Energy-Efficient Resource Allocation for OFDM-Based Cognitive Radio Networks

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***Abstract*—In this paper, we investigate the energy-efficient resource allocation in orthogonal frequency division multiplexing (OFDM)-based cognitive radio (CR) networks, where we try to maximize the system energy-efficiency under the consideration of many practical limitations, such as transmission power budget of the CR system, interference threshold of primary users and traffic demands of secondary users. Our general objective formulation leads to a challenging mixed integer programming problem that is hard to solve. To make it computationally tractable, we employ a time-sharing method to transform it into a non-linear fractional programming problem, which can be further converted into an equivalent convex optimization problem by using its *hypogragh* form. Based on these transformations, it is possible to obtain (near) optimal solution by standard optimization technique. However, the complexity of the standard technique is too high for this real-time optimization task. By exploiting the structure of the problem extensively, we develop an efficient *barrier* method to work out the (near) optimal solution with a reasonable complexity, significantly better than the standard technique. Numerical results show that our proposal can maximize the energy efficiency of the CR system, whilst the proposed algorithm performs quickly and stably.**

***Index Terms*—Cognitive radio, energy-efficiency, OFDM, optimization and resource allocation.**

I. Introduction

MOST of the available spectral resources have already been licensed as a result of the regulatory spectrum allocation policy. There is little room to develop new wireless services to meet the ever increasing demands of users. However, investigations show that large portions of the licensed spectrum are far away from highly utilization [1]. In other words, a significant amount of spectrum is used sporadically for given time or region, although it has already been assigned to the licensed primary users (PUs) for exclusive usage. As a promising paradigm, cognitive radio (CR) arises to improve the usage efficiency of radio spectrum with great potential [2]. CR users, also referred to as secondary users (SUs), sense the radio environment and access the licensed spectrum, as long as the interference to the PUs is kept below a preset threshold, such as the level of interference temperature [3]. In order to meet the requirements of opportunistic access, the physical layer of a CR system should be very flexible, which necessitates multicarrier methods to operate in CR networks. Orthogonal frequency division multiplexing (OFDM), which offers a high flexibility in radio resource management, is deemed as an appropriate air interface of a CR system [4]. Consequently, resource allocation arises as an important issue for OFDM-based CR systems.

Adaptive resource allocation (RA) for the OFDM systems has been studied extensively for more than a decade. A survey can be found in [5] and references therein. For the arising OFDM-based CR systems, adaptive RA has also attracted much attention starting from the broaching of the CR and [6] with the references therein provides a comprehensive survey. For single SU case, RA in an OFDM-based CR system degenerates into power distribution. In [7], optimal and suboptimal power allocation schemes are proposed to maximize the sum capacity of the CR system under the interference constraints of the PUs. In [8], the authors studied the optimal power allocation to achieve ergodic capacity and outage capacity in fading channel. In [9], a greedy max-min algorithm is proposed to maximize the throughput of the CR system with a given power budget.

In [10], an efficient algorithm is developed by jointly considering the transmission power budget of the CR system and the interference constraints of the PUs, which achieves a good tradeoff between capacity performance and complexity. In [11], a low-complexity optimal power allocation algorithm is derived by exploiting the structure of the considered optimization problem. RA in multiuser OFDM-based CR systems have been investigated in [12–15]. Both real-time and non-real-time services in multiuser OFDM-based CR networks are studied in [12], where two fast RA algorithms are derived to work out the optimal or suboptimal solutions, respectively. In [13], the sum capacity of a multiuser OFDM-based CR system is maximized while satisfying SUs' proportional rate requirements. In [14, 15], general RA frameworks in CR networks are proposed for the single and multiple primary users cases, respectively, as well as efficient algorithms, which hint that RA in OFDM-based CR networks can be tackled effectively and efficiently.

In comparison with a large amount of work devoted to enhance the throughput of the OFDM-based CR systems, energy-efficiency (EE), which is also an important issue in wireless networks, attracts little attention as far as the authors known. With the ever growth of high-data-rate applications, the energy consumption is also growing at a staggering rate nowadays, which results in a large amount of greenhouse gas and high operation expenditure for wireless service providers [16]. Therefore, green radio, which places great emphasis on energy-efficiency in wireless networks, is

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becoming increasingly important and prompting new waves of research and standard development activities [17], and energy-efficient RA has been put on the agenda in both industry and academia recently, especially for the OFDM-based system which is the most promising modulation technique for future wireless networks. Different from the two conventional classes of dynamic RA in OFDM systems: rate adaptation and margin adaptation [5], energy-efficient RA is a special case where the objective is generally to maximize the EE of the wireless system. Hence, an adequate energy-efficient metric should be given primary importance in overall energy-efficient network design since it is related to the optimized decisions directly. The most popular one is called '*bits-per-Joule*', which is defined as the system throughput to unit-energy consumption.

The EE of wireless communications is surveyed in [18], which depicts the technical roadmaps of several major international projects for energy-efficient wireless networks and discusses state-of-the-art researches on this topic. In [19], the authors developed a low-complexity energy-efficient scheme by taking the time-averaged *bits-per-Joule* as a metric for an uplink OFDMA system. In [20], the fundamental tradeoff between EE and spectral-efficiency in downlink OFDMA networks is investigated. For interference limited wireless communications, a non-cooperative game for energy-efficient power optimization is studied in [21], where the tradeoff between EE and spectral-efficiency is also considered. In [22], energy-efficient RA in OFDM Systems with large numbers of base station is investigated, where imperfect channel state information and different quality of service (QoS) requirements are taken into account. Energy-efficient RA in multi-cell OFDM systems with limited backhaul capacity is studied in [23], where the trade-off between energy efficiency, network capacity, and backhaul capacity is uncovered.

As mentioned above, it is noteworthy that there is little work on the energy-efficient RA of the OFDM-based CR networks. Actually, dynamic energy-efficient RA is of crucial importance for the CR scenario, because it not only involves greenhouse problem and operation expenditure, but is a prerequisite to achieve highly utilization of the limited transmission power which is consumed to support additional signal processing requirements compared with non-CR scenarios, such as spectrum sensing. In [24], energy-efficient channel sensing in the CR networks is proposed, where the optimal sensing-access strategy and the sensing order are designed to achieve the maximum energy efficiency. In addition to the EE of spectrum sensing, energy-efficient RA in the CR networks can also reduce the interference to the PUs and improve the QoS of both CR systems and primary ones. In brief, energy efficiency is as important as spectrum efficiency for the CR networks.

In this paper, we investigate the energy-efficient RA problem in an OFDM-based CR system that operates in a centralized manner, e.g., an access point serves all SUs, which shares spectrum with a licensed primary system. We take the mutual interference between the two systems into consideration, trying to maximize the overall EE of the CR system with given power budgets and the rate requirements of the SUs. The main contributions of this work are summarized as follows: First, we formulate a general energy-efficient RA model which covers essential constraints for the OFDM-based CR systems, which can be extended to many practical scenarios with necessary modifications; second, we propose effective equivalences to transform the formulated intractable mixed integer programming task into a convex optimization problem by using time-sharing method and *hypograph* form, making it possible to work out (near) optimal solutions; third, we develop a fast *barrier* method by exploiting the special structure of the problem to speed up the computation of Newton step which is time-consuming, decreasing computational complexity dramatically to meet the requirement for the considered energy-efficient RA problem that should be handled real time. Simulations results validate the promising performance of our proposal. The energy efficiency of the CR system is very close to the upper bound and the proposed algorithm converges quickly and stably.

The rest of this paper is organized as follows. In Section II, we illustrate system model and formulate our optimization task. In Section III, we propose crucial equivalent transformations to make the frustrating optimization problem tractable, followed by an efficient *barrier* method developed by exploiting the structure of the problem. In Section IV, integer subchannel allocation and optimal power distribution algorithms are presented. Simulation results are given in Section V, as well as discussions. Finally, we conclude the paper in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider the downlink of an OFDM-based CR system with K SUs, denoted by $\mathcal{K} = \{1, 2, \dots, K\}$, coexisting with L PUs in a licensed system. The SUs are allowed to access the radio spectrum registered by the PUs via an access point (AP). The whole available bandwidth W is divided into N subchannels in the CR system, denoted by $\mathcal{N} = \{1, 2, \dots, N\}$. The bandwidth of the nth subchannel spans from $f_0 + (n-1)B$ to $f_0 + nB$, where f_0 is the starting frequency and $B = W/N$.

Throughout this paper, we assume that AP in the CR system has the perfect knowledge of the channel state information (CSI) between the transmitter of the AP and the receivers of the SUs, as well as perfect CSI between the transmitter of the AP and the receivers of the active PUs. The former can be approximated by assuming channel reciprocity. The latter can be periodically measured by a band manager and sent to the CR AP via a common control channel, or estimated by listening to a beacon signal and then fed-back to the CR AP [25]. The CSI can also be estimated by using the statistical information of channel gains, which may yield a chance constrained optimization model [26] that emphasizes on long-term energy efficiency of the CR system. In this paper, we consider instantaneous energy efficiency with perfect CSI. Hence the results obtained by our proposed scheme can serve as an upper bound on the achievable EE with channel estimation errors.

To prohibit the unacceptable performance degradation of the PUs, the interference introduced to the PUs must be carefully controlled in a tolerable range. Some promising methods can be employed by the CR system to eliminate the interference to the PUs, such as sub-carrier weighting [27] and non-continuous OFDM scheme [28]. In this paper, we consider a general OFDM system which may not adopt non-continuous OFDM or sub-carrier weighting techniques. Assume the lth PU's nominal band ranges from f_l to $f_l + B_l$, where f_l and B_l are the lth PU's starting frequency and bandwidth, respectively, the interference introduced to the lth PU by SUs' access on the nth subchannel with unit transmission power is

$$I_{n,l}^{SP} = \int_{f_l - f_0 - (n-1/2)B}^{f_l + B_l - f_0 - (n-1/2)B} g_{n,l}^{SP} \phi(f) df, \qquad (1)$$

where $g_{n,l}^{SP}$ represents the power gain from the CR AP in the CR system to the lth PU's receiver on the nth subchannel. $\phi(f)$ is the baseband power spectral density (PSD) of OFDM signal with $\phi(f) = T(\frac{\sin \pi f T}{\pi f T})^2$, where T is OFDM symbol duration.

Similarly, the interference generated by the lth PU into the nth subchannel used by the kth SU is

$$I_{k,n,l}^{PS} = \int_{f_0 + (n-1)B - f_l - B_l/2}^{f_0 + nB - f_l - B_l/2} g_{k,n,l}^{PS} \phi_l(f) df, \qquad (2)$$

where $\phi_l(f)$ is the PSD of the lth PU's signal and $g_{k,n,l}^{PS}$ is the power gain from the lth PU's transmitter to the receiver of the kth SU over the nth subchannel.

Let $r_{k,n}$ denote the transmission rate of the nth subchannel used by the kth SU, we have

$$r_{k,n} = \log\left(1 + \frac{p_{k,n}|h_{k,n}|^2}{\Gamma(N_0 B + \sum_{l=1}^{L} I_{k,n,l}^{PS})}\right), \qquad (3)$$

where $p_{k,n}$ is the power allocated to the nth subchannel used by the kth SU and $h_{k,n}$ is the channel gain from the CR AP to the kth SU's receiver over the nth subchannel. N_0 is the PSD of additive white Gaussian noise and Γ is the SNR gap. For an uncoded MQAM, Γ is related to a given bit-error-rate (BER) requirement with $\Gamma = -\ln(5BER)/1.5$ as derived in [29]. The interference caused by the PUs' signals on the nth subchannel used by the kth SU can be calculated by $\sum_{l=1}^{L} I_{k,n,l}^{PS}$ using (2), or regarded as noise and measured by the receiver of the kth SU [30]. For notation brevity, denote the signal-to-noise ratio (SNR) of the nth subchannel used by the kth SU with unit power as follows,

$$H_{k,n} = \frac{|h_{k,n}|^2}{\Gamma(N_0 B + \sum_{l=1}^{L} I_{k,n,l}^{PS})}. \qquad (4)$$

The rate of the kth SU is

$$R_k = \sum_{n=1}^{N} \rho_{k,n} \log(1 + p_{k,n} H_{k,n}), \qquad (5)$$

where $\rho_{k,n}$ can either be 1 or 0 informing whether the nth subchannel is occupied by the kth SU or not. And the transmission power of the kth SU is

$$P_k = \sum_{n=1}^{N} \rho_{k,n} p_{k,n}. \qquad (6)$$

Besides the transmission power, the energy consumption also includes circuit energy consumption in the active mode of transmitter, incurred by signal processing, active circuit blocks, etc. Without loss of generality, the associated circuit energy consumption is generally modeled as a constant P_c [31, 32]. Our results can be extended to other circuit energy consumption models as discussed in [31, 32].

In [20, 22, 23, 32], an adequate EE metric is defined as the system throughput for unit-energy consumption. Both the transmission power and the circuit energy consumption should be taken into consideration for energy-efficient communication, where the former is used for reliable data transmission and the latter represents average energy consumption of device electronics. Thus, the EE in *bits/Joule* of the CR system is given by

$$\eta_{EE} = \frac{\sum_{k=1}^{K} R_k}{\sum_{k=1}^{K} P_k + P_c}. \qquad (7)$$

In this work, we aim to maximize the downlink EE of the CR system under the transmission power budget. Moreover, each SU requires an individual minimal rate $R_{k,min}$ to support its reliable communication, while the interference to the PUs should be strictly kept below their thresholds. Hence, the energy-efficient RA optimization problem can be formulated as

$$\begin{aligned}
\max_{p_{k,n}, \rho_{k,n}} \quad & \eta_{EE} = \frac{\Sigma_{n=1}^{N} \Sigma_{k=1}^{K} \rho_{k,n} r_{k,n}}{\Sigma_{n=1}^{N} \Sigma_{k=1}^{K} \rho_{k,n} p_{k,n} + P_c} \\
s.t.\ C1 \quad & \sum_{n=1}^{N} \rho_{k,n} r_{k,n} \geq R_{k,min}, \forall k \\
C2 \quad & \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_{k,n} p_{k,n} \leq P_t \\
C3 \quad & \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_{k,n} p_{k,n} I_{n,l}^{SP} \leq I_l^{th}, l = 1, \ldots, L \\
C4 \quad & p_{k,n} \geq 0, \forall k, n \\
C5 \quad & \rho_{k,n} \in \{0, 1\}, \forall k, n \\
C6 \quad & \sum_{k=1}^{K} \rho_{k,n} = 1, \forall n,
\end{aligned} \qquad (8)$$

where P_t is the total transmission power limit of the CR AP, while the I_l^{th} is the interference threshold of the lth PU. C1 guarantees the minimal rate requirements of the SUs. C2 and C3 are constraints on transmission power budget and interference level, respectively. C5 and C6 indicate that each OFDM subchannel is prohibited from sharing by multiple SUs.

III. Optimal Energy-Efficient RA Based on Time-Sharing

Obviously, (8) is a mixed integer programming problem since both binary variables $\rho_{k,n}$'s and real variables $p_{k,n}$'s are involved. The main difficulty of solving (8) lies in the integer constraint C5, which generates K^N possible subchannel assignments if employing an intuitive exhaustive search. It is obviously impractical even for a small number of OFDM subchannels. A general method to tackle the integer programming is time-sharing, by which integer variables are relaxed into continuous ones so that efficient linear/nonlinear optimization methods can be employed. The optimal solution to the relaxation form is always an upper bound of the problem (8) because all feasible solutions to the original problem fall into the solution space of the relaxed one.

Here, we try to introduce the time-sharing method to tackle the frustrating problem (8). Redefine $\rho_{k,n} \in [0, 1]$ as the faction of the nth subchannel allocated to the kth SU, temporarily permitting that each subchannel is shared by multiple SUs. Furthermore, we introduce a new variable $s_{k,n}$, where $s_{k,n} = \rho_{k,n} p_{k,n}$, to characterize the actual power consumption of the kth user on the nth subchannel in a time frame interval. Then the EE η_{EE} of the system can be rewritten as

$$\eta_{EE} = \frac{\Sigma_{n=1}^{N}\Sigma_{k=1}^{K}\rho_{k,n}\log(1+s_{k,n}H_{k,n}/\rho_{k,n})}{\Sigma_{n=1}^{N}\Sigma_{k=1}^{K}s_{k,n}+P_c}. \quad (9)$$

Thus, the relaxation form of the (8) is given by

$$\begin{aligned}
\max_{s_{k,n},\rho_{k,n}} \quad & \eta_{EE} \\
s.t.\ C1 \quad & \sum_{n=1}^{N}\rho_{k,n}\log(1+\tfrac{s_{k,n}H_{k,n}}{\rho_{k,n}}) \geq R_{k,min}, \forall k \\
C2 \quad & \sum_{k=1}^{K}\sum_{n=1}^{N}s_{k,n} \leq P_t \\
C3 \quad & \sum_{k=1}^{K}\sum_{n=1}^{N}s_{k,n}I_{n,l}^{SP} \leq I_l^{th}, l=1,\ldots,L \\
C4 \quad & s_{k,n} \geq 0, \forall k,n \\
C5 \quad & 0 < \rho_{k,n} \leq 1, \forall k,n \\
C6 \quad & \sum_{k=1}^{K}\rho_{k,n} = 1, \forall n.
\end{aligned} \quad (10)$$

A. Equivalent transformation in hypograph form

(10) defines a non-linear fractional programming problem that is still difficult to solve [33]. Nevertheless, (10) has an equivalent transformation via its *hypograph* form [33], since the denominator of the objective function is jointly concave in $\{s_{k,n}, \rho_{k,n}\}$'s and the numerator is linear. The *hypogragh* form of (10) is following,

$$\begin{aligned}
\max_{s_{k,n},\rho_{k,n},y} \quad & y \\
s.t. \quad & \eta_{EE} \geq y \\
& C1 \sim C6 \text{ in } (10) \\
& y \geq 0,
\end{aligned} \quad (11)$$

where the domain $y \geq 0$ is determined by the inequality $\eta_{EE} \geq 0$. Such a transformation can guarantee the equivalence relation of (10) and (11). Collect $s_{k,n}$'s and $\rho_{k,n}$'s into two vectors $\boldsymbol{s}$ and $\boldsymbol{\rho}$ with $\boldsymbol{s} = (s_{1,1}, s_{1,2}, \ldots, s_{K,N})$ and $\boldsymbol{\rho} = (\rho_{1,1}, \rho_{1,2}, \ldots, \rho_{K,N})$. (11) can be analyzed geometrically as an optimization problem in the 'graph space' of $(\boldsymbol{s}, \boldsymbol{\rho}, y)$. That is, we maximize y over the *hypograph* of η_{EE}, subject to the constraints in (10), which is equivalent to solve the problem (10) directly.

Thus, (11) can be transformed into a convex optimization problem as follows,

$$\eta_{EE}(\boldsymbol{s}, \boldsymbol{\rho}) \geq y \Longleftrightarrow \varphi(\boldsymbol{s}, \boldsymbol{\rho}, y) \geq 0, \quad (12)$$

where $\varphi(\boldsymbol{s}, \boldsymbol{\rho}, y) = \sum_{n=1}^{N}\sum_{k=1}^{K}\rho_{k,n}\log(1+\frac{s_{k,n}H_{k,n}}{\rho_{k,n}}) - y(\sum_{n=1}^{N}\sum_{k=1}^{K}s_{k,n}+P_c)$. Then the equivalent *hypograph* form of (11) can be alternatively written as follows,

$$\begin{aligned}
\max_{s_{k,n},\rho_{k,n},y} \quad & y \\
s.t. \quad & \varphi(\boldsymbol{s}, \boldsymbol{\rho}, y) \geq 0 \\
& C1 \sim C6 \text{ in } (10) \\
& y \geq 0.
\end{aligned} \quad (13)$$

It is easy to prove that (13) defines a convex optimization problem, to which the optimal solutions, s^* and ρ^*, are the same as (10). Since there are fully developed algorithms to tackle such kind of problems, it becomes optimistical to work out the optimal solution to (10) by solving (13).

B. Fast barrier method

Barrier method is treated as a standard technique to solve convex optimization problems. The computational complexity of the *barrier* method mainly lies in the computation of Newton step that needs matrix inversion with a complexity of $O((2KN+N)^3)$ for our considered problem. For a practical OFDM system, the number of subchannels is always several thousand and such a complexity is too high to apply, especially for the RA problem that should be tackled in an online manner. In this work, we develop an efficient algorithm to compute the Newton step by exploiting the special structure of the problem, making the *barrier* method promising to perform the concerned optimization task.

To employ the *barrier* method, a preparatory procedure is necessary to transform the objective function y in (13) into a twice differentiable function $U(y)$. We rewrite the transformation form of the optimization problem as follows,

$$\begin{aligned}
\max_{s_{k,n},\rho_{k,n},y} \quad & U(y) \\
s.t.\ C1 \quad & \varphi(\boldsymbol{s}, \boldsymbol{\rho}, y) \geq 0 \\
C2 \quad & \sum_{n=1}^{N}\rho_{k,n}\log(1+\tfrac{s_{k,n}H_{k,n}}{\rho_{k,n}}) \geq R_{k,min}, \forall k \\
C3 \quad & \textstyle\sum_{k=1}^{K}\sum_{n=1}^{N}s_{k,n} \leq P_t \\
C4 \quad & \sum_{k=1}^{K}\sum_{n=1}^{N}s_{k,n}I_{n,l}^{SP} \leq I_l^{th}, l=1,\ldots,L \\
C5 \quad & s_{k,n} \geq 0, \forall k,n \\
C6 \quad & 0 < \rho_{k,n} \leq 1, \forall k,n \\
C7 \quad & \sum_{k=1}^{K}\rho_{k,n} = 1, \forall n, \\
C8 \quad & y \geq 0,
\end{aligned} \quad (14)$$

where U: $\Re \rightarrow \Re$ is monotone increasing. Evidently, the associated problem (14) and the original form (13) are equivalent; indeed, their feasible solution sets are identical, as well as the optimal points [33]. In this paper, we take $U(y) = \log(1+y)$ to guarantee the equivalence of (13) and (14).

Then we convert all inequality constraints into a logarithmic

TABLE I
THE BARRIER METHOD

Algorithm
1. ***Initialization***
2. Feasible point $\boldsymbol{x} \in \mathbf{R}^{2KN+1\times 1}$, $\epsilon > 0, \epsilon_n > 0$, $t = t^{(0)} > 0$, $\mu > 1, \alpha \in (0, 1/2), \beta \in (0, 1)$.
3. **repeat**
4. ***Newton method***
5. Starting point $\boldsymbol{x}$, subject to $\boldsymbol{Bx} = \mathbf{1}$
6. **repeat**
7. Compute $\triangle \boldsymbol{x}_{nt}$ and $\lambda^2 = -\nabla\psi_t(\boldsymbol{x})^T \triangle \boldsymbol{x}_{nt}$
8. ***Backtracking line search***
9. $s = 1$;
10. **while** $\psi_t(\boldsymbol{x} + s\Delta\boldsymbol{x}_{nt}) > \psi_t(\boldsymbol{x}) - \alpha s \lambda^2$
11. $s = \beta s$
12. **end while**
13. Update $\boldsymbol{x} = \boldsymbol{x} + s\triangle \boldsymbol{x}_{nt}$
14. **until** $\lambda^2/2 \le \epsilon_n$
15. $t = \mu t$
16. **until** $(3KN + K + L + 3)/t < \epsilon$
17. **return** $\boldsymbol{x}$

barrier function $\phi(\boldsymbol{x})$,

$$\begin{aligned}\phi(\boldsymbol{x}) = \quad & -\log\varphi(\boldsymbol{x}) - \log y - \log(P_t - \sum_{n=1}^{N}\sum_{k=1}^{K} s_{k,n}) \\ & - \sum_{k=1}^{K}\log(\sum_{n=1}^{N}\rho_{k,n}\log(1 + \frac{s_{k,n}H_{k,n}}{\rho_{k,n}}) - R_{k,min}) \\ & - \sum_{l=1}^{L}\log(I_l^{th} - \sum_{n=1}^{N}\sum_{k=1}^{K} s_{k,n} I_{n,l}^{SP}) \\ & - \sum_{k=1}^{K}\sum_{n=1}^{N}(\log s_{k,n} + \log\rho_{k,n} + \log(1-\rho_{k,n})),\end{aligned} \tag{15}$$

where all variables $\{\boldsymbol{s}, \boldsymbol{\rho}, y\}$ are collected into one vector $\boldsymbol{x}$, i.e.$\boldsymbol{x} = \{s_{1,1}, \rho_{1,1}, s_{1,2}\ldots, \rho_{K,N}, y\}$.

Thus, the optimal solution to (13) can be approximated by solving the following minimization problem,

$$\begin{aligned} \min \quad & \psi_t(\boldsymbol{x}) = -t\log(1+y) + \phi(\boldsymbol{x}) \\ s.t. \quad & \boldsymbol{Bx} = \mathbf{1}, \end{aligned} \tag{16}$$

where t is a parameter to control the accuracy of solution. The C7 in (14) is reformed into the constraint in (16), where $\boldsymbol{B}$ is an $N \times (2KN+1)$ matrix and $\mathbf{1} \in \Re^N$ with

$$\boldsymbol{B}_{n,m} = \begin{cases} 1 & m = 2(k-1)N + 2n, \forall k, n \\ 0 & \text{otherwise} \end{cases} . \tag{17}$$

The optimal solution to (16) is an approximation of the problem (14). As t increases, the approximation becomes more and more close to the optimal solution. The outline of the *barrier* method is summarized in Table I. ϵ and ϵ_n are the tolerances of the *barrier* method and the Newton step, respectively. α and β are two constants utilized in backtracking line search with $\alpha \in (0, 0.5)$ and $\beta \in (0, 1)$. The step size of the backtracking line search is s with $s > 0$. t and μ are parameters which are associated with the tradeoff between outer iterations and inner iterations.

In the inner loop of the *barrier* method, Newton method is always preferred to compute the central point because of its quadratic convergence property. With a given parameter t, Newton step $\Delta\boldsymbol{x}_{nt}$ and its associated dual variables $\boldsymbol{\nu}$ satisfy the following Karush-Kuhn-Tucker (KKT) systems,

$$\begin{bmatrix} \nabla^2\psi_t(\boldsymbol{x}) & \boldsymbol{B}^T \\ \boldsymbol{B} & \mathbf{0}_n \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{x}_{nt} \\ \boldsymbol{\nu} \end{bmatrix} = \begin{bmatrix} -\nabla\psi_t(\boldsymbol{x}) \\ \mathbf{0}_v \end{bmatrix}, \tag{18}$$

where $\Delta\boldsymbol{x}_{nt} \in \Re^{2KN+1}$ $\mathbf{0}_n \in \Re^{N\times N}$ and $\mathbf{0}_v \in \Re^N$. $\nabla^2\psi_t(\boldsymbol{x})$ and $\nabla\psi_t(\boldsymbol{x})$ are the Hessian and the gradient of $\psi_t(\boldsymbol{x})$, respectively.

If calculating the Newton step by solving (18) directly, it will generate a complexity of $O((2KN+N)^3)$, which is too high to apply in practice. We analyze the problem (16) and develop an efficient procedure to compute the Newton step quickly by exploiting its special structure. For simplicity, denote

$$\begin{aligned} f_0 &= P_t - \sum_{k=1}^{K}\sum_{n=1}^{N} s_{k,n}, \\ f_k &= \sum_{n=1}^{N}\rho_{k,n}\log(1 + \tfrac{s_{k,n}H_{k,n}}{\rho_{k,n}}) - R_{k,min}, \forall k \\ g_l &= I_l^{th} - \sum_{n=1}^{N}\sum_{k=1}^{K} s_{k,n} I_{n,l}^{SP}, l = 1, \ldots, L, \end{aligned} \tag{19}$$

then the gradient of $\psi_t(\boldsymbol{x})$ is

$$\begin{aligned} \frac{\partial\psi_t}{\partial s_{k,n}} &= -(\frac{1}{\varphi} + \frac{1}{f_k})\frac{\rho_{k,n}H_{k,n}}{\rho_{k,n} + s_{k,n}H_{k,n}} + \frac{y}{\varphi} \\ & \quad + \sum_{l=1}^{L}\frac{I_{n,l}^{SP}}{g_l} + \frac{1}{f_0} - \frac{1}{s_{k,n}} \\ \frac{\partial\psi_t}{\partial\rho_{k,n}} &= -(\frac{1}{\varphi} + \frac{1}{f_k})(\log(1 + \frac{s_{k,n}H_{k,n}}{\rho_{k,n}}) \\ & \quad - \frac{s_{k,n}H_{k,n}}{\rho_{k,n} + s_{k,n}H_{k,n}}) - \frac{1}{\rho_{k,n}} + \frac{1}{1-\rho_{k,n}} \\ \frac{\partial\psi_t}{\partial y} &= \frac{t}{1+y} + \frac{1}{y} - (\sum_{k=1}^{K}\sum_{n=1}^{N} s_{k,n} + P_c)/\varphi. \end{aligned} \tag{20}$$

And the Hessian of $\psi_t(\boldsymbol{x})$ can be calculated by (21). Hence it follows,

$$\begin{aligned} \nabla^2\psi_t(\boldsymbol{x}) &= \begin{bmatrix} \boldsymbol{D}_{1,1} & & & \\ & \ddots & & \\ & & \boldsymbol{D}_{K,N} & \\ & & & Y \end{bmatrix} \\ & \quad + \frac{\nabla f_0 \nabla f_0^T}{f_0^2} + \sum_{k=1}^{K}\frac{\nabla f_k \nabla f_k^T}{f_k^2} \\ & \quad + \frac{\nabla\varphi\nabla\varphi^T}{\varphi^2} + \sum_{l=1}^{L}\frac{\nabla g_l \nabla g_l^T}{g_l^2} \\ &= \boldsymbol{D} + \sum_{i=1}^{K+L+2}\boldsymbol{q}_i\boldsymbol{q}_i^T, \end{aligned} \tag{22}$$

$$\begin{aligned}\bigtriangledown^2\psi_t(\boldsymbol{x}) \;=\; & \begin{bmatrix} \frac{1}{s_{k,n}^2} & & & & & \\ & \frac{1}{\rho_{k,n}^2}+\frac{1}{(1-\rho_{k,n})^2} & & & & \\ & & \ddots & & & \\ & & & \frac{1}{s_{K,N}^2} & & \\ & & & & \frac{1}{\rho_{K,N}^2}+\frac{1}{(1-\rho_{K,N})^2} & \\ & & & & & \frac{t}{(1+y)^2}+\frac{1}{y^2}\end{bmatrix} \\ & +\frac{\bigtriangledown f_0 \bigtriangledown f_0^T}{f_0^2}+\frac{\bigtriangledown^2 f_0}{f_0}+\sum_{k=1}^{K}\frac{\bigtriangledown f_k \bigtriangledown f_k^T}{f_k^2}+\sum_{k=1}^{K}\frac{\bigtriangledown^2 f_k}{f_k} \\ & +\frac{\bigtriangledown \varphi \bigtriangledown \varphi^T}{\varphi^2}+\frac{\bigtriangledown^2 \varphi}{\varphi}+\sum_{l=1}^{L}\frac{\bigtriangledown g_l \bigtriangledown g_l^T}{g_l^2}+\sum_{l=1}^{L}\frac{\bigtriangledown^2 g_l}{g_l}\end{aligned} \tag{21}$$

Denote $\boldsymbol{D}=\mathrm{diag}(\boldsymbol{D}_{1,1},\boldsymbol{D}_{1,2},\ldots,\boldsymbol{D}_{K,N},Y)$, where

$$\begin{aligned}\boldsymbol{D}_{k,n} &= \begin{bmatrix}\frac{1}{s_{k,n}^2} & 0\\ 0 & \frac{1}{\rho_{k,n}^2}+\frac{1}{(1-\rho_{k,n})^2}\end{bmatrix} \\ &+\left(\frac{1}{\varphi}+\frac{1}{f_k}\right)\begin{bmatrix}\frac{\partial^2 f_k}{\partial s_{k,n}^2} & \frac{\partial^2 f_k}{\partial s_{k,n}\partial\rho_{k,n}}\\ \frac{\partial^2 f_k}{\partial \rho_{k,n}\partial s_{k,n}} & \frac{\partial^2 f_k}{\partial\rho_{k,n}^2}\end{bmatrix}, \\ Y &= t/(1+y)^2+1/y^2,\end{aligned} \tag{23}$$

and $\boldsymbol{q}_i\in\Re^{2KN+1}$ with

$$\boldsymbol{q}_i=\begin{cases}\frac{\bigtriangledown f_0}{f_0}, & i=1\\ \frac{\bigtriangledown f_k}{f_k}, & k=1,\ldots,K, i=k+1\\ \frac{\bigtriangledown\varphi}{\varphi}, & i=K+2\\ \frac{\bigtriangledown g_l}{g_l}, & l=1,\ldots,L, i=K+l+2,\end{cases} \tag{24}$$

we can find that the Hessian is positive definite because the matrix $\boldsymbol{D}$ is positive definite and all $\boldsymbol{q}_i\boldsymbol{q}_i^T\geq 0$. Moreover, since $\boldsymbol{B}$ is a full row rank matrix, the KKT matrix at the left-side of (18) is invertible. However, there are always thousands of OFDM subchannels in practical wireless systems, the arising complexity of matrix inversion for solving the KKT matrix is $O((2KN+N)^3)$, which frustrates its application.

Consider the Hessian of $\psi_t(\boldsymbol{x})$, based on the matrix inversion lemma [34], we derive a fast algorithm to speed up the computation of Newton step. According to the decomposition in (22), the KKT matrix in the left of (18) can be written into

$$\boldsymbol{\Lambda}_0=\begin{bmatrix}\boldsymbol{D} & \boldsymbol{B}^T\\ \boldsymbol{B} & \boldsymbol{0}_n\end{bmatrix}+\sum_{i=1}^{M}\boldsymbol{\lambda}_i\boldsymbol{\lambda}_i^T, \tag{25}$$

where $\boldsymbol{\Lambda}_0=\begin{bmatrix}\bigtriangledown^2\psi_t(\boldsymbol{x}) & \boldsymbol{B}^T\\ \boldsymbol{B} & \boldsymbol{0}_n\end{bmatrix}$, $\boldsymbol{\lambda}_i=\begin{bmatrix}\boldsymbol{q}_i\\ \boldsymbol{0}_v\end{bmatrix}, i=1,\ldots,M$ with $M=K+L+2$. Denote $\boldsymbol{\lambda}_0=\begin{bmatrix}-\nabla\psi_t\\ \boldsymbol{0}_v\end{bmatrix}$, we propose an $(M+1)$-step iterative algorithm to calculate the Newton step quickly. The detail of the algorithm is given Table II.

TABLE II
FAST CALCULATING OF NEWTON STEP

Step 1	Decompose $\boldsymbol{\Lambda}_0$, $\boldsymbol{\Lambda}_0=\boldsymbol{\Lambda}_1+\boldsymbol{\lambda}_1\boldsymbol{\lambda}_1^T$. Then we have $\begin{bmatrix}\Delta\boldsymbol{x}_{nt}\\ \boldsymbol{\nu}\end{bmatrix}=\boldsymbol{v}_1^1-\frac{\boldsymbol{\lambda}_1\boldsymbol{v}_1^1}{1+\boldsymbol{\lambda}_1\boldsymbol{v}_2^1}\boldsymbol{v}_2^1$, Where $\boldsymbol{\Lambda}_1\boldsymbol{v}_1^1=\boldsymbol{\lambda}_0$ and $\boldsymbol{\Lambda}_1\boldsymbol{v}_2^1=\boldsymbol{\lambda}_1$. After Step 1, we can figure out the $\Delta\boldsymbol{x}_{nt}$ by solving $\boldsymbol{v}_1^1$ and $\boldsymbol{v}_2^1$, instead of using (18).
Step 2	Decompose $\boldsymbol{\Lambda}_1$ with $\boldsymbol{\Lambda}_1=\boldsymbol{\Lambda}_2+\boldsymbol{\lambda}_2\boldsymbol{\lambda}_2^T$, Similarly, the two variables in Step 1 can be updated by $\boldsymbol{v}_i^1=\boldsymbol{v}_i^2-\frac{\boldsymbol{\lambda}_2\boldsymbol{v}_i^2}{1+\boldsymbol{\lambda}_2\boldsymbol{v}_3^2}\boldsymbol{v}_3^2, i=1,2$, where $\boldsymbol{\Lambda}_2\boldsymbol{v}_i^2=\boldsymbol{\lambda}_{i-1}, i=1,2,3$. $\vdots$ Without loss of generality, we consider step m,
Step m	Let $\boldsymbol{\Lambda}_{m-1}=\boldsymbol{\Lambda}_m+\boldsymbol{\lambda}_m\boldsymbol{\lambda}_m^T$. We can update the m variables in Step $m-1$ by $\boldsymbol{v}_i^{m-1}=\boldsymbol{v}_i^m-\frac{\boldsymbol{\lambda}_m^T\boldsymbol{v}_i^m}{1+\boldsymbol{\lambda}_m\boldsymbol{v}_{m+1}^m}\boldsymbol{v}_{m+1}^m, i=1,\ldots,m$, where $\boldsymbol{\Lambda}_m\boldsymbol{v}_i^m=\boldsymbol{\lambda}_{i-1}, i=1,\ldots,m+1$ with $\boldsymbol{\Lambda}_i=\boldsymbol{D}+\sum_{j=i+1}^{M}\boldsymbol{\lambda}_j\boldsymbol{\lambda}_j^T$

After the Mth step, it produces $M+1$ matrix systems $\boldsymbol{\Lambda}_M\boldsymbol{v}_i^M=\boldsymbol{\lambda}_{i-1}, i=1,\ldots,M+1$. From the decomposition procedure shown above, we can find that the m variables $\boldsymbol{v}_i^{m-1}, i=1,\ldots,m$ in Step $m-1$ can be obtained by the $m+1$ variables $\boldsymbol{v}_i^m, i=1,\ldots,m+1$ in Step m. Hence, if we figure out the $M+1$ variables $\boldsymbol{v}_i^M, i=1,\ldots,M+1$, the Newton step and the associated dual variable in (18) can be indirectly obtained by an M-step reverse computation.

The computation process to solve the matrix system $\boldsymbol{\Lambda}_M\boldsymbol{v}_i^M=\boldsymbol{\lambda}_{i-1}$ is as follows. Without loss of generality, we convert the equations into a unified form since we have $\boldsymbol{\Lambda}_M=\begin{bmatrix}\boldsymbol{D} & \boldsymbol{B}^T\\ \boldsymbol{B} & \boldsymbol{0}_n\end{bmatrix}$,

$$\begin{bmatrix}\boldsymbol{D} & \boldsymbol{B}^T\\ \boldsymbol{B} & \boldsymbol{0}_n\end{bmatrix}\begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{v}\end{bmatrix}=\begin{bmatrix}\boldsymbol{q}\\ \boldsymbol{0}_v\end{bmatrix} \tag{26}$$

where $\boldsymbol{u},\boldsymbol{q}\in\Re^{2KN+1}$ and $\boldsymbol{v}\in\Re^N$. It follows that

$$\begin{aligned}&\boldsymbol{D}_{k,n}\left[\begin{array}{c}\boldsymbol{u}_{\theta(k,n)-1}\\ \boldsymbol{u}_{\theta(k,n)}\end{array}\right]=\left[\begin{array}{c}\boldsymbol{q}_{\theta(k,n)-1}\\ \boldsymbol{q}_{\theta(k,n)}-\boldsymbol{v}_n\end{array}\right],\\ &\sum_{k=1}^{K}\boldsymbol{u}_{\theta(k,n)}=0,\forall n,\\ &\boldsymbol{u}_{2KN+1}=\boldsymbol{q}_{2KN+1}/Y,\end{aligned} \tag{27}$$

where $\theta(k,n)=2(k-1)N+2n$. According to the (27), it follows

$$\begin{aligned}\sum_{k=1}^{K}\boldsymbol{u}_{\theta(k,n)} &= \sum_{k=1}^{K}\boldsymbol{D}_{k,n_{2,1}}^{-1}\boldsymbol{q}_{\theta(k,n)-1}\\ &\quad+\sum_{k=1}^{K}\boldsymbol{D}_{k,n_{2,2}}^{-1}(\boldsymbol{q}_{\theta(k,n)}-\boldsymbol{v}_n)\\ &= 0.\end{aligned} \tag{28}$$

Then, we can obtain

$$\boldsymbol{v}_n=\frac{1}{\sum\limits_{k=1}^{K}\boldsymbol{D}_{k,n_{2,2}}^{-1}}\left(\sum_{k=1}^{K}\boldsymbol{D}_{k,n_{2,1}}^{-1}\boldsymbol{q}_{2[k,n]-1}+\sum_{k=1}^{K}\boldsymbol{D}_{k,n_{2,2}}^{-1}\boldsymbol{q}_{2[k,n]}\right). \tag{29}$$

By substituting (29) into (27), we have

$$\left[\begin{array}{c}\boldsymbol{u}_{\theta(k,n)-1}\\ \boldsymbol{u}_{\theta(k,n)}\end{array}\right]=\boldsymbol{D}_{k,n}^{-1}\left[\begin{array}{c}\boldsymbol{q}_{\theta(k,n)-1}\\ \boldsymbol{q}_{\theta(k,n)}-\boldsymbol{v}_n\end{array}\right]. \tag{30}$$

Thus, the matrix system (26) can be solved based on (29) and (30).

C. Warm start procedure

In the initialization of the *barrier* method in Section III-B, a strictly feasible starting point is required. Thus, a preparatory procedure is necessary to obtain a feasible point or prove its nonexistence. We execute the warm start procedure with two steps. First, we try to find a feasible point $(\boldsymbol{s}^0,\boldsymbol{\rho}^0)$, satisfying the constraints $C2\sim C7$ in (14). Then we can take any value in the interval $(0,\eta_{EE}(\boldsymbol{s}^0,\boldsymbol{\rho}^0))$ as a feasible y, denoted as y^0.

In the first step, finding a feasible solution is equivalent to solve a minimization problem by introducing a crucial indicator parameter z as discussed in [33]. The optimization problem for the warm start procedure is following,

$$\begin{aligned}\min_{s_{k,n},\rho_{k,n},z}\quad & z\\ s.t.\ C1\quad & \textstyle\sum_{n=1}^{N}\rho_{k,n}\log(1+\dfrac{s_{k,n}H_{k,n}}{\rho_{k,n}})\geq R_{min}-z\\ & C3\sim C7\ \text{ in }\ (14),\end{aligned} \tag{31}$$

where z can be interpreted as a bound on the maximum infeasibility of the inequality C1 in (31) and our goal is to drive it below zero. Since it is easy to find $(\boldsymbol{s},\boldsymbol{\rho})$ to satisfy C3$\sim$C7 in (14), we can choose a feasible z to satisfy C1 in (31). Thus the feasible solution to (31) always exists.

Note that (31) also defines a convex problem of which the structure is similar to (14). Due to its special structure, we can also apply the fast algorithm developed in Section III-B to solve (31). By solving (31), a strictly feasible point $(\boldsymbol{s}^0,\boldsymbol{\rho}^0,y^0)$ may be obtained, or there is no feasible point. If the optimal solution to (31) satisfies $z\leq 0$, the associated solution of s and ρ can be used as the starting point of the *barrier* method employed to solve (14). Otherwise, no feasible point exists for (14), and we regard such a case as system outage.

D. Complexity analysis

The computational complexity of our proposed algorithm can be counted as follows. The derived fast algorithm to solve (14) requires M decompositions, while each decomposition yields an additional matrix equation. First, we need to solve $M+1$ matrix systems according to (29) and (30) with a complexity $O(KN)$ for each system. Then, a reverse substitution with M steps is required. Thus, we can conclude the complexity to work out the optimal solution to (10) is measured by $O(M^2KN)$.

Since we can also apply the proposed fast algorithm to solve the warm start problem, the computation complexity of the procedure is roughly equal to that of solving (12) because of the similar structure. Therefor, we conclude that the complexity of the proposed algorithm for the considered problem is $O(M^2KN)$. If employing standard method to compute the Newton step directly by matrix inversion, the complexity is $O((2KN+N)^3)$, which is much higher than our proposed algorithm since $M\ll KN$ generally holds in practical systems.

IV. Integer Subchannel Allocation and Optimal Power Allocation

A. Integer-tone assignment

Generally, each subchannel is kept from being shared among multiple users in practical OFDM systems. So each subchannel can be assigned to only one SU, which means that the allocation indexes $\rho_{k,n}$'s are constrained to be 0's or 1's.

Consider the optimal solution of the relaxation form (10), the fractional $\rho_{k,n}$ can be regarded as a metric to determine the exact assignment of subchannels. Actually, for the optimal solution of (10), only few subchannels are shared among users as $\rho_{k,n}$ is mostly either 1 or 0 for $K\ll N$ [35]. Based on this fact, it is appropriate to allocate the nth subchannel to the kth SU with the maximum $\rho_{k,n}$, that is,

$$\rho_{k,n}^{*}=\begin{cases}1, & k=\text{argmax }\rho_{k,n}\\ 0, & \text{otherwise.}\end{cases} \tag{32}$$

(32) guarantees that each subchannel is strictly assigned to only one SU. Consequently, the tie on each subchannel is broken in this way. We will show that such a simple rounding technique can obtain good solution close to the upper bound in the next Section.

B. Power distribution with given subchannel assignment

Given a subchannel assignment, the binary variables $\rho_{k,n}$'s in the original problem (8) are fixed to 0's or 1's, the integer constraints vanish and power distribution across subchannels follows. Let Ω_k be the set of subchannels allocated to the kth relay, the power distribution problem can be given by

$$\begin{aligned} \max_{p_{k,n}} \quad & \eta_{EE} = \frac{\Sigma_{k=1}^{K} \Sigma_{n\in\Omega_k} r_{k,n}}{\Sigma_{k=1}^{K} \Sigma_{n\in\Omega_k} p_{k,n} + P_c} \\ s.t.\ C1 \quad & \sum_{n\in\Omega_k} r_{k,n} \geq R_{k,min}, \forall k \\ C2 \quad & \sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n} \leq P_t \\ C3 \quad & \sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n} I_{n,l}^{SP} \leq I_l^{th}, l = 1, \ldots, L \\ C4 \quad & p_{k,n} \geq 0, \forall k, n. \end{aligned} \tag{33}$$

(33) also defines a non-linear fractional programming problem, which can also be converted into a convex optimization problem via its *hypograph* form as discussed in Section III-A. Additionally, the proposed fast algorithm in Section III-B is also applicable for solving (33). We omit the derivation process in detail because it is the same as solving (10). Thus, the computational load for the optimal power allocation with a given subchannel assignment is bounded by $O(M^2N)$, which can be obtained by similar analysis in Section III-D.

V. Simulation Results

The performance of our proposed resource allocation scheme is evaluated by a series of numerical experiments. Consider a multiuser OFDM-based CR system, where all users randomly located in a 3×3km area, and each receiver uniformly distributed in the circle within 0.5km from its transmitter. The path loss exponent is 4 [36], the variance of shadowing effect is 10dB and the amplitude of multipath fading is Rayleigh. We assume that each PU's bandwidth is randomly generated by uniform distribution and the maximum value is $2W/3L$. The noise power is 10^{-13}W.

As discussed in Section III, the optimal solution of the relaxation form (10) serves as an upper bound of the original problem[1]. We also give the integer subchannel assignment with optimal power allocation (INT-OP) in Section IV. If no feasible point exists, we regard it as system outage.

Fig.1 illustrates the EE of the CR system versus the transmission power limit for different numbers of subchannels. The numbers of SUs and PUs are set to 4 and 2, respectively. The interference threshold of each PU is 5×10^{-12}W and the rate requirement of each SU is 20bits/symbol. The static circuit power is fixed to 0.25W. There are N=32 and N=64 subchannels in the two cases. As can be seen in Fig.1, the EE of the CR system varies acutely at the beginning (growing) because the CR system outage can be reduced as the increase of the transmission power budget. When the transmission power budget is sufficient enough, all SUs' rate requirements can be always satisfied and the EE of the CR system keeps almost invariable as seen in Fig.1. Additionally, the EE can be improved when there are more OFDM subchannels, which is a result of channel diversity in wireless environment. Notice that the INT-OP achieves more than 98% of the Upper Bound,

[1]Note that the upper bound can not be a feasible solution because the relaxed form of the original problem ignores the integer constraints.

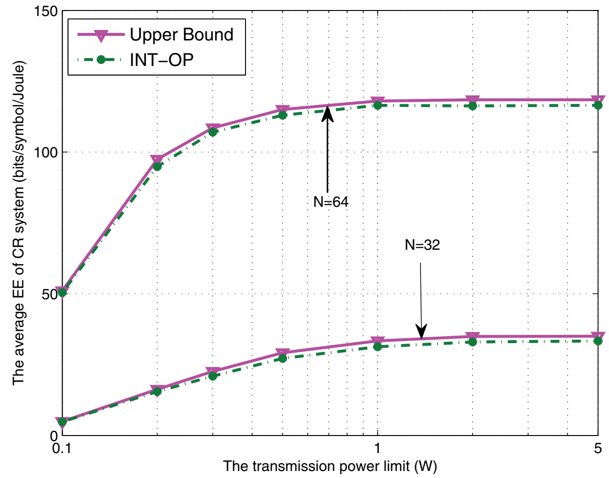

Fig. 1. The EE of CR system as a function of the transmission power limit. K=4, L=2.

indicating our proposed algorithm performs quite well for the considered problem.

We also depict the EE of the CR system versus the interference threshold for different numbers of PUs ($L = 1$, L=2 and L=4) in Fig.2. The number of subchannels is N=64. There are 4 SUs with uniform rate requirement $R_{k,min} = 20$bits/symbol. The transmission power limit is 1W and the static circuit power is 0.25W. It is shown in Fig.2 that the EE increases with the increasing of the interference threshold. The reason is that the lower the interference threshold is, the more frequently the CR system suffers outage. It can be also seen from Fig.2 that more PUs can decrease the EE of the CR system, which can be explained that more subchannels are interference limited [10] in these cases and the subchannels with poor channel gains consume much power to maintain the required rates of the SUs. Again, we can see our proposed INT-OP can obtain solutions close to the Upper Bound.

Fig.3 shows the EE of the CR system as a function of the minimal rate requirements of SUs for different numbers of SUs. There are 64 subchannels with the total transmission power limit $P_t = 1$W and static circuit power $P_c = 0.25$W. The number of PUs is 2 with the interference threshold $I_l^{th} = 5 \times 10^{-12}$W. The INT-OP always performs quite close to the Upper Bound. We can observe that the EE of the CR system decreases with the growth of the rate requirements for both $K = 2$ and $K = 4$. Because the growth of rate requirements not only results in exponentially increase of power consumption, but also more frequently exhausts the radio resource (subchannels and power), even leads to system outage. Comparing the curves of the two cases, we can find the EE becomes larger with the growth of the number of SUs when the rate requirement is relatively small. However, when the rate requirement increases, more SUs can contrarily lower the EE of the CR system, which occurs as a cut-off of the rate requirement. This phenomenon can be explained as follows.

When the SUs' rate requirements are small, the CR network benefits from multiuser diversity for more SUs case because a subchannel is more likely to be allocated to an SU who

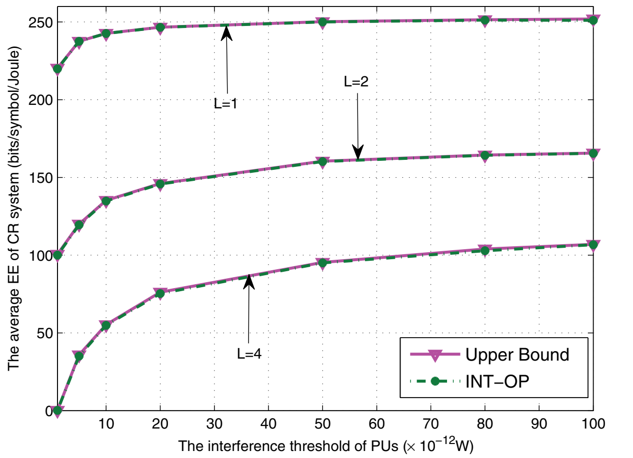

Fig. 2. The EE of CR system as a function of interference threshold of PUs. N=64, K=4, P_t=1W.

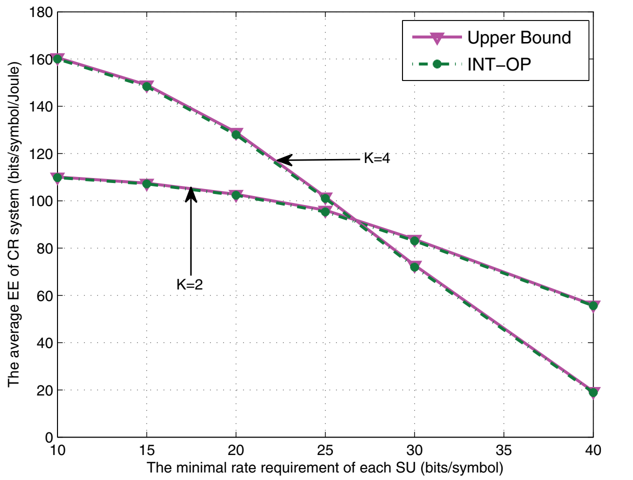

Fig. 3. The EE of CR system as a function of the minimal rate requirements of SUs. N=64, L=2, P_t=1W.

has good channel gain over it. Thus, less power is required to maintain the SUs' rate requirements and the EE of the CR system is larger than less SUs case. However, when an SU's rate requirement is high, more subchannels and much power are mandatorily allocated to the SU in order to meet the rate requirement. At this time, the subchannels with poor link gains inevitably consume much power to meet all SUs' rate requirements, which rather results in the decreasing of the EE. In other words, the degree of freedom for the SUs to get sunchannels with good channel gains reduces from the viewpoint of the CR system for the high rate requirements case.

Then we investigate the convergence of our proposed algorithms in Fig.4 and Fig.5. As discussed in Section III, the computation load of the *barrier* method mainly lies in the computation of Newton step. If the number of Newton iterations is large or varies in a wide range, the algorithm would be difficult to be applied in practical wireless systems. Fig.4 and Fig.5 show that it is not the case for our proposed

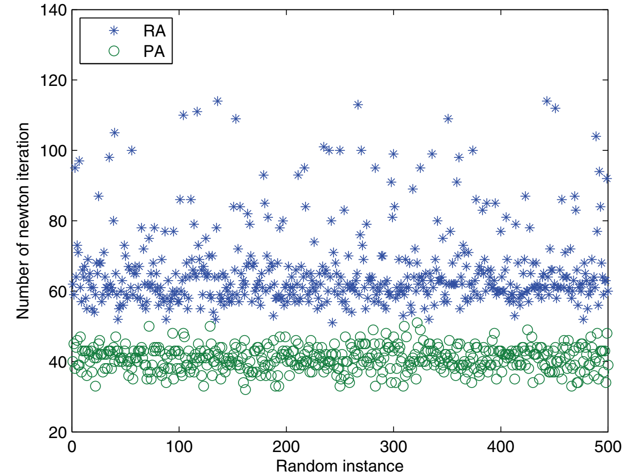

Fig. 4. Number of Newton iterations required for convergence with 500 channel realizations. N=64, K=4, L=2.

algorithm for all concerned settings. Fig.4 shows the number of Newton iterations of the *barrier* method to converge in 500 random instances for both solving the relaxed RA problem and the optimal power allocation (PA). Fig.5 gives the cumulative distribution function (CDF) of the number of Newton iterations for the optimal RA based on time-sharing in (a) and optimal power allocation in (b) with different settings of N. Both Fig.4 and Fig.5 show that the number of Newton iterations varies in a narrow range with a given N. All these observations validate that our proposed algorithm is effective and efficient.

We also give the time cost of our proposed algorithm and the standard one which computes Newton step by matrix inversion directly. Fig.6 shows the average time cost (in *second*) as a function of number of subchannels over 1000 instances, for both the cases of the optimal RA based on time-sharing and the optimal power allocation. The elapsed time is counted by in-built *tic-toc* function in *Matlab*. From Fig.6 we can see the time cost of our proposed algorithm is much less than the standard technique. Even the number of subchannels is 256, the time cost is less than 0.2s for the optimal power distribution. We can conservatively conclude that the consumed time could be further reduced for specialized computing platform. So our proposal is promising for applications.

VI. Conclusion

In this paper, we studied the energy-efficient resource allocation in an OFDM-based CR network, which is an urgent task for green communication design. Our model is general and covers many practical constraints, leading to an intractable mixed integer programming problem. We perform a series of equivalent transformations by analyzing the formulated problem intensively, converting it into a convex optimization problem which can be solved by standard optimization technique. Furthermore, we develop an efficient algorithm to work out the (near) optimal solution by exploiting its special structure to update Newton step in an ingenious way, reducing the computation complexity dramatically and making

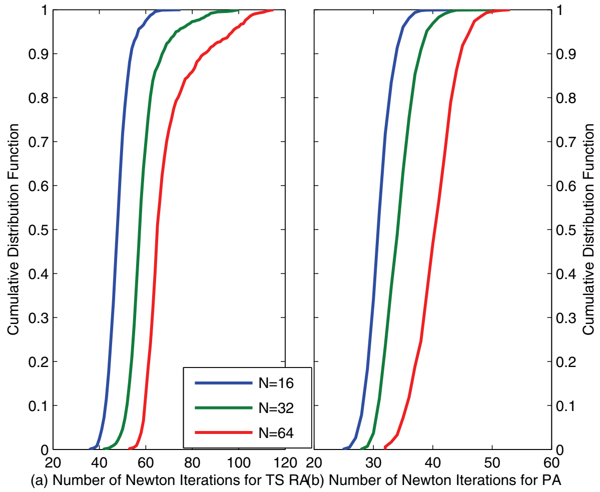

Fig. 5. CDF of the number of Newton iterations required for convergence for 1000 channel realizations. K=4, L=2. (a) Fast barrier method for optimal RA based on time-sharing; (b) fast barrier method for optimal power allocation.

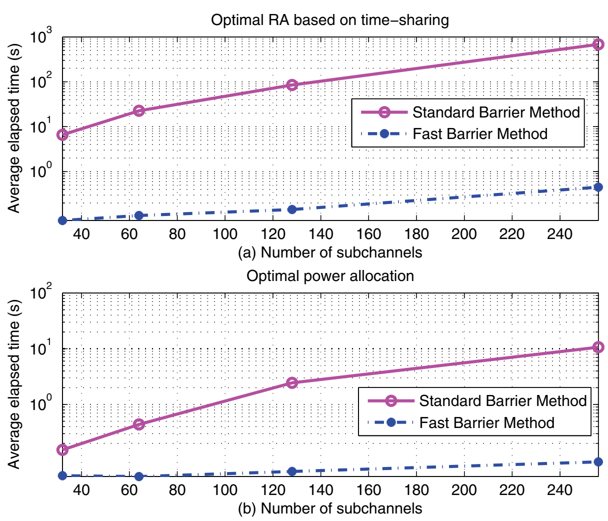

Fig. 6. Average time cost as a function of the number of subchannels. (a) Optimal RA based on time-sharing; (b) optimal power allocation.

its applications possible. Numerical results show that our resource allocation proposal can achieve near optimal energy efficiency, while the algorithm developed in this paper converges quickly and stably. For future work, imperfect channel state information case should be considered. Efficient heuristic methods with lower complexity are also promising for this real-time optimization problem, especially for the subchannel assignment that introduces intractable integer variables.

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