellular network with given rate requirements of DN compared to other cell planning schemes.

## I. INTRODUCTION

Due to rapidly growing demands for mobile services, high installation cost and scarcity of radio resources, spectral and cost-efficient cell planning strategy appears to be of utmost importance for the next generation cellular systems. Generally, the planning target for a cellular network is to employ a number of base stations (BSs) at candidate sites and configure their parameters to obtain predefined coverage at the least cost. For the 2nd generation (2G) cellular systems, cell planning usually consists of two stages. Firstly, a set of macro BSs is selected from a candidate list, and configured with a series of radio parameters to cover the predefined service area [1–7]. The planning objective at this stage is usually to minimize the total deployment cost while ensuring the signal strength high enough for terminal users in the coverage area [1–6]. Secondly, according to the traffic demand distribution of the service area, radio frequency is grouped and the adjacent BSs are assigned different frequency to minimize the interference among these

BSs, so that the signal-to-interference-plus-noise-ratio (SINR) of the received signal of each user is above a threshold that can guarantee the QoS of the cellular system [3–7].

For the 3rd generation (3G) cellular systems, frequency grouping is no longer required since the whole spectrum is used by each cell or BS. To obtain the required SINRs, interference between adjacent cells should be carefully addressed because the measured SINR of a user served by a given cell is heavily affected by its received interference power from the neighboring cells. Thus power control/allocation is important for the 3G cellular systems. Generally, cell planning for 3G cellular systems is formulated as a capacitated facility location problem which is NP-hard [8]. Heuristic methods, such as tabu search, simulated annealing, genetic algorithms are preferred in the literature [9–11].

In the standardization process of the next generation cellular network, such as 3GPP Long Term Evolution-Advanced (LTE-A), heterogeneous network (HetNet), which generally consists of macro BSs and low-power access points, such as pico BSs, femto BSs, and relays, is receiving significant attention and deemed as a cost-effective way to keep up with the increasing traffic demands of users [12–16]. By overlaying low-power and low-cost access points on the coverage holes or the capacity starved locations, a HetNet can enhance coverage and improve system capacity by offloading the macro BSs in the cellular network. A femto BS is usually employed by users in an un-planned way to enhance indoor coverage. However, a pico BS is operator-installed and usually adopts the same backhaul and access features as a macro BS, except for serving much smaller areas than the macro BS. Pico BSs are usually deployed at hot spots or coverage holes. A relay serves the similar size of footprint as a pico BS. However, a backhaul link between the relay and its donor macro BS is required. Thus the deployment of relays consumes parts of radio resources of their donor macro BSs. Relays are generally used to extend the coverage of macro BSs. Compared to macro BSs, the installation and maintenance operation cost of low-power BSs is much cheaper. Moreover, due to the much lower transmission power and smaller physical size, a low-power BS usually requires flexible site acquisition, which is very important for service providers.

However, how to plan macro BSs, pico BSs and relays for

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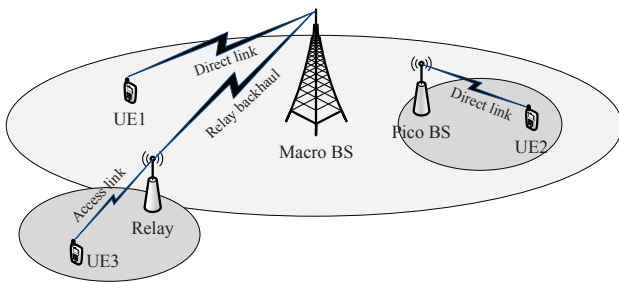


Fig. 1. A heterogeneous cellular network example.

a HetNet has not been studied extensively in the literature. Most of previous researches focus on macro only deployment scheme, as well as considering the bandwidth and power allocation to meet given traffic requirements. In this paper, we propose a novel HetNet planning model, involving macro BSs, pico BSs and relays. The transmission power budgets of BSs are limited with different levels and all spectrum can be used by any cell. If a macro BS is selected for opening, its associated relays can be also selected. Each selected relay will consume a part of radio resource of its donor macro BS. We try to select a subset of BSs (relays) among all candidate sites with the minimum cost to provide required traffic demands for all demand nodes (DNs). We call our target optimization task as the minimum cost cell planning problem. First, we prove that the general case of this problem is APX-hard if every DN only can be served by only one BS. Then we focus on addressing a special case that is also reasonable for practical cellular networks, where multiple BSs can serve a single DN. We show that such a problem is still NP-hard but not APX-hard. We propose an  $O(\log R)$ -approximation algorithm to solve the problem efficiently, where  $R$  is the maximum achievable transmission rate of the BSs. Theoretical analysis and numerical results validate the effectiveness of our proposed cell planning algorithm.

The remainder of this paper is organized as follows. In Section II, system model and the minimum cell planning problem are illustrated, as well as the proof of the inapproximation of the formulated problem. In Section III, an approximation algorithm for a meaningful case in practical cellular systems is developed, including the proof of the approximation ratio and the implementation of the algorithm. In Section IV, numerical results are given with discussions. Conclusions are drawn in Section V.

## II. PROBLEM FORMULATION

### A. System Model

Some frequently used notations are listed in Table I. Fig. 1 gives a simple illustration of a cellular system employing HetNet framework, which involves one macro BS, one pico BS, one relay and three user equipments (UEs). UE1 and UE2 are directly associated with the macro BS and the pico BS, respectively. UE3 locates far away from the macro BS and can be served indirectly by the macro BS via the relay. The radio link between the macro BS and the relay is referred to

TABLE I  
NOTATIONS

Symbol	Semantics
$\mathcal{N}_m$	The set of candidate sites of macro BSs
$\mathcal{N}_p$	The set of candidate sites of pico BSs
$\mathcal{N}_r$	The set of candidate sites of relays
$\mathcal{M}_n$	The set of relays that can be connected to macro BS $n$
$\mathcal{N}_n$	The set of macro BS $n$ and its associated relays
$\mathcal{N}$	The set of candidate sites of all BSs
$N$	The number of all BSs
$c_n$	The cost of BS $n$
$z_n$	The selection variable of BS $n$
$P_n^{max}$	The maximum transmission power of BS $n$
$B$	The total bandwidth
$\mathcal{K}$	The set of DNs
$K$	The number of DNs
$R_k^{min}$	The required rate of DN $k$
$b_{k,n}$	The bandwidth of BS $n$ allocated to DN $k$
$p_{k,n}$	The power of BS $n$ allocated to DN $k$
$h_{k,n}$	The channel gain between BS $n$ to DN $k$
$r_{k,n}$	The achievable rate from BS $n$ to DN $k$
$\Gamma$	The SNR gap
$N_0$	The PSD of AWGN
$\gamma_n$	The resource consumption proportion by backhaul of relay $n$

as backhaul link. Consider an area served by such kind of cellular system, the sets of candidate sites of macro BSs, pico BSs and relays are denoted as  $\mathcal{N}_m$ ,  $\mathcal{N}_p$  and  $\mathcal{N}_r$ , respectively. Denote  $\mathcal{N} = \mathcal{N}_m \cup \mathcal{N}_p \cup \mathcal{N}_r = \{1, 2, \dots, N\}$  as the set of all candidate sites. For macro BS  $n \in \mathcal{N}_m$ , denote  $\mathcal{M}_n$  as the set of relays which can be connected to the macro BS  $n$  via backhaul links. We assume that  $|\mathcal{M}_n|$  is limited to a constant number and  $\mathcal{M}_n \cap \mathcal{M}_{n'} = \emptyset$ , where  $n' \in \mathcal{N}_m$  and  $n \neq n'$ . Denote  $\mathcal{N}_n = \{n\} \cup \mathcal{M}_n$  as the set of macro BS  $n$  and its associated relays. When a relay is selected for opening, the relay consumes radio resources from its donor macro BS. We assume that such a resource consumption is related to the total allocated bandwidth and power of the relay, and define  $\gamma_n$  as the resource consumption proportion by the backhaul link between the relay and its donor macro BS.  $c_n$  is the employment cost of the BS  $n$ . The transmission power budget of the BS  $n$  is  $P_n^{max}$ . Generally, the maximum transmission power of a macro BS (e.g., 46 dBm) is much higher than that of a low-power BS (e.g., 30 dBm). As a result, the coverage area of a low-power BS is usually much smaller than that of a macro BS. The total bandwidth is  $B$  (e.g., 100MHz) for the considered cellular system, which is available for macro BSs, pico BSs and relays.

The set of DNs is denoted as  $\mathcal{K} = \{1, 2, \dots, K\}$ . Each DN  $k \in \mathcal{K}$  has a rate requirement of  $R_k^{min}$ . Obviously,  $R_k^{min} > 0$  for each DN. Denote  $h_{k,n}$  as the channel gain between BS  $n$  and DN  $k$ , which is a function of the distance between the BS and the DN with a predefined path-loss propagation model. Denote  $b_{k,n}$  and  $p_{k,n}$  as the bandwidth and power of BS  $n$  allocated to DN  $k$ , respectively, the achievable rate of the BS  $n$  to the DN  $k$  can be calculated as

$$r_{k,n} = b_{k,n} \log_2 \left( 1 + \frac{p_{k,n} |h_{k,n}|^2}{\Gamma N_0 b_{k,n}} \right),$$

where  $\Gamma$  is the SNR gap and related to a given bit-error-rate

(BER) for a specific modulation/demodulation scheme, e.g.,  $\Gamma = -\ln(5\text{BER})/1.6$  for an uncoded multilevel quadrature amplitude modulation (MQAM) modulation system [17].  $N_0$  is the power spectral density (PSD) of additive white Gaussian noise (AWGN). For notation brevity, denote  $H_{k,n} = |h_{k,n}|^2/\Gamma N_0$ , then the rate of DN  $k$  is

$$R_k = \sum_{n \in \mathcal{N}} b_{k,n} \log_2 \left( 1 + \frac{p_{k,n} H_{k,n}}{b_{k,n}} \right).$$

### B. Minimum Cost Cell Planning

Our optimization objective is to select a subset of  $\mathcal{N}$  with the minimum employment cost to satisfy all rate requirements of  $\mathcal{K}$ , while the allocated power and bandwidth of each selected BS cannot exceed its budget. Denote  $z_n$  as a selection variable of BS  $n$

$$z_n = \begin{cases} 1 & \text{BS } n \text{ is selected for opening,} \\ 0 & \text{otherwise,} \end{cases} \quad \forall n \in \mathcal{N}.$$

Assume that each DN can be assigned to only one BS, our optimization problem can be formulated as follows,

$$\begin{aligned} & \min_{\substack{z_n, y_{k,n} \\ p_{k,n}, b_{k,n}}} \sum_{n \in \mathcal{N}} c_n z_n \\ \text{s.t. } & C_1: \sum_{k \in \mathcal{K}} b_{k,n} \leq z_n B, \forall n \in \mathcal{N}_p \cup \mathcal{N}_r, \\ & C_2: \sum_{k \in \mathcal{K}} b_{k,n} + \sum_{\substack{n' \in \mathcal{M}_n \\ k \in \mathcal{K}}} \gamma_{n'} b_{k,n'} \leq z_n B, \forall n \in \mathcal{N}_m, \\ & C_3: \sum_{k \in \mathcal{K}} p_{k,n} \leq z_n P_n^{\max}, \forall n \in \mathcal{N}_p \cup \mathcal{N}_r, \\ & C_4: \sum_{k \in \mathcal{K}} p_{k,n} + \sum_{\substack{n' \in \mathcal{M}_n \\ k \in \mathcal{K}}} \gamma_{n'} p_{k,n'} \leq z_n P_n^{\max}, \\ & \quad \forall n \in \mathcal{N}_m, \\ & C_5: R_k = R_k^{\min}, \forall k \in \mathcal{K}, \\ & C_6: z_{n'} \leq z_n, \forall n \in \mathcal{N}_m, n' \in \mathcal{M}_n, \\ & C_7: 0 \leq b_{k,n} \leq y_{k,n} B, \forall k \in \mathcal{K}, n \in \mathcal{N}, \\ & C_8: 0 \leq p_{k,n} \leq y_{k,n} P_n^{\max}, \forall k \in \mathcal{K}, n \in \mathcal{N}, \\ & C_9: y_{k,n} \leq z_n, \forall k \in \mathcal{K}, n \in \mathcal{N}, \\ & C_{10}: \sum_{n \in \mathcal{N}} y_{k,n} = 1, \forall k \in \mathcal{K}, \\ & C_{11}: z_n, y_{k,n} \in \{0, 1\}, \forall k \in \mathcal{K}, n \in \mathcal{N}, \end{aligned} \quad (1)$$

where  $y_{k,n}$  is an associate index of DN  $k$  and BS  $n$ , that is,  $y_{k,n} = 1$  means that DN  $k$  is served by BS  $n$ .  $C_1$  to  $C_4$  make sure the allocated bandwidth and power of each selected BS  $n$  are limited to  $B$  and  $P_n^{\max}$ , respectively.  $C_2$  and  $C_4$  indicate that each selected relay consumes a part of resources of its donor macro BS.  $C_5$  is the rate requirement of DN  $k$ . The required rates of all DNs should be satisfied by the selected BSs.  $C_6$  ensures that relay  $n' \in \mathcal{M}_n$  can be selected for opening only if macro BS  $n$  is selected.  $C_7$  and  $C_8$  mean the allocated bandwidth and power are both zero when the DN is not assigned to the BS.  $C_9$  means DN  $k$  only can be allocated to BS  $n$  when BS  $n$  is selected.  $C_{10}$  indicates that a DN can be served by at most one BS.

The problem defined by Eq.(1) is NP-hard. Moreover, it is a strong NP-hard problem which means even finding a feasible solution is also difficult.

**Theorem 1.** *There is no  $\rho$ -approximation algorithm ( $\rho > 1$ ) for Eq.(1) unless  $P = NP$ .*

*Proof:* The proof is referred to Appendix A.  $\blacksquare$

Thus, we focus on a special case, where each DN can be served by multiple BSs<sup>1</sup>. Mathematically, the problem can be formulated as follows,

$$\begin{aligned} & \min_{z_n, p_{k,n}, b_{k,n}} \sum_{n \in \mathcal{N}} c_n z_n \\ \text{s.t. } & 0 \leq b_{k,n}, 0 \leq p_{k,n}, \forall k \in \mathcal{K}, n \in \mathcal{N}, \\ & z_n \in \{0, 1\}, \forall n \in \mathcal{N}, \\ & C_1 \sim C_6 \text{ in Eq.(1)}. \end{aligned} \quad (2)$$

Eq.(2) also defines an NP-hard problem. However, it is not APX-hard and we can develop approximation algorithm for Eq.(2).

### III. $O(\log R)$ -APPROXIMATION ALGORITHM

Motivated by the algorithm proposed in [18], we first formulate a bandwidth and power allocation optimization task for a given set of BSs to maximize the sum rate of the BSs while keeping the achievable rate of each DN not exceeding its rate requirement; then we design an approximation algorithm by generalizing the result of [18] to solve Eq.(2).

#### A. Bandwidth and Power Allocation

The bandwidth and power allocation problem for a given set of BSs is as follows: Given a set of macro BSs  $\mathcal{N}_m'$ , a set of pico BSs  $\mathcal{N}_p'$  and a set of relays  $\mathcal{N}_r'$ , we try to work out the achievable rate that can be sustained by those BSs while keeping the bandwidth and power constraints of the BSs satisfied. Let  $\mathcal{N}' = \mathcal{N}_m' \cup \mathcal{N}_p' \cup \mathcal{N}_r'$  be the set of given BSs and  $N' = |\mathcal{N}'|$  be the number of the BSs. Let  $\mathcal{M}'_n = \mathcal{M}_n \cap \mathcal{N}'_r$  be the set of relays which can be associated to macro BS  $n$ . Mathematically, the optimization problem is:

$$\begin{aligned} & \max_{b_{k,n}, p_{k,n}} \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} r_{k,n} \\ \text{s.t. } & C_1: 0 \leq b_{k,n}, 0 \leq p_{k,n}, \forall k \in \mathcal{K}, n \in \mathcal{N}', \\ & C_2: \sum_{k \in \mathcal{K}} b_{k,n} \leq B, \forall n \in \mathcal{N}'_p \cup \mathcal{N}'_r, \\ & C_3: \sum_{k \in \mathcal{K}} b_{k,n} + \sum_{\substack{n' \in \mathcal{M}'_n \\ k \in \mathcal{K}}} \gamma_{n'} b_{k,n'} \leq B, \forall n \in \mathcal{N}'_m, \\ & C_4: \sum_{k \in \mathcal{K}} p_{k,n} \leq P_n^{\max}, \forall n \in \mathcal{N}'_p \cup \mathcal{N}'_r, \\ & C_5: \sum_{k \in \mathcal{K}} p_{k,n} + \sum_{\substack{n' \in \mathcal{M}'_n \\ k \in \mathcal{K}}} \gamma_{n'} p_{k,n'} \leq P_n^{\max}, \forall n \in \mathcal{N}'_m, \\ & C_6: \sum_{n \in \mathcal{N}'} r_{k,n} \leq R_k^{\min}, \forall k \in \mathcal{K}. \end{aligned} \quad (3)$$

<sup>1</sup>DN can be deemed as an abstraction of the average traffic demand in a small area, which enables the assumption that a DN located in an overlapped area can be served by multiple BSs [2].

If the optimal value of the objective function of Eq.(3) is  $\sum_{k \in \mathcal{K}} R_k^{min}$ ,  $\mathcal{N}'$  is obviously a feasible solution to Eq.(2). The feasible set of Eq.(3), however, is not convex for both  $b_{k,n}$  and  $p_{k,n}$ . We use a general transformation to yield an equivalent problem whose feasible set is convex. Since

$$p_{k,n} = \frac{b_{k,n}}{H_{k,n}} \cdot \left( 2^{r_{k,n}/b_{k,n}} - 1 \right),$$

the equivalent problem of Eq.(3) can be written as

$$\begin{aligned} \max_{b_{k,n}, r_{k,n}} \quad & \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} r_{k,n} \\ \text{s.t.} \quad & 0 \leq b_{k,n}, 0 \leq r_{k,n}, \forall k \in \mathcal{K}, n \in \mathcal{N}', \\ & C_2 \sim C_6 \text{ in Eq.(3)}. \end{aligned} \quad (4)$$

It is easy to prove that Eq.(4) defines a convex optimization problem [19] since the objective function and the constraints  $C_2, C_3, C_6$  are affine for both  $b_{k,n}, r_{k,n}$ , and other constraints,  $C_4$  and  $C_5$ , are convex. Generally, barrier method is treated as a standard technique to solve convex optimization problems, which makes all inequality constraints implicit in the optimization objective [19]. In this paper, we develop a fast barrier method by exploiting the special structure of Eq.(4) as suggested in [20–22]. For each  $n \in \mathcal{N}'_m$ , denote

$$\begin{aligned} b_n &= B - \sum_{k \in \mathcal{K}} b_{k,n} - \sum_{\substack{n' \in \mathcal{M}'_n \\ k \in \mathcal{K}}} \gamma_{n'} b_{k,n'}, \\ p_n &= P_n^{max} - \sum_{k \in \mathcal{K}} p_{k,n} - \sum_{\substack{n' \in \mathcal{M}'_n \\ k \in \mathcal{K}}} \gamma_{n'} p_{k,n'}, \end{aligned}$$

and for each  $n \in \mathcal{N}' \setminus \mathcal{N}'_m$ , denote

$$\begin{aligned} b_n &= B - \sum_{k \in \mathcal{K}} b_{k,n}, \\ p_n &= P_n^{max} - \sum_{k \in \mathcal{K}} p_{k,n}. \end{aligned}$$

For each  $k \in \mathcal{K}$ , denote

$$r_k = R_k^{min} - \sum_{n \in \mathcal{N}'} r_{k,n}.$$

First, we collect all variables into one vector  $\mathbf{x} \in \mathbf{R}^{2KN'}$ ,  $\mathbf{x} = (r_{1,1}, b_{1,1}, r_{1,2}, b_{1,2}, \dots, r_{K,N'}, b_{K,N'})$ . Then, we convert all inequality constraints into a logarithmic barrier function  $\phi(\mathbf{x})$ :

$$\begin{aligned} \phi(\mathbf{x}) &= - \sum_{n \in \mathcal{N}'} \log b_n - \sum_{n \in \mathcal{N}'} \log p_n - \sum_{k \in \mathcal{K}} \log r_k \\ &\quad - \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} \log r_{k,n} - \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} \log b_{k,n}. \end{aligned}$$

The optimization problem can be converted into a sequence of minimization problems by introducing a logarithmic barrier function with a parameter  $t$ . For Eq.(4), its optimal solution can be approximated by solving the following minimization problem:

$$\min_{\mathbf{x}} \psi_t(\mathbf{x}) = -t \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} r_{k,n} + \phi(\mathbf{x}).$$

TABLE II  
THE BARRIER METHOD FOR BANDWIDTH AND POWER ALLOCATION

Algorithm 1	
1:	$\mathbf{x} = \mathbf{0}, t > 0, \epsilon > 0, \epsilon_{nt} > 0, \mu > 1, \alpha \in (0, 1/2), \beta \in (0, 1)$ .
2:	<b>while</b> $(2KN' + K + 2N' + 1)/t > \epsilon$
3:	<b>while</b> True
4:	Compute $\Delta \mathbf{x}$ by using Eq.(5) and $\lambda^2 = \nabla \psi_t(\mathbf{x}) \Delta \mathbf{x}; s = 1$ ;
5:	<b>if</b> $\lambda^2/2 \leq \epsilon_{nt}$
6:	break;
7:	<b>end if</b>
8:	<b>while</b> $\psi_t(\mathbf{x} + s\Delta \mathbf{x}) > \psi_t(\mathbf{x}) - \alpha s \lambda^2$
9:	$s = \beta s$ ;
10:	<b>end while</b>
11:	Update $\mathbf{x} = \mathbf{x} + s\Delta \mathbf{x}$ ;
12:	<b>end while</b>
13:	$t = \mu t$ ;
14:	<b>end while</b>
15:	<b>return</b> $\mathbf{x}$

As  $t$  increases, such an approximation becomes more and more close to the optimal solution to Eq.(4). Generally, Newton method is preferred to solve the minimization problem mentioned above because of its quadratic convergence property [19]. For a given parameter  $t$ , Newton step  $\Delta \mathbf{x}$  can be computed by solving the following equation:

$$\nabla^2 \psi_t(\mathbf{x}) \Delta \mathbf{x} = -\nabla \psi_t(\mathbf{x}), \quad (5)$$

where  $\nabla^2 \psi_t(\mathbf{x})$  and  $\nabla \psi_t(\mathbf{x})$  are the Hessian and the gradient of  $\psi_t(\mathbf{x})$ , respectively. The Hessian of  $\psi_t(\mathbf{x})$  can be written as:

$$\begin{aligned} \nabla^2 \psi_t(\mathbf{x}) &= D + \sum_{n \in \mathcal{N}'} \frac{\nabla b_n \nabla b_n^T}{b_n^2} \\ &\quad + \sum_{n \in \mathcal{N}'} \frac{\nabla p_n \nabla p_n^T}{p_n^2} + \sum_{k \in \mathcal{K}} \frac{\nabla r_k \nabla r_k^T}{r_k^2}, \end{aligned}$$

where  $D = \text{diag}(D_{1,1}, \dots, D_{K,N'}) \in \mathbf{R}^{2KN'}$  with

$$D_{k,n} = \begin{bmatrix} \frac{1}{r_{k,n}^2} + \frac{1}{p_n} \frac{\partial^2 p_{k,n}}{\partial r_{k,n}^2} & \frac{1}{p_n} \frac{\partial^2 p_{k,n}}{\partial r_{k,n} \partial b_{k,n}} \\ \frac{1}{p_n} \frac{\partial^2 p_{k,n}}{\partial b_{k,n} \partial r_{k,n}} & \frac{1}{b_{k,n}^2} + \frac{1}{p_n} \frac{\partial^2 p_{k,n}}{\partial b_{k,n}^2} \end{bmatrix},$$

$\forall n \in \mathcal{N}'_m \cup \mathcal{N}'_p$ , and

$$\begin{aligned} D_{k,n} &= \left( \frac{1}{p_n} + \frac{\gamma_n}{p_{n'}} \right) \begin{bmatrix} \frac{\partial^2 p_{k,n}}{\partial r_{k,n}^2} & \frac{\partial^2 p_{k,n}}{\partial r_{k,n} \partial b_{k,n}} \\ \frac{\partial^2 p_{k,n}}{\partial b_{k,n} \partial r_{k,n}} & \frac{\partial^2 p_{k,n}}{\partial b_{k,n}^2} \end{bmatrix} \\ &\quad + \begin{bmatrix} \frac{1}{r_{k,n}^2} & 0 \\ 0 & \frac{1}{b_{k,n}^2} \end{bmatrix}, \forall n \in \mathcal{M}'_n. \end{aligned}$$

Due to the special structure of the Hessian, we can solve Eq.(5) by using the method proposed in [20] efficiently. The outline of the barrier method is summarized in Table II, where  $\mathbf{0} \in \mathbf{R}^{2KN'}$  whose elements are all zeros.  $\epsilon$  and  $\epsilon_{nt}$  are the tolerances of the barrier method and the Newton method,

respectively.  $\alpha$  and  $\beta$  are two constants utilized in backtracking line search with  $\alpha \in (0, 0.5)$  and  $\beta \in (0, 1)$ . The step size of the backtracking line search is  $s$  with  $s > 0$ .  $t$  and  $\mu$  are parameters associated with a tradeoff between outer iterations and inner iterations. Based on the optimal solution to Eq.(4), we can design approximation algorithm to solve the minimum cost cell planning problem defined by Eq.(2).

### B. An Approximation Algorithm

Given a set  $\mathcal{N}_1 \subseteq \mathcal{N}$  of BSs, let  $w(\mathcal{N}_1)$  be the optimal value of the objective function of Eq.(4). Denote  $c(\mathcal{N}_1)$  as the total deployment cost of  $\mathcal{N}_1$ . For each  $\mathcal{N}_2 \subseteq \mathcal{N}$ , we define

$$w_{\mathcal{N}_1}(\mathcal{N}_2) = w(\mathcal{N}_1 \cup \mathcal{N}_2) - w(\mathcal{N}_1),$$

and

$$W_{\mathcal{N}_1}(\mathcal{N}_2) = \frac{w_{\mathcal{N}_1}(\mathcal{N}_2)}{c(\mathcal{N}_2)}.$$

Specifically, we define  $w_{\mathcal{N}_1}(\{n\}) = 0$  if the BS  $n$  is a relay but its donor macro BS is not in  $\mathcal{N}_1$ .

The outline of our proposed approximation algorithm for Eq.(2) is described in Table III, where  $X \setminus Y = \{x | x \in X, x \notin Y\}$ . The key idea of our proposed algorithm lies in a greedy strategy: We always select a candidate BS to increase the system capacity with the least cost. That is, the BS with the maximum average capacity per unit deployment cost is selected with priority. Mathematically, we try to add a set of BSs  $G_l$  at the  $l$ th iteration to satisfy

$$\begin{aligned} W_{\mathcal{N}_{l-1}}(G_l) &\geq \max_{\substack{n \in \mathcal{N}_m \setminus \mathcal{N}_{l-1} \\ G \subseteq \mathcal{M}_n}} W_{\mathcal{N}_{l-1}}(\{n\} \cup G), \\ W_{\mathcal{N}_{l-1}}(G_l) &\geq \max_{n \in \mathcal{N}_p \setminus \mathcal{N}_{l-1}} W_{\mathcal{N}_{l-1}}(\{n\}), \\ W_{\mathcal{N}_{l-1}}(G_l) &\geq \max_{\substack{n \in \mathcal{N}_m \cap \mathcal{N}_{l-1} \\ G \subseteq \mathcal{M}_n \setminus \mathcal{N}_{l-1}}} W_{\mathcal{N}_{l-1}}(G), \end{aligned} \quad (6)$$

where  $\mathcal{N}_l$  is all selected BSs at the  $l$ th iteration. Hence, we test all remaining candidate BSs at each iteration, including pico BSs, macro BSs and relays. Such a procedure repeats until the rate requirements of all DNs are satisfied. Then the set of selected BSs can be output as a feasible solution.

To analyze the approximation ratio of the proposed approximation algorithm (Algorithm 2), we need to describe the optimal solution to Eq.(2) at first. Given a cell planning instance, let  $z_n^*, b_{k,n}^*, p_{k,n}^*$  be an optimal solution to Eq.(2), and  $r_{k,n}^* = b_{k,n}^* \log_2(1 + p_{k,n}^* H_{k,n} / b_{k,n}^*)$  is the achievable transmission rate. Denote  $\mathcal{N}_m^* = \{n | z_n^* = 1, n \in \mathcal{N}_m\}$ ,  $\mathcal{N}_p^* = \{n | z_n^* = 1, n \in \mathcal{N}_p\}$  and  $\mathcal{N}_r^* = \{n | z_n^* = 1, n \in \mathcal{N}_r\}$  as the sets of selected macro BSs, pico BSs and relays that exist in the optimal solution, respectively.  $\mathcal{M}_n^* = \mathcal{N}_r^* \cap \mathcal{M}_n$  is the set of relays in  $\mathcal{N}_r^*$  that can be associated to macro BS  $n$ .

We temporarily treat the macro BS and its associated relays as a unity. Denote  $\mathcal{N}_n^* = \{n\} \cup \mathcal{M}_n^*$  as the set of macro BS  $n$  and its associated relays that exist in the optimal solution,  $\forall n \in \mathcal{N}_m^*$ . Define  $\mathcal{N}^* = \mathcal{N}_m^* \cup \mathcal{N}_p^*$ . For all  $n \in \mathcal{N}^*$ , denote

TABLE III  
 $O(\log R)$ -APPROXIMATION ALGORITHM

#### Algorithm 2

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```

1:  $\mathcal{N}_l \leftarrow \emptyset, l = 0.$ 
2: while  $w(\mathcal{N}_l) < \sum_{k \in \mathcal{K}} R_k^{min}$  do
3:    $l = l + 1, W = 0;$ 
4:   for all  $n \in \mathcal{N}_m \setminus \mathcal{N}_{l-1}$  do
5:     Find  $G^* \subseteq \mathcal{M}_n$  that maximizes  $W_n = W_{\mathcal{N}_{l-1}}(\{n\} \cup G^*);$ 
6:     if  $W_n > W$ 
7:        $G_l \leftarrow \{n\} \cup G^*, W = W_n;$ 
8:     end if
9:   end for
10:  for all  $n \in \mathcal{N}_p \setminus \mathcal{N}_{l-1}$  do
11:     $W_n = W_{\mathcal{N}_{l-1}}(\{n\});$ 
12:    if  $W_n > W$ 
13:       $G_l \leftarrow \{n\}, W = W_n;$ 
14:    end if
15:  end for
16:  for all  $n \in \mathcal{N}_m \cap \mathcal{N}_{l-1}$  do
17:    Find  $n^* \in \mathcal{M}_n \setminus \mathcal{N}_{l-1}$  that maximizes  $W_n = W_{\mathcal{N}_{l-1}}(\{n^*\});$ 
18:    if  $W_n > W$ 
19:       $G_l \leftarrow \{n^*\}, W = W_n;$ 
20:    end if
21:  end for
22:   $\mathcal{N}_l \leftarrow \mathcal{N}_{l-1} \cup G_l;$ 
23: end while
24: return  $\mathcal{N}_l$ 

```

---

$R_n^*$  as the total transmission rates of BS  $n$ , where

$$R_n^* = \begin{cases} \sum_{k \in \mathcal{K}} r_{k,n}^* + \sum_{n' \in \mathcal{M}_n^*} \sum_{k \in \mathcal{K}} r_{k,n'}^*, & n \in \mathcal{N}_m^*, \\ \sum_{k \in \mathcal{K}} r_{k,n}^*, & \text{otherwise.} \end{cases}$$

Each macro BS  $n \in \mathcal{N}_m^*$  sustains a rate  $R_n^*$  at the employment cost of  $c(\mathcal{N}_n^*)$ . Each pico BS  $n' \in \mathcal{N}_p^*$  sustains a rate  $R_{n'}^*$  at the cost of  $c_{n'}$ .

**Theorem 2.** *If Algorithm 2 can always find a set of BSs  $G_l$  which satisfies Eq.(6) at the  $l$ th iteration, Algorithm 2 achieves an approximation factor of  $O(\log R)$  for the minimum cost cell planning problem defined by Eq.(2), where*

$$R = \max \left\{ \max_{n \in \mathcal{N}_m} w(\mathcal{N}_n), \max_{n \in \mathcal{N}_p} w(\{n\}) \right\}.$$

*Proof:* Let  $z_n', b_{k,n}', p_{k,n}'$  be the solution of Algorithm 2. Denote  $L$  as the number of iterations and  $\mathcal{N}_L = \{n | z_n' = 1, n \in \mathcal{N}\}$  as the corresponding set of selected BSs by Algorithm 2.

We mainly focus on the BSs in the optimum that are not selected by Algorithm 2 since otherwise the optimum also pays for them. Suppose  $a_l(n)$  is the remaining rate that should be covered by  $R_n^*$  at the  $l$ th iteration. If we add BS  $n$  to  $\mathcal{N}_l$  (if  $n$  is macro BS, the associated relays are also added), the rate requirement  $a_l(n)$  can be obviously satisfied. So we always have

$$a_l(n) \leq \begin{cases} w_{\mathcal{N}_l}(\mathcal{N}_n^*), & \forall n \in \mathcal{N}_m^* \setminus \mathcal{N}_L, \\ w_{\mathcal{N}_l}(\{n\}), & \forall n \in \mathcal{N}_p^* \setminus \mathcal{N}_L, \\ w_{\mathcal{N}_l}(\mathcal{M}_n^* \setminus \mathcal{N}_l), & \forall n \in \mathcal{N}_m^* \cap \mathcal{N}_L. \end{cases} \quad (7)$$

Based on Eq.(6) and Eq.(7), at the  $l$ th iteration, it holds that

$$\begin{aligned} W_{\mathcal{N}_{l-1}}(G_l) &\geq \max_{G \subseteq \mathcal{M}_n} W_{\mathcal{N}_{l-1}}(\{n\} \cup G) \\ &\geq W_{\mathcal{N}_{l-1}}(\mathcal{N}_n^*) \\ &\geq \frac{a_{l-1}(n)}{c(\mathcal{N}_n^*)}, \end{aligned}$$

for each macro BS  $n \in \mathcal{N}_m^* \setminus \mathcal{N}_{l-1}$ . In addition, if macro BS  $n \in \mathcal{N}_m^*$  has been selected before the  $l$ th iteration, but some relays in  $\mathcal{M}_n^*$  are not selected, we still have

$$\begin{aligned} W_{\mathcal{N}_{l-1}}(G_l) &\geq \max_{G \subseteq \mathcal{M}_n \setminus \mathcal{N}_i} W_{\mathcal{N}_{l-1}}(G) \\ &\geq W_{\mathcal{N}_{l-1}}(\mathcal{M}_n^* \setminus \mathcal{N}_i) \\ &\geq \frac{a_{l-1}(n)}{c(\mathcal{M}_n^* \setminus \mathcal{N}_i)} \\ &\geq \frac{a_{l-1}(n)}{c(\mathcal{N}_n^*)}. \end{aligned}$$

Similarly, we can obtain

$$W_{\mathcal{N}_{l-1}}(G_l) \geq \frac{a_{l-1}(n')}{c_{n'}},$$

for each pico BS  $n' \in \mathcal{N}_p^* \setminus \mathcal{N}_l$ .

Now, we charge the cost of each BS in  $\mathcal{N}^*$  at the  $l$ th iteration. Since  $W_{\mathcal{N}_{l-1}}(G_l) = w_{\mathcal{N}_{l-1}}(G_l)/c(G_l)$ , the cost of BS  $n$  in  $\mathcal{N}^*$  at the  $l$ th iteration is

$$c_l(n) = \frac{a_{l-1}(n) - a_l(n)}{W_{\mathcal{N}_{l-1}}(G_l)}.$$

The total cost of  $\mathcal{N}_n^*$  is

$$\begin{aligned} \sum_{l=1}^L c_l(n) &= \sum_{l=1}^L \frac{a_{l-1}(n) - a_l(n)}{W_{\mathcal{N}_{l-1}}(G_l)} \\ &\leq c(\mathcal{N}_n^*) \cdot \sum_{l=1}^L \frac{a_{l-1}(n) - a_l(n)}{a_{l-1}(n)} \\ &\leq c(\mathcal{N}_n^*) \cdot H(R_n^*) \\ &= c(\mathcal{N}_n^*) \cdot O(\log R_n^*) \\ &\leq c(\mathcal{N}_n^*) \cdot O(\log R), \end{aligned} \quad (8)$$

where  $H(r)$  is the  $r$ th harmonic number. For each pico BS, we also can obtain

$$\sum_{l=1}^L c_l(n') \leq c_{n'} \cdot O(\log R), n' \in \mathcal{N}_p^*.$$

Therefore, it always holds that

$$c(\mathcal{N}_L) \leq O(\log R) \cdot c(\mathcal{N}_m^* \cup \mathcal{N}_p^* \cup \mathcal{N}_r^*). \quad \blacksquare$$

Note that the first inequality of Eq.(8) gives a much tighter bound than the third inequality. Unfortunately, we cannot get the information of the optimal solution. However, we can conclude that the solutions obtained by Algorithm 2 perform better than the proved worst-case theoretical bound.

### C. Relay Selection

To implement our proposed  $O(\log R)$ -approximation algorithm, we need to find  $G_l$  as shown in Eq.(6). We decompose it into two subproblems. First, we find  $G^* \subseteq \mathcal{M}_n$  to maximize  $W_n = W_{\mathcal{N}_{l-1}}(\{n\} \cup G^*)$  when an unselected macro BS  $n \in \mathcal{N}_m \setminus \mathcal{N}_l$  is tested at the  $l$ th iteration. Note that there are  $2^{|\mathcal{M}_n|}$  relay combinations for each macro BS  $n$ . We can use the following procedure to find the optimal relay selection efficiently.

**Step 1.**  $W_0 = W_{\mathcal{N}_{l-1}}(\{n\})$  and  $W_{n'} = W_{\mathcal{N}_{l-1} \cup \{n\}}(\{n'\})$ , if  $W_0 > W_{n'}$ ,  $\forall n' \in \mathcal{M}_n$ , we can conclude that  $G^* = \emptyset$ ; otherwise, find  $n^* = \arg \max_{n' \in \mathcal{M}_n} W_{\mathcal{N}_{l-1}}(\{n, n'\})$  and collect all  $n'$ s that  $W_{\mathcal{N}_{l-1}}(\{n, n^*\}) < W_{n'}$  in the set  $\mathcal{A}_n$ . Then go to step 2.

Before proving the conclusion, we need the following mathematical facts.

**Fact 1.** Given positive numbers  $a_1, \dots, a_n$  and  $b_1, \dots, b_n$ , then

$$\max_{i=1, \dots, n} \frac{a_i}{b_i} \geq \frac{\sum_{i=1}^n a_i}{\sum_{i=1}^n b_i} \geq \min_{i=1, \dots, n} \frac{a_i}{b_i}.$$

**Fact 2.** Given positive numbers  $a_1, \dots, a_n$ ,  $b_1, \dots, b_n$  and  $A$ , if

$$\frac{a_i}{b_i} \leq A, i = 1, \dots, n,$$

it always holds

$$\frac{\sum_{i=1}^j a_i}{\sum_{i=1}^j b_i} \leq A, \forall 2 \leq j \leq n.$$

**Lemma 1.** At the  $l$ th iteration, if  $W_0 > W_{n'}$ ,  $\forall n' \in \mathcal{M}_n$ , it always holds that

$$W_0 \geq W_{\mathcal{N}_{l-1}}(\{n\} \cup \mathcal{M}'_n), \forall \mathcal{M}'_n \subseteq \mathcal{M}_n.$$

*Proof:* The proof is referred to Appendix B.  $\blacksquare$

**Lemma 2.** At the  $l$ th iteration, if  $W_0 > W_{n'}$ ,  $n' \in \mathcal{M}_n$  and  $G^* \neq \emptyset$  can maximize  $W_n = W_{\mathcal{N}_{l-1}}(\{n\} \cup G^*)$ , then  $n' \notin G^*$ .

*Proof:* The proof is referred to Appendix C.  $\blacksquare$

**Step 2.** Find

$$(n_1^*, n_2^*) = \arg \max_{n_1, n_2 \in \mathcal{A}_n} W_{\mathcal{N}_{l-1}}(\{n, n_1, n_2\}).$$

If  $W_{\mathcal{N}_{l-1}}(\{n, n_1^*, n_2^*\}) > W_{\mathcal{N}_{l-1}}(\{n\} \cup G^*)$  update  $G^* = \{n_1^*, n_2^*\}$  and remove all  $n'$ s that  $W_{n'} < W_{\mathcal{N}_{l-1}}(\{n, n_1^*, n_2^*\})$  from the set  $\mathcal{A}_n$ . If  $|\mathcal{A}_n| = 2$ , the procedure terminates; otherwise, go to step 3.

**Lemma 3.** At the  $l$ th iteration, if  $W_{\mathcal{N}_{l-1}}(\{n, n_1^*, n_2^*\}) > W_{n'}$ ,  $n' \in \mathcal{A}_n$ , then  $n' \notin G^*$ .

*Proof:* The lemma is intuitive since it obviously holds

$$W_{\mathcal{N}_{l-1}}(\{n\} \cup G^*) \geq W_{\mathcal{N}_{l-1}}(\{n, n_1^*, n_2^*\}) > W_{n'},$$

which also implies  $W_{\mathcal{N}_{l-1}}(\{n\} \cup G^* \setminus \{n'\}) > W_{\mathcal{N}_{l-1}}(\{n\} \cup G^*)$  by using Fact 1.  $\blacksquare$

**Step m.** Find

$$(n_1^*, \dots, n_m^*) = \arg \max_{n_1, \dots, n_m \in \mathcal{A}_n} W_{\mathcal{N}_{l-1}}(\{n, n_1, \dots, n_m\}).$$

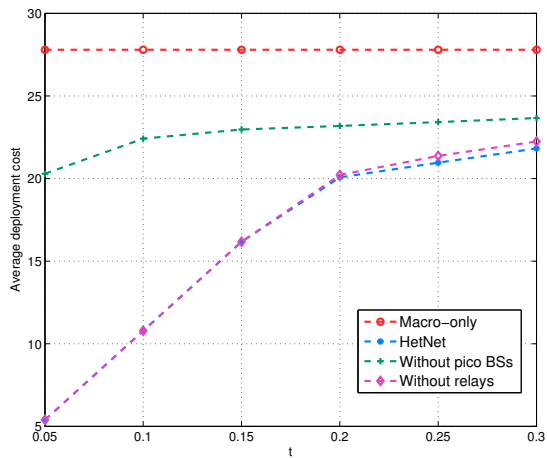


Fig. 2. Average deployment cost as a function of  $t$ .  $N = 200$ ,  $K = 100$ ,  $B = 100\text{MHz}$ ,  $R_k^{\min} = 1\text{Mbps}$  for all  $k$ ,  $P_n^{\max} = 40\text{W}$ ,  $\forall n \in \mathcal{N}_m$ ,  $P_n^{\max} = 1\text{W}$ ,  $\forall n \in \mathcal{N}_p \cup \mathcal{N}_r$ .

If  $W_{\mathcal{N}_{l-1}}(\{n, n_1^*, \dots, n_m^*\}) > W_{\mathcal{N}_{l-1}}(\{n\} \cup G^*)$ , update  $G^* = \{n_1^*, \dots, n_m^*\}$  and remove all  $n$ 's that  $W_{n'} < W_{\mathcal{N}_{l-1}}(\{n\} \cup G^*)$  from the set  $\mathcal{A}_n$ . If  $|\mathcal{A}_n| = m$ , the procedure terminates; otherwise, go to step  $m + 1$ .

**Lemma 4.** At the  $l$ th iteration, if  $W_{\mathcal{N}_{l-1}}(\{n, n_1^*, \dots, n_m^*\}) > W_{n'}$ ,  $n' \in \mathcal{A}_n$ , then  $n' \notin G^*$ .

*Proof:* The proof is similar to Lemma 3. ■

We can continue this procedure to find  $G^*$  to maximize  $W_n = W_{\mathcal{N}_{l-1}}(\{n\} \cup G^*)$  efficiently.

Second, for unselected relays of selected macro BS  $n \in \mathcal{N}_m \cap \mathcal{N}_l$ , we need find  $G^* \subseteq \mathcal{M}_n \setminus \mathcal{N}_l$  to maximize  $W'_n = W_{\mathcal{N}_{l-1}}(G^*)$ . We can find such a set efficiently based on the following lemma.

**Lemma 5.** At the  $l$ th iteration, if relay  $n^*$  can maximize  $W'_n = W_{\mathcal{N}_{l-1}}(\{n^*\})$ , it always holds that

$$W_{\mathcal{N}_{l-1}}(\{n^*\}) \geq W_{\mathcal{N}_{l-1}}(G), \forall G \subseteq \mathcal{M}_n \setminus \mathcal{N}_l.$$

*Proof:* The proof is referred to Appendix D. ■

Thus, we only need to find  $n^*$  that maximize  $W_n = W_{\mathcal{N}_{l-1}}(\{n^*\})$  for this case. To conclude, we have the following theorem.

**Theorem 3.** At the  $l$ th iteration of Algorithm 2, it can always find  $G_l$  which satisfies Eq.(6).

#### IV. NUMERICAL RESULTS

In this section, we give numerical results to evaluate the performance of our proposed algorithm. The system parameters of the considered cellular network, such as path-loss model, maximum transmission power of BS, system bandwidth, etc., are based on the specifications proposed in [23]. All results are averaged over 500 Monte Carlo simulations. The service area is  $3 \times 3 \text{ km}^2$ . There are 20 candidate sites for macro BSs, 100 candidate sites for pico BSs and 80 candidate sites for relays.

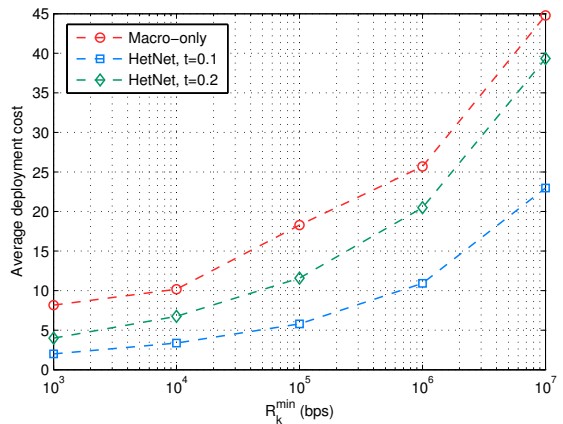


Fig. 3. Average deployment cost as a function of  $R_k^{\min}$ .  $N = 200$ ,  $K = 100$ ,  $B = 100\text{MHz}$ ,  $P_n^{\max} = 40\text{W}$ ,  $\forall n \in \mathcal{N}_m$ ,  $P_n^{\max} = 1\text{W}$ ,  $\forall n \in \mathcal{N}_p \cup \mathcal{N}_r$ .

Each candidate site is uniformly distributed in the deployment area. Each macro BS can provide backhaul for 4 relays and each relay is distributed at the edge of its donor macro BS with uniform distribution. The maximum transmission power of a macro BS is 46dBm and its deployment cost is distributed uniformly in the interval (8, 12). For a pico BS, the transmission power is limited by 30dBm and the deployment cost is distributed uniformly in the interval  $(8 \cdot t, 12 \cdot t)$ , where  $t$  is the average cost ratio of the pico BS to the macro BS. The total available bandwidth is 100 MHz. The transmission power of a relay is also limited by 30dBm, and its cost is distributed uniformly in the interval  $(4 \cdot t, 6 \cdot t)$ . The resource consumption  $\gamma_n$  for each relay is set to 0.05. DNs are distributed uniformly in the deployment area. Path loss (in dB) from the macro BS to a DN is calculated as  $128.1 + 37.6 \log_{10}(D)$  while the path loss from a low-power BS to a DN is  $140.7 + 36.7 \log_{10}(D)$ , where  $D$  is the distance between transmitter and receiver (in km), the standard deviation of lognormal shadowing is 10 dB. The noise PSD is -184 dBm/Hz and  $\Gamma$  is set to 7.6288 ( $BER = 10^{-6}$ ).

First, we study the average deployment cost versus parameter  $t$ . There are 100 DNs in the deployment area and the rate requirement of each DN is 1Mbps. We consider four representative planning schemes: deployment with all kinds of BSs, without pico BSs, without relays and macro-only scheme. As shown in Fig. 2, the deployment cost of HetNet, that is, all kinds of BSs are available, is the most cost-efficient scheme to satisfy the rate requirements of all users, even for the case that the cost of a low-power BS is relatively high compared to a macro BS. We can also observe from Fig. 2 that the planning schemes without relays or without pico BSs, also outperform the macro-only one. The average cost of HetNet is at least 50% and 20% lower than that of the macro-only scheme for the cases of  $t = 0.1$  and  $t = 0.2$ , respectively. Intuitively, more low-power BSs can be selected when  $t$  decreases. As a result, spatial spectrum reuse efficiency is improved, which can satisfy more DNs at a relative low cost. When  $t \geq 0.2$ , the performance gap between HetNet scheme and others becomes

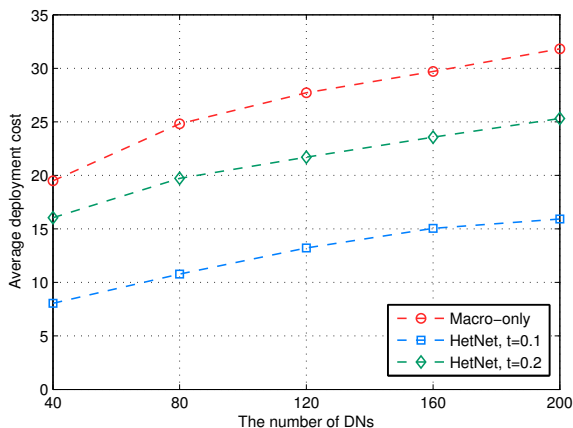


Fig. 4. Average deployment cost as a function of  $K$ .  $N = 200$ ,  $B = 100\text{MHz}$ ,  $R_k^{\min} = 1\text{Mbps}$  for all  $k$ ,  $P_n^{\max} = 40\text{W}$ ,  $\forall n \in \mathcal{N}_m$ ,  $P_n^{\max} = 1\text{W}$ ,  $\forall n \in \mathcal{N}_p \cup \mathcal{N}_r$ .

moderate. It is reasonable because a large  $t$  means that the cost of the deployed small coverage low power BSs certainly increases to obtain capacity gain resulting from improving spatial spectrum efficiency.

Second, we investigate the changes of the average deployment cost for different  $R_k^{\min}$ . There are 100 DNs in the service area and the  $R_k^{\min}$  varies from 1Kbps to 10Mbps. As can be seen in Fig. 3, when  $t = 0.1$ , the deployment cost of the HetNet scheme is at least 50% lower than that of the macro-only one for all considered  $R_k^{\min}$ 's. For the case of  $t = 0.2$ , the cost gap between HetNet and the macro-only scheme is still more than 10%. It is worthy to notice that, as  $R_k^{\min}$  increases, the deployment cost increases sharply for both HetNet and macro-only planning schemes. The HetNet demonstrates great absolute cost advantage compared to the macro-only scheme for high traffic demand cellular networks. So HetNet is of great importance to improve system capacity for the next generation cellular networks.

Fig.4 shows the average deployment cost as a function of the number of DNs. The rate requirement of each DN is 1Mbps. The number of DNs varies from 40 to 200. It can be observed from Fig.4 that the average deployment cost of HetNet is about 50% and 20% lower than that of macro-only scheme for the cases of  $t = 0.1$  and  $t = 0.2$ , respectively. Moreover, as the number of DNs increases, the deployment cost of HetNet scheme increases slowly. HetNet can partially address the changes of traffic demand in a cellular system.

## V. CONCLUSIONS

In this paper, we studied the minimum cost cell planning for a heterogeneous cellular network. Given a candidate set of macro BSs, pico BSs and relays, the cell planning task for a heterogeneous network (HetNet) is to select a subset of BSs (relays) with the minimum deployment cost to fulfill the traffic demands of all users. Since the formulated problem is NP-hard, we proposed an  $O(\log R)$ -approximation algorithm to solve a special case of the problem which is reasonable

for practical cellular networks. Numerical results are given and validate the capacity effectiveness of the HetNet and the efficiency of our proposed approximation algorithm, throwing some insights on how to plan heterogeneous networks.

## APPENDIX A PROOF OF THEOREM 1

If there exists a  $\rho$ -approximation algorithm for the case that every DN can be assigned to one BS, we can find a feasible solution in polynomial time. However, we will show that it is NP-hard to find a feasible solution to such a problem. The proof can be deduced via a reduction from the generalized assignment problem [24]. For simplicity, we consider a macro-only cellular network. For any given subset of BSs  $\mathcal{N}' \subseteq \mathcal{N}$ , the problem of finding a feasible solution can be formulated as follows:

$$\begin{aligned}
 & \text{find } y_{k,n}, p_{k,n}, b_{k,n} \\
 \text{s.t. } & C_1 : \sum_{n \in \mathcal{N}} y_{k,n} r_{k,n} = R_k^{\min}, \forall k \in \mathcal{K}, \\
 & C_2 : 0 \leq b_{k,n}, 0 \leq p_{k,n}, \forall k \in \mathcal{K}, n \in \mathcal{N}', \\
 & C_3 : \sum_{k \in \mathcal{K}} b_{k,n} \leq B, \forall n \in \mathcal{N}', \\
 & C_4 : \sum_{k \in \mathcal{K}} p_{k,n} \leq P_n^{\max}, \forall n \in \mathcal{N}', \\
 & C_5 : y_{k,n} \in \{0, 1\}, \forall k \in \mathcal{K}, n \in \mathcal{N}', \\
 & C_6 : \sum_{n \in \mathcal{N}} y_{k,n} \leq 1, \forall k \in \mathcal{K}.
 \end{aligned} \tag{9}$$

An equivalent form of the above problem can be described as

$$\begin{aligned}
 & \max_{y_{k,n}, b_{k,n}} \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} y_{k,n} R_k^{\min} \\
 \text{s.t. } & C_1 : 0 \leq b_{k,n} \leq y_{k,n} B, \\
 & C_2 : p_{k,n} = \frac{b_{k,n}}{H_{k,n}} \cdot \left( 2^{R_k^{\min}/b_{k,n}} - 1 \right), \\
 & C_3 \sim C_6 \text{ in Eq.(9)}.
 \end{aligned} \tag{10}$$

If the optimal value of the objective function of Eq.(10) is  $\sum_{k \in \mathcal{K}} R_k^{\min}$ , Eq.(9) is feasible. If a feasible solution to Eq.(9) can be found in polynomial time, Eq.(10) can be also solved in polynomial time since the feasible solution to Eq.(9) must be the optimal solution to Eq.(10).

A special case for Eq.(10) is the generalized assignment problem [24] which is NP-hard, so Eq.(10) also defines an NP-hard problem. Therefore, there does not exist any approximation algorithm for Eq.(1) unless implies  $P = NP$ .

## APPENDIX B PROOF OF LEMMA 1

If  $W_0 > W_{n'}, \forall n' \in \mathcal{M}_n$ , we can obtain

$$\begin{aligned}
 \frac{w_{\mathcal{N}_{i-1}(\{n\})}}{c_n} & \geq \max_{|G|=1, G \subseteq \mathcal{M}'_n} \frac{w_{\mathcal{N}_{i-1} \cup \{n\}}(G)}{c(G)} \\
 & \geq \frac{\sum_{|G|=1, G \subseteq \mathcal{M}'_n} w_{\mathcal{N}_{i-1} \cup \{n\}}(G)}{\sum_{|G|=1, G \subseteq \mathcal{M}'_n} c(G)} \\
 & \geq \frac{w_{\mathcal{N}_{i-1} \cup \{n\}}(\mathcal{M}'_n)}{c(\mathcal{M}'_n)}, \forall \mathcal{M}'_n \subseteq \mathcal{M}_n,
 \end{aligned}$$

which follows by Fact 1 and

$$\sum_{|G|=1, G \subseteq \mathcal{M}'_n} w_{\mathcal{N}_{i-1} \cup \{n\}}(G) \geq w_{\mathcal{N}_{i-1} \cup \{n\}}(\mathcal{M}'_n).$$

By using Fact 1 again, we finally have

$$W_{\mathcal{N}_{i-1}}(\{n\}) \geq W_{\mathcal{N}_{i-1}}(\{n\} \cup \mathcal{M}'_n), \forall \mathcal{M}'_n \subseteq \mathcal{M}_n.$$

#### APPENDIX C

##### PROOF OF LEMMA 2

If  $n' \in G^*$ , we have

$$W_{\mathcal{N}_{i-1}}(\{n\} \cup G^*) \geq W_{\mathcal{N}_{i-1}}(G'),$$

where  $G' = \{n\} \cup G^* \setminus \{n'\}$ . However, according to the definition of  $W_{\mathcal{N}_{i-1}}(\{n\} \cup G^*)$ , we can obtain

$$\begin{aligned} W_{\mathcal{N}_{i-1}}(\{n\} \cup G^*) &= \frac{w_{\mathcal{N}_{i-1}}(G') + w_{\mathcal{N}_{i-1} \cup G'}(\{n'\})}{c(G') + c_{n'}} \\ &\leq \frac{w_{\mathcal{N}_{i-1}}(G') + w_{\mathcal{N}_{i-1} \cup \{n\}}(\{n'\})}{c(G') + c_{n'}}. \end{aligned}$$

On the other hand, according to Fact 2, if  $W_{\mathcal{N}_{i-1}}(G') < W_0$ , it certainly holds

$$W_{\mathcal{N}_{i-1}}(\{n\} \cup G^*) = \frac{w_{\mathcal{N}_{i-1}}(G') + w_{\mathcal{N}_{i-1} \cup G'}(\{n'\})}{c(G') + c_{n'}} \leq W_0.$$

Thus, it can be conclude that

$$W_{\mathcal{N}_{i-1}}(G') \geq W_0 > W_{\mathcal{N}_{i-1} \cup \{n\}}(\{n'\}),$$

Then, we have

$$\begin{aligned} W_{\mathcal{N}_{i-1}}(\{n\} \cup G^*) &\leq \frac{w_{\mathcal{N}_{i-1}}(G') + w_{\mathcal{N}_{i-1} \cup \{n\}}(\{n'\})}{c(G') + c_{n'}} \\ &< \frac{w_{\mathcal{N}_{i-1}}(G')}{c(G')} = W_{\mathcal{N}_{i-1}}(G'). \end{aligned}$$

Thus, we can conclude that  $n' \notin G^*$ .

#### APPENDIX D

##### PROOF OF LEMMA 4

According to Fact 1, for every  $G \subseteq \mathcal{M}_n \setminus \mathcal{N}_i$ , we can obtain

$$\begin{aligned} \max_{n' \in G} W_{\mathcal{N}_{i-1}}(\{n'\}) &\geq \frac{\sum_{n' \in G} w_{\mathcal{N}_{i-1}}(\{n'\})}{\sum_{n' \in G} c_{n'}} \\ &\geq \frac{w_{\mathcal{N}_{i-1}}(G)}{\sum_{n' \in G} c_{n'}} = W_{\mathcal{N}_{i-1}}(G), \end{aligned}$$

where the second inequity follows  $\sum_{n' \in G} w_{\mathcal{N}_{i-1}}(\{n'\}) \geq w_{\mathcal{N}_{i-1}}(G)$ . So it always holds

$$W_{\mathcal{N}_{i-1}}(\{n^*\}) \geq \max_{n' \in G} W_{\mathcal{N}_{i-1}}(\{n'\}) \geq W_{\mathcal{N}_{i-1}}(G).$$

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