Resource Sharing Scheme for Device-to-Device Communication Underlaying Cellular Networks

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***Abstract*—Device-to-device (D2D) communication is deemed as a promising technology to improve the spectrum efficiency of the cellular systems. In this paper, we study a resource sharing scheme for the D2D communication underlaying cellular networks, where multiple D2D pairs can share subchannels with multiple cellular users (CUs). Our optimization task is to maximize the sum rate of D2D pairs while satisfying the rate requirements of all CUs. The formulated problem falls naturally into a mixed integer programming form that is intractable. We first develop a subchannel sharing protocol, by which we can determine whether or not a subchannel can be reused by two D2D pairs. Then, we prove that the problem can be well approximated by ignoring the mutual interference among the D2D pairs that share the same subchannels without deteriorating the performance of both the CUs and the D2D pairs. Based on the analysis results, we propose an efficient subchannel allocation scheme by employing a simple greedy strategy, as well as a power distribution algorithm that can work out almost the optimal solution to the original problem. Numerical results show that our proposal can significantly increase spectrum efficiency of the cellular system. Furthermore, our proposed subchannel sharing protocol is effective and efficient for practical communication scenarios.**

***Index Terms*—Cellular network, device-to-device (D2D) communication, mixed integer programming, resource allocation.**

I. Introduction

DEVICE-TO-DEVICE (D2D) communication underlaying cellular network, which provides direct data transmission among user equipments with the support of base stations (BSs) in a cellular network, has been proposed as a candidate technology to improve the spectrum- and energy-efficiency of the cellular system [1], [2]. The proximity of the D2D communication pairs brings a lot of benefits for users, such as high transmission rates, low delays and low power consumptions, which can improve the quality of service (QoS) of the cellular system and extend the battery lifetime of user equipment [3], [4]. Since the D2D links generally share radio resources with the cellular system, the spectrum utilization efficiency of the cellular system can also be enhanced in this way. Moreover, D2D communication can offload the traffic of the BSs significantly, which can improve the performance of the cellular system [5], [6].

However, D2D communication sharing radio spectrum with the cellular system poses new challenges that should be addressed properly if we expect to obtain the potential performance gains. One of the key issues in D2D communication is the interference management problem caused by the resource sharing between the D2D pairs and the cellular system. In other words, intra-cell interference is no longer negligible for the D2D communication underlaying cellular network scenarios. D2D pairs can share spectrum with the cellular users (CUs) in orthogonal or non-orthogonal ways [7]–[9]. The former is that the CUs use part of the spectrum and leave the remaining spectrum to the D2D pairs. Obviously, intra-cell interference can be eliminated completely for the orthogonal spectrum sharing scheme. However, the spectrum reuse efficiency of this scheme is not as high as that of the non-orthogonal one which allows the D2D pairs to share the whole spectrum with the CUs. The non-orthogonal sharing scheme for the D2D communication has been receiving much attention in recent years, mainly focusing on the arising the interference management issue.

The case that one D2D pair shares a single channel with one CU is investigated in [10]–[12], where the key issue is to design an efficient radio resource sharing mode to control the interference to the CU. Three resource sharing modes are proposed in [10], where the D2D pair can share the resource with the CU in orthogonal or non-orthogonal way. Additionally, the BS acts as a relay to help the D2D pair achieve better capacity gain. Mode switching of the three resource sharing modes is further investigated in [11]. By contrast, in [12], the D2D pair acts as a relay to help BS communicate with the CU when they locate far apart. A two-time-slot protocol is proposed, where the D2D pair receives the signals of the CU in the first time slot and transfers them to the BS in the second time slot. In [13], multiple subchannels are shared by one D2D pair and one CU, where a low complexity power allocation algorithm is developed. The scenario that one D2D link shares spectrum resources with multiple CUs is studied in [14], [15]. In [14], the authors suggest that the D2D link should use the channels that are not used by the CUs falling into a protection area of the D2D link which is defined by a circle with given radius. In [15], the transmission rate of the D2D link is maximized by reusing all subchannels while guaranteeing the CUs' rates.

The case that multiple D2D pairs share radio resources with multiple CUs is investigated in [16]–[22]. In [16], [17], the

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optimization objective is to maximize the sum rate of both the D2D pairs and the CUs, while satisfying all users' rate requirements. Each D2D pair can only use one subchanel. Meanwhile, each CU shares the subchannel with at most one D2D pair. In [16], the authors proposed a suboptimal subchannel allocation algorithm, which can roughly guarantee each user's minimal rate when all users transmit data with the maximum powers. In [17], the subchannel and power allocation problem is formulated as an assignment problem, which is solved by Kuhn-Munkres algorithm. [17] and [18] have the similar problem formulations expect that two resource sharing modes are both taken into consideration in [18]. A dynamic distributed resource sharing scheme is developed to jointly assign subchannel and power among the CUs and the D2D pairs. Fairness among D2D pairs is considered in [19]. In [20], a robust resource allocation for relay-assisted D2D communication underlaying cellular networks is studied, where relay is introduced to help the D2D pair communicate with each other. Coalition game based resource sharing for D2D communication in cellular networks is also an important topic, which has been studied in [21]. In [22], an alternating optimization method is proposed to maximize the sum throughput of the CUs and the D2D pairs.

For all problem formulations mentioned in [14]–[21], a subchannel is strictly constrained to be used by at most one D2D pair. In such scenario, there is no interference among the D2D pairs. However, such subchannel reuse mode is not spectrum-efficient and not suitable for many practical application scenarios. Obviously, the spectrum efficiency of the cellular systems can be improved further if multiple D2D pairs can use the same subchannels [23]–[28]. On the other hand, most of previous researches on multiple D2D pairs sharing the same subchannels concentrated on fixed power allocation among the CUs and the D2D pairs [23]–[26], which cannot achieve optimal solutions from the viewpoint of the system. In [27], a D2D link can reuse a subchannel only if the interference to other D2D pairs that also use this subchannel is below a given threshold. However, the CUs and the D2D pairs use different subchannels, which lowers spectrum utilization efficiency as discussed above. In [28], dynamic resource allocation is studied, where the D2D pairs are allowed to utilize all subchannels. However, the D2D pairs in close proximity will inevitably suffer heavy mutual interference.

In this paper, we study the resource allocation problem for D2D communication underlaying cellular networks, where multiple D2D pairs can use the same subchannels. We try to maximize the sum rate of the D2D pairs while satisfying all CUs' rate requirements. To achieve the goal, we answer two questions: First, under what condition a subchannel can be used by two D2D pairs with acceptable mutual interference; second, for each subchannel, how to determine the set of D2D pairs which can share this subchannel so that we can maximize the spectrum reuse efficiency of the cellular system. We develop a subchannel sharing protocol for two D2D pairs which can suppress the mutual interference between them to a negligible level. Then we design a greedy algorithm to allocate each subchannel to the D2D pairs that can share this subchannel without heavy mutual interference among them. Finally, we propose a power distribution algorithm for both the D2D pairs and the CUs. We show that the proposed algorithm converges rapidly to a stationary point which is often globally optimal. Numerical results show that our proposed resource sharing scheme can significantly improve the capacity of the cellular system. Moreover, experiment results also verify that our proposed subchannel sharing protocol is effective and efficient. The main contributions of this paper can be summarized as follows:

- We investigate the optimal power allocation problem for the case that two D2D pairs use the same subchannel while guaranteeing the QoS of the CU who also uses this subchannel. We find out that the optimal solution always exists in a set with few feasible solutions that can be obtained by straight-forward search.
- We develop a subchannel sharing protocol for two D2D pairs which potentially use the same subchannel. The proposed protocol can keep the mutual interference between the D2D pairs under an acceptable level. Moreover, it offers insight into how to reuse a subchannel among multiple D2D pairs in cellular systems.
- We propose a greedy-based algorithm to allocate a subchannel to one or more D2D pairs while keeping the interference among all D2D pairs under negligible levels, as well as an efficient power distribution algorithm. Our proposed resource allocation scheme can improve the throughput of the D2D pairs without deteriorating the QoSs of the CUs.

The remainder of this paper is organized as follows. In Section II, we present system model and formulate optimization task. In Section III, we present our resource sharing scheme in detail. In Section IV, numerical results are reported with discussions. Conclusion is drawn in Section V.

II. System Model and Problem Formulation

To make the rest of this paper easy to follow, we list some frequently used notations in Table I.

Consider a cellular network with D2D communication, which consists of a BS, N CUs and M D2D pairs. The D2D pairs reuse the uplink resource under the control of the BS. The sets of the CUs and the D2D pairs are denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$ and $\mathcal{M} = \{1, 2, \ldots, M\}$, respectively. The available spectrum is divided into N orthogonal subchannels. Each CU uses one of the subchannels. Without loss of generality, we assume that nth subchannel is allocated to CU n, so $\mathcal{N}$ also denotes the set of the subchannels. Each subchannel can be shared by both the CUs and the D2D pairs. $\rho_{m,n}$ informs whether D2D pair m uses the nth subchannel or not. If $\rho_{m,n} = 1$, the D2D pair m uses the nth subchannel; $\rho_{m,n} = 0$, otherwise. Let p_n and $p_{m,n}$ be the transmission powers of the CU n and the D2D transmitter (D2D-Tx) m on the nth subchannel. Recall that the set of the subchannels shares the same order of the set of the CUs. Denote H_n and $H_{m,n}$ as the channel power gains of the CU n and the D2D pair m on the nth subchannel.

For the CU n, the maximum transmission power is limited to P_n^C. $\widetilde{H}_{m,n}^D$ is the power gain of the interference link from

TABLE I
SYMBOL NOTATIONS

Symbol	Semantics
$\mathcal{N}$	The set of the CUs
N	The number of the CUs
p_n	The transmission power of the CU n
P_n^C	The maximum transmission power of the CU n
H_n	The channel power gain from the CU n to the BS on the nth subchannel
γ_n^C	The SINR of the BS on the nth subchannel
$R_{n,min}^C$	The sate requirement of the CU n
$\mathcal{M}$	The set of the D2D pairs
M	The number of the D2D pairs
$p_{m,n}$	The transmission power of the D2D-Tx m on the nth subchannel
P_m^D	The maximum transmission power of the D2D-Tx m
$H_{m,n}$	The channel power gain from the D2D-Tx m to the D2D-Rx m on the nth subchannel
$\widetilde{H}_{m,n}^D$	The power gain of the interference link from the D2D-Tx m to the BS
$\widetilde{H}_{m,n}^C$	The power gain of the interference link from the CU n to the D2D-Rx m on the nth subchannel
$\widetilde{H}_{m',m}^n$	The power gain of the interference link from the D2D-Tx m' to the D2D-Rx m on the nth subchannel
$\gamma_{m,n}^D$	The SINR of the D2D-Rx m on the nth subchannel
$\rho_{m,n}$	The D2D pairs assignment index
$\mathcal{M}_n$	The set of D2D pairs which are assigned to the nth subchannel
σ_0^2	The noise power on each subchannel

the D2D-Tx m to the BS. For simplicity, we use $\mathcal{M}_n = \{m \in \mathcal{M} | \rho_{m,n} = 1\}$ to denote the set of the D2D pairs which are assigned to the nth subchannel. The signal to interference plus noise ratio (SINR) of the BS on the nth subchannel can be written as

$$\gamma_n^C = \frac{p_n H_n}{\sum\limits_{m \in \mathcal{M}_n} p_{m,n} \widetilde{H}_{m,n}^D + \sigma_0^2}, \quad (1)$$

where σ_0^2 is the noise power. For simplicity, we assume that the noise powers are the same for the CUs and the D2D links on each subchannel.

If the D2D-Tx m uses the nth subchannel, both the CU n and other D2D-Tx's that use this subchannel can generate interference to the D2D receiver (D2D-Rx) m. Denote $\widetilde{H}_{m,n}^C$ and $\widetilde{H}_{m',m}^n$ as the power gain of the interference link from the CU n to the D2D-Rx m and the power gain of the interference link from the D2D-Tx m' to the D2D-Rx m on nth subchannel, respectively, the SINR of the D2D-Rx m on the nth subchannel can be written as

$$\gamma_{m,n}^D = \frac{p_{m,n} H_{m,n}}{\sum\limits_{m' \in \mathcal{M}_n \backslash \{m\}} p_{m',n} \widetilde{H}_{m',m}^n + p_n \widetilde{H}_{m,n}^C + \sigma_0^2}, \quad (2)$$

where $\mathcal{X} \backslash \mathcal{Y} = \{x | x \in \mathcal{X}, x \notin \mathcal{Y}\}$. The maximum transmission power is P_m^D for the D2D-Tx m.

Our goal is to maximize the total throughput of the D2D pairs while satisfying the rate requirements of all CUs. Mathematically, such a resource sharing problem can be formulated as follows,

$$\begin{aligned} \max_{\rho_{m,n}, p_n, p_{m,n}} \quad & \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} \rho_{m,n} \log_2 \left(1 + \gamma_{m,n}^D\right) \\ s.t.\ C_1: \quad & \log_2 \left(1 + \gamma_n^C\right) \geq R_{n,min}^C, \forall n \in \mathcal{N}, \\ C_2: \quad & P_m^D \geq p_{m,n} \geq 0, \forall m \in \mathcal{M}, n \in \mathcal{N}, \\ C_3: \quad & P_n^C \geq p_n \geq 0, \forall n \in \mathcal{N}, \\ C_4: \quad & \rho_{m,n} \in \{0, 1\}, \forall m \in \mathcal{M}, n \in \mathcal{N}, \\ C_5: \quad & \sum_{n \in \mathcal{N}} \rho_{m,n} \leq 1, \forall m \in \mathcal{M}, \end{aligned} \quad (3)$$

where $R_{n,min}^C$ is the required rate of the CU n. Without loss of generality, we assume that $R_{n,min}^C > 0$. C_1 guarantees the rate requirements of the CUs, which can be rewritten as the following form:

$$\sum_{m \in \mathcal{M}_n} p_{m,n} \widetilde{H}_{m,n}^D \leq I_{th}(p_n), \forall n \in \mathcal{N}, \quad (4)$$

where

$$I_{th}(p_n) = \frac{p_n H_n^C}{2^{R_{n,min}^C} - 1} - \sigma_0^2. \quad (5)$$

C_2 and C_3 are the transmission power budgets for the D2D-Tx's and the CUs. C_4 and C_5 indicate that each D2D pair can use at most one subchannel.

III. EFFICIENT RESOURCE SHARING SCHEME

A. Motivation of Our Proposal

Problem (3) is generally hard to solve because it defines a mixed binary integer programming problem. There are N^M

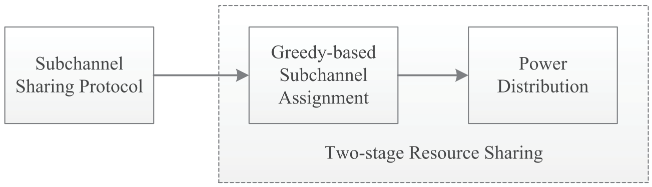

Fig. 1. The overall procedure of the proposed resource sharing scheme.

possible combinations of subchannel allocations for the D2D pairs, which makes the optimal solution impossible to obtain even though the scale of the problem is moderate. Moreover, problem (3) is a nonconcave function of both $p_{m,n}$ and p_n, which means that it might have multiple local optima [29].

To reduce complexity and make the problem tractable, one approach is to reduce the solution space of (3) with additional constraints. We introduce a 0–1 variable $\xi^n_{m,m'}$. $\xi^n_{m,m'} = 1$ informs that the D2D pair m can share the nth subchannel with the D2D pair m'; otherwise, $\xi^n_{m,m'} = 0$. Consider the following problem,

$$\begin{aligned} \max_{\rho_{m,n}, p_n, p_{m,n}} \quad & \sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{M}} \log_2\left(1+\gamma^D_{m,n}\right) \\ s.t. \quad & C_1 \sim C_5 \text{ in (3)}, \\ & C_6: \rho_{m,n} + \rho_{m',n} \le 1 + \xi^n_{m,m'}, \forall m \ne m', n. \end{aligned} \tag{6}$$

C_6 indicates that the D2D pairs m and m' cannot share the subchannel if one of them generates heavy mutual interference to the other on the nth subchannel. Problem (6) is a special case of (3). By carefully designing $\xi^n_{m,m'}$, problem (6) can provide a good approximation to (3) and the mutual interference among the D2D pairs can be ignored. Then, problem (3) can be approximated further as follows, where the mutual interference term between the D2D pairs is disappeared:

$$\begin{aligned} \max_{\rho_{m,n}, p_n, p_{m,n}} \quad & \sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{M}} \rho_{m,n} \log_2\left(1+\frac{p_{m,n}H_{m,n}}{p_n\widetilde{H}^C_{m,n}+\sigma_0^2}\right) \\ s.t. \quad & C_1 \sim C_6 \text{ in (6)}. \end{aligned} \tag{7}$$

Problem (7) is also a mixed binary integer programming problem, which is still hard to solve. However, we can address it efficiently by using a two-stage procedure: subchannel assignment and power allocation, which is verified by many existing similar problems [30], [31].

Based on the analysis above, we propose an efficient resource sharing algorithm to address the problem defined in (3). First, we design $\xi^n_{m,m'}$'s to ensure the mutual interference of two D2D pairs is negligible when they use the same subchannel. Then, we consider a two-stage approach to solve (7) efficiently. Specifically, the resource sharing for (7) is divided into two individual procedures, subchannel assignment and power distribution. In the first stage, we assign each subchannel to one or more D2D pairs. In the second stage, we allocate powers among all CUs and D2D pairs to maximize the sum rate of the D2D pairs. The overall procedure of our resource sharing scheme is illustrated in Fig. 1.

B. Subchannel Sharing Protocol

In this subsection, we first investigate the optimal power allocation problem for the case that two D2D pairs use the same subchannel with the CU n. Then we propose a subchannel sharing protocol to decide $\xi^n_{m,m'}$'s by exploiting the optimal power allocation. The protocol ensures that the D2D pairs m and m' can share the nth subchannel while the mutual interference between them can be negligible.

Consider two D2D pairs, m and m', sharing the nth subchannel with CU n. The problem of maximizing the sum rate of m and m' can be formulated as follows,

$$\begin{aligned} \max_{p_{m,n}, p_{m',n}, p_n} \quad & \log_2\left(1+\gamma^D_{m,n}\right) + \log_2\left(1+\gamma^D_{m',n}\right) \\ s.t. \; C_1: \; & p_{m,n}\widetilde{H}^D_{m,n} + p_{m',n}\widetilde{H}^D_{m',n} \le I_{th}\left(p_n\right), \\ C_2: \; & P^D_m \ge p_{m,n} \ge 0, \; P^D_{m'} \ge p_{m',n} \ge 0, \\ C_3: \; & P^C_n \ge p_n \ge 0. \end{aligned} \tag{8}$$

Note that (8) is different from the problems formulated in [32], [33] since the power allocation for the CU will affect the SINR of each D2D pair, e.g., lower p_n can decrease the total received interference at each D2D-Rx, as well as the feasible region of $(p_{m,n}, p_{m',n})$.

In order to obtain the optimal solution to (8), we first prove that there exists a transmitter (among CU or D2D pairs) which transmits at the maximum power on the nth subchannel.

Lemma 1: Given a set $\mathcal{M}_n$ of D2D pairs reusing the nth subchannel, at least one D2D-Tx $m \in \mathcal{M}_n$ or the CU n will transmit at the maximum transmission power.

Proof: The proof is presented in Appendix. ■

Let $(p^*_{m,n}, p^*_{m',n}, p^*_n)$ be the optimal solution to (8). Based on Lemma 1, the optimal solution to (8) falls into the following cases: $p^*_{m,n} = P^D_m$ or $p^*_{m',n} = P^D_{m'}$ or $p^*_n = P^C_n$. Furthermore, we have

$$p^*_{m,n}\widetilde{H}^D_{m,n} + p^*_{m',n}\widetilde{H}^D_{m',n} = I_{th}\left(p^*_n\right), \tag{9}$$

since otherwise we can decrease the transmission power of the CU n to decrease the interference to D2D-Rx's to increase the throughput of the D2D pairs.

Then, we can classify the case that two D2D pairs reuse a subchannel into two categories: one of the D2D pairs transmits at its maximum power, or the CU n transmits at his maximum power.

First, we focus on the case that $p^*_{m,n} = P^D_m$. Note that the methods proposed in [32], [33] do not work in such case. For simplicity, define

$$p^{req}_n\left(p_{m,n}, p_{m',n}\right) = \frac{p_{m,n}\widetilde{H}^D_{m,n} + p_{m',n}\widetilde{H}^D_{m',n} + \sigma_0^2}{G_n} \tag{10}$$

as the required transmission power of the CU n with given $(p_{m,n}, p_{m',n})$, where $G_n = H^C_n/(2^{R^C_{n,min}} - 1)$. Define

$$p^{max}_{m,n}\left(p_{m',n}\right) = \min\left\{P^D_m, \frac{I_{th}\left(P^C_n\right) - p_{m',n}\widetilde{H}^D_{m',n}}{\widetilde{H}^D_{m,n}}\right\} \tag{11}$$

as the maximum possible power of the D2D-Tx m on the nth subchannel with given $p_{m',n}$. According to Lemma 1, p_n can be expressed as a function of $p_{m',n}$,

$$p_n = p_n^{req}\left(P_m^D, p_{m',n}\right) = p_{m',n}A_0 + B_0, \tag{12}$$

where $A_0 = \widetilde{H}_{m',n}^D/G_n$ and $B_0 = (P_m^D \widetilde{H}_{m,n}^D + \sigma_0^2)/G_n$. Then (8) can be rewritten as follows,

$$\begin{aligned}\max_{p_{m',n}}\ & Q\left(p_{m',n}\right) \triangleq \log_2\left(p_{m',n}A_1 + B_1\right)\\ & - \log_2\left(p_{m',n}A_2 + B_2\right) + \log_2\left(p_{m',n}A_3 + B_3\right)\\ & - \log_2\left(p_{m',n}A_4 + B_4\right)\\ s.t.\ \ & p_{m',n}^{max}\left(P_m^D\right) \geq p_{m',n} \geq 0,\end{aligned} \tag{13}$$

where

$$\begin{aligned}A_1 &= A_2 = \widetilde{H}_{m',m}^n + \widetilde{H}_{m,n}^C A_0,\\ A_3 &= H_{m',n} + \widetilde{H}_{m',n}^C A_0,\\ A_4 &= \widetilde{H}_{m',n}^C A_0,\\ B_1 &= H_{m,n}P_m^D + \widetilde{H}_{m,n}^C B_0 + \sigma_0^2,\\ B_2 &= \widetilde{H}_{m,n}^C B_0 + \sigma_0^2,\\ B_3 &= B_4 = \widetilde{H}_{m,m'}^n P_m^D + \widetilde{H}_{m',n}^C B_0 + \sigma_0^2.\end{aligned} \tag{14}$$

Differentiate $Q(p_{m',n})$ and set the derivative to 0, we have

$$\begin{aligned}Q'\left(p_{m',n}\right) =& \frac{1}{\ln 2}\cdot\left(\frac{1}{p_{m',n}+\frac{B_1}{A_1}} - \frac{1}{p_{m',n}+\frac{B_2}{A_2}}\right.\\ &\left.+\frac{1}{p_{m',n}+\frac{B_3}{A_3}} - \frac{1}{p_{m',n}+\frac{B_4}{A_4}}\right) = 0.\end{aligned} \tag{15}$$

With simple mathematical operations, (15) can be rewritten as follow,

$$Ap_{m',n}^2 + Bp_{m',n} + C = 0, \tag{16}$$

where

$$\begin{aligned}A =& \frac{B_1}{A_1} - \frac{B_2}{A_2} + \frac{B_3}{A_3} - \frac{B_4}{A_4},\\ B =& 2\cdot\left(\frac{B_1B_3}{A_1A_3} - \frac{B_2B_4}{A_2A_4}\right),\\ C =& \frac{B_1B_3}{A_1A_3}\left(\frac{B_2}{A_2}+\frac{B_4}{A_4}\right) - \frac{B_2B_4}{A_2A_4}\left(\frac{B_1}{A_1}+\frac{B_3}{A_3}\right).\end{aligned} \tag{17}$$

If $B^2 - 4AC < 0$, which implies $Q(p_{m',n})$ is monotone in the interval $[0, p_{m',n}^{max}(P_m^D)]$, $p_{m',n}^* = 0$ if $Q'(0) \leq 0$ and $p_{m',n}^* = p_{m',n}^{max}(P_m^D)$ if $Q'(0) > 0$. If $B^2 - 4AC \geq 0$, the optimal solution to (13) can be obtained as follows,

$$p_{m',n}^{opt1} = \begin{cases}\left[\dfrac{-B \pm \sqrt{B^2-4AC}}{2A}\right]_0^{p_{m',n}^{max}(P_m^D)} & A \neq 0,\\ -\dfrac{C}{B} & A = 0,\end{cases} \tag{18}$$

where $[x]_0^{p_{m',n}^{max}(P_m^D)}$ represents the projection onto the interval $[0, p_{m',n}^{max}(P_m^D)]$. If there are two optima in the interval, we select the solution with better performance as $p_{m',n}^{opt1}$.

As a result, the optimal solution $(p_{m,n}^*, p_{m',n}^*, p_n^*)$ in this case can be found in the set of Ω_1 defined as follows:

$$\begin{aligned}\Omega_1 = \Big\{&\left(P_m^D, 0, B_0\right), \left(P_m^D, p_{m',n}^{opt1}, p_n^{req}\left(P_m^D, p_{m',n}^{opt1}\right)\right),\\ &\left(P_m^D, p_{m',n}^{max}\left(P_m^D\right), p_n^{req}\left(P_m^D, p_{m',n}^{max}\left(P_m^D\right)\right)\right)\Big\}.\end{aligned} \tag{19}$$

Similar conclusion holds for the case that $p_{m',n}^* = P_{m'}^D$. By swapping m' and m, we can the local optimum $p_{m,n}^{opt2}$ with the same method. Thus, $(p_{m,n}^*, p_{m',n}^*, p_n^*)$ for the case of $p_{m',n}^* = P_{m'}^D$ can be found in the set of Ω_2, where

$$\begin{aligned}\Omega_2 = \Big\{&\left(0, P_{m'}^D, p_n^{req}\left(0, P_{m'}^D\right)\right), \left(p_{m,n}^{opt2}, P_{m'}^D, p_n^{req}\left(p_{m,n}^{opt2}, P_{m'}^D\right)\right),\\ &\left(p_{m,n}^{max}\left(P_{m'}^D\right), P_{m'}^D, p_n^{req}\left(p_{m,n}^{max}\left(P_m^D\right), P_{m'}^D\right)\right)\Big\}.\end{aligned} \tag{20}$$

Second, we analyze the case that $p_n^* = P_n^C$. Without loss of generality, we assume $P_m^D\widetilde{H}_{m,n}^D + P_{m'}^D\widetilde{H}_{m',n}^D \geq I_{th}(P_n^C)$ because otherwise CU n can decrease his transmission power to lower the interference to D2D-Rx's. In this case, $p_{m',n}$ can be expressed as a function of $p_{m,n}$,

$$p_{m',n} = \left[I_{th}\left(P_n^C\right) - p_{m,n}\widetilde{H}_{m,n}^D\right]/\widetilde{H}_{m',n}^D = D_0 p_{m,n} + E_0. \tag{21}$$

where $D_0 = -\widetilde{H}_{m,n}^D/\widetilde{H}_{m',n}^D$ and $E_0 = I_{th}(P_n^C)/\widetilde{H}_{m',n}^D$. Then (8) can be rewritten as follows,

$$\begin{aligned}\max_{p_{m,n}}\ & U\left(p_{m,n}\right) \triangleq \log_2\left(p_{m,n}D_1 + E_1\right)\\ & - \log_2\left(p_{m,n}D_2 + E_2\right) + \log_2\left(p_{m,n}D_3 + E_3\right)\\ & - \log_2\left(p_{m,n}D_4 + E_4\right)\\ s.t.\ \ & p_{m,n}^{max}(0) \geq p_{m,n} \geq 0,\end{aligned} \tag{22}$$

where

$$\begin{aligned}D_1 &= \widetilde{H}_{m',m}^n D_0 + H_{m,n},\\ D_2 &= \widetilde{H}_{m',m}^n D_0,\\ D_3 &= H_{m',n}D_0 + \widetilde{H}_{m,m'}^n,\\ D_4 &= \widetilde{H}_{m,m'}^n,\\ E_1 &= E_2 = \widetilde{H}_{m',m}^n E_0 + \widetilde{H}_{m,n}^C P_n^C + \sigma_0^2,\\ E_3 &= H_{m',n}E_0 + \widetilde{H}_{m',n}^C P_n^C + \sigma_0^2,\\ E_4 &= \widetilde{H}_{m',n}^C P_n^C + \sigma_0^2.\end{aligned} \tag{23}$$

Similar to the analysis of the first case, we differentiate $Q(p_{m,n})$ and set the derivative to 0, we have

$$\begin{aligned}U'(p_{m,n}) =& \frac{1}{\ln 2}\cdot\left(\frac{1}{p_{m,n}+\frac{D_1}{E_1}} - \frac{1}{p_{m,n}+\frac{D_2}{E_2}}\right.\\ &\left.+\frac{1}{p_{m,n}+\frac{D_3}{E_3}} - \frac{1}{p_{m,n}+\frac{D_4}{E_4}}\right) = 0.\end{aligned} \tag{24}$$

Denote

$$\begin{aligned} D =& \frac{E_1}{D_1} - \frac{E_2}{D_2} + \frac{E_3}{D_3} - \frac{E_4}{D_4}, \\ E =& 2 \cdot \left(\frac{E_1 E_3}{D_1 D_3} - \frac{E_2 E_4}{D_2 D_4} \right), \\ F =& \frac{E_1 E_3}{D_1 D_3} \left(\frac{E_2}{D_2} + \frac{E_4}{D_4} \right) - \frac{E_2 E_4}{D_2 D_4} \left(\frac{E_1}{D_1} + \frac{E_3}{D_3} \right), \end{aligned} \tag{25}$$

equation (24) can be written as $Dp_{m',n}^2 + Ep_{m',n} + F = 0$. If $D^2 - 4EF < 0$, $U(p_{m,n})$ is monotone in the interval $[0, p_{m,n}^{max}(0)]$. We can directly obtain $p_{m,n}^* = 0$ if $U'(0) \leq 0$ and $p_{m,n}^* = p_{m,n}^{max}(0)$ if $U'(0) > 0$. If $D^2 - 4EF \geq 0$, the feasible local optimum can be obtained as follows,

$$p_{m,n}^{opt3} = \begin{cases} \left[\dfrac{-E \pm \sqrt{E^2 - 4DF}}{2D} \right]_0^{p_{m,n}^{max}(0)} & D \neq 0, \\ -\dfrac{F}{E} & D = 0. \end{cases} \tag{26}$$

Similarly, we select the solution with better performance for the case of two optima. Thus, $(p_{m,n}^*, p_{m',n}^*, p_n^*)$ for the case that $p_n^* = P_n^C$ can be found in the set defined as follows:

$$\begin{aligned} \Omega_3 = \Bigg\{ & \left(\frac{I_{th}(P_n^C) - p_{m',n}^{max}(0)\widetilde{H}_{m',n}^D}{\widetilde{H}_{m,n}^D}, p_{m',n}^{max}(0), P_n^C \right), \\ & \left(p_{m,n}^{max}(0), \frac{I_{th}(P_n^C) - p_{m,n}^{max}(0)\widetilde{H}_{m,n}^D}{\widetilde{H}_{m',n}^D}, P_n^C \right), \\ & \left(p_{m,n}^{opt3}, \frac{I_{th}(P_n^C) - p_{m,n}^{opt3}\widetilde{H}_{m,n}^D}{\widetilde{H}_{m',n}^D}, P_n^C \right) \Bigg\}. \end{aligned} \tag{27}$$

Based on the discussion above, we can conclude that the optimal solution can be found in a set of feasible solutions.

Theorem 1: The optimal solution to (8) lies in the set $\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3$.

Proof: According to Lemma 1, the optimal solutions to (8) can be classified into three cases: $p_{m,n}^* = P_m^D$, $p_{m',n}^* = P_{m'}^D$ and $p_n^* = P_n^C$. The optimal solution to each case can be found in Ω_1, Ω_2 and Ω_3, respectively, which implies that the optimal solution can be found in the set of $\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3$. ■

In summary, the optimal solution to (8) can be searched in a finite set of feasible solutions with the complexity of $O(1)$. We can develop a subchannel sharing protocol by exploiting the optimal power allocation.

Denote $R_n(\mathcal{M}_n)$ as the maximum achievable sum rate of the D2D pairs in $\mathcal{M}_n$ on the nth subchannel. Based on Lemma 1, when only D2D pair m uses the nth subchannel, the D2D-Tx m will transmit at its maximum possible transmission power. Recall that

$$p_{m,n}^{max} = \min[P_m^D, I_{th}(P_n^C)/\widetilde{H}_{m,n}^D] \tag{28}$$

and

$$p_n^{req} = (p'_{m,n}\widetilde{H}_{m,n}^D + \sigma_0^2)/G_n \tag{29}$$

are the maximum possible power of the D2D-Tx m on the nth subchannel and the required power of the CU n in this case. The maximum achievable rate $R_n(m)$ can be calculated as follows,

$$R_n(m) = \log_2 \left(1 + \frac{p_{m,n}^{max} H_{m,n}^D}{p_n^{req} \widetilde{H}_{m,n}^C + \sigma_0^2} \right), \tag{30}$$

Note that the capacity gain brought by any D2D pair reusing any subchannel can be roughly measured by $R_n(m)$, which plays an important role in our proposed subchannel allocation.

When D2D pair m and D2D pair m' share the nth subchannel, the sum rate of them is

$$\begin{aligned} R_n(m, m') = & \log_2 \left(1 + \frac{p_{m,n}^* H_{m,n}^D}{p_n^* \widetilde{H}_{m,n}^C + p_{m',n}^* \widetilde{H}_{m',m}^n + \sigma_0^2} \right) \\ & + \log_2 \left(1 + \frac{p_{m',n}^* H_{m',n}^D}{p_n^* \widetilde{H}_{m',n}^C + p_{m,n}^* \widetilde{H}_{m,m'}^n + \sigma_0^2} \right). \end{aligned} \tag{31}$$

If $p_{m,n}^* = 0$ or $p_{m',n}^* = 0$, obviously, only one of the D2D pairs can use the nth subchannel because there is no throughput for the other D2D pair. Moreover, since there exists signal overhead when two D2D pairs share the nth subchannel, which can counteract the capacity gain brought by subchannel reuse, we should restrict subchannel sharing to the case that the capacity gain is more than a threshold defined as follows:

$$\xi_{m,m'}^n = \begin{cases} 1 & p_{m,n}^* \neq 0, p_{m',n}^* \neq 0, \\ & R_n(m, m') \geq \gamma \cdot [R_n(m) + R_n(m')], \\ 0 & \text{otherwise}, \end{cases} \tag{32}$$

where γ is usually in the interval of $[0.5, 1)$.[1] A reasonable value for γ might be $\gamma = 0.8$ or $\gamma = 0.9$, depending on the application scenario. The proposed subchannel sharing protocol is summarized as follows:

Step 1: Obtain the set of possible solutions Ω;

Step 2: For each $(p_{m,n}, p_{m',n}, p_n) \in \Omega$, calculate the corresponding sum rate of the D2D pairs m and m'; then find the $(p_{m,n}^*, p_{m',n}^*, p_n^*)$ which yields the maximum sum rate;

Step 3: Calculate $R_n(m)$ and $R_n(m, m')$ by using (30) and (31), respectively;

Step 4: Obtain $\xi_{m,m'}^n$ by using (32), $\forall m, m', n$.

C. Greedy-Based Subchannel Assignment

With the $\xi_{m,m'}^n$ defined in (32), the mutual interference among the D2D pairs can be safely ignored. Then, problem (3) can be approximated to (7), where the mutual interference term between the D2D pairs is disappeared. We address the problem defined in (7) efficiently by using a two-stage procedure: subchannel assignment and power allocation.

In this subsection, we propose a greedy-based subchannel assignment algorithm to allocate each D2D pair to the subchannels on which the D2D pair can yield the maximum achievable

[1] Note that $\xi_{m,m'}^n = 1$ is satisfied when $\gamma = 0.5$ for any two D2D pairs and the CU n.

TABLE II
GREEDY-BASED SUBCHANNEL ASSIGNMENT

Algorithm 1

Initialization: Set $\mathcal{C}_n = \mathcal{M}, \mathcal{M}_n = \emptyset, \forall n$; Obtain all $\xi^n_{m,m'}$'s and $R_n(m)$'s by using the subchannel sharing procotol.
repeat
  $(m^*, n^*) = \arg\max_{(m,n): m \in \mathcal{C}_n, n \in \mathcal{N}} R_n(m)$;
  $\mathcal{M}_{n^*} = \mathcal{M}_{n^*} \cup \{m^*\}$;
  $\mathcal{C}_n = \mathcal{C}_n \backslash \{m^*\}, \forall n \in \mathcal{N}$;
  $\mathcal{C}_{n^*} = \mathcal{C}_{n^*} \backslash \{m \neq m^* | \xi^n_{m,m^*} = 0\}$;
until $\mathcal{C}_n$ is empty set, $\forall n \in \mathcal{N}$
return $\mathcal{M}_n$

rate. The pseudo code of the subchannel assignment algorithm is shown in Table II, where $\mathcal{C}_n$ is the set of D2D pairs remained that can use the nth subchannel. As mentioned before, the capacity gain brought by any D2D pair reusing any subchannel can be roughly measured by $R_n(m)$ defined in (30). Therefore, the D2D pair that has the maximum $R_n(m)$ has the priority to use the nth subchannel in our algorithm. When the nth subchannel is allocated to the D2D pair m, it will not be used by other D2D pairs that cannot share the subchannel with the D2D pair m. Such procedure repeats until all D2D pairs have been assigned or cannot be assigned due to the subchannel sharing protocol.

D. Power Distribution With a Given Subchannel Assignment

Given a subchannel assignment, the binary variables $\rho_{m,n}$'s in (7) are fixed to 0's or 1's, and the integer constraints vanish. We can maximize the sum rate of all D2D pairs by maximizing the throughput yielded by each subchannel. Recall that $\mathcal{M}_n$ is the set of the D2D pairs that use the nth subchannel, the power allocation for the nth subchannel can be converted into the following optimization problem:

$$\begin{aligned} \min_{p_n, p_{m,n}} \quad & -\sum_{m \in \mathcal{M}_n} \log_2\left(1 + \frac{H_{m,n} p_{m,n}}{\widetilde{H}^C_{m,n} p_n + \sigma_0^2}\right) \\ s.t. \; C_1: \; & \sum_{m \in \mathcal{M}_n} p_{m,n} \widetilde{H}^D_{m,n} \leq I_{th}(p_n), \\ C_2: \; & P^D_m \geq p_{m,n} \geq 0, \forall m \in \mathcal{M}_n, \\ C_3: \; & P^C_n \geq p_n \geq P^{min}_n, \end{aligned} \tag{33}$$

where $p^{min}_n = \sigma_0^2 / G_n$ is the minimum required power of the CU n. The Lagrangian of (33) is

$$\begin{aligned} J = & -\sum_{m \in \mathcal{M}_n} \log_2\left(1 + \frac{H_{m,n} p_{m,n}}{\widetilde{H}^C_{m,n} p_n + \sigma_0^2}\right) \\ & + \lambda \left[\sum_{m \in \mathcal{M}_n} p_{m,n} \widetilde{H}^D_{m,n} - I_{th}(p_n)\right] + \chi\left(p_n - P^C_n\right) \\ & - \psi p_n + \sum_{m \in \mathcal{M}_n}\left[\nu_m\left(p_{m,n} - P^D_m\right) - \mu_m p_{m,n}\right], \end{aligned} \tag{34}$$

where λ, χ, ψ, ν_m's and μ_m's are Lagrange multipliers.

The constraints of problem (33) are all affine, and thus the KKT conditions holds [29]. We redefine $p^*_{m,n}$ and p^*_n as the optimal power allocation to the D2D pair m on the nth subchannel and the corresponding optimal power allocation to the CU n, respectively. We have the following equations:

$$-\frac{1}{\ln 2} \cdot \frac{1}{\frac{\widetilde{H}^C_{m,n} p^*_n + \sigma_0^2}{H_{m,n}} + p^*_{m,n}} + \lambda^* \widetilde{H}^D_{m,n} + \nu^*_m - \mu^*_m = 0, \forall m, \tag{35}$$

$$\begin{aligned} & \frac{1}{\ln 2} \sum_{m \in \mathcal{M}_n}\left(\frac{\widetilde{H}^C_{m,n}}{\widetilde{H}^C_{m,n} p^*_n + \sigma_0^2} - \frac{\widetilde{H}^C_{m,n}}{H_{m,n} p^*_{m,n} + \widetilde{H}^C_{m,n} p^*_n + \sigma_0^2}\right) \\ & - \lambda^* G_n + \chi^* - \psi^* = 0, \end{aligned} \tag{36}$$

$$\lambda^*\left[\sum_{m \in \mathcal{M}_n} p^*_{m,n} \widetilde{H}^D_{m,n} - I_{th}(p^*_n)\right] = 0, \tag{37}$$

$$\chi^*\left(p^*_n - P^C_n\right) = 0, \psi^* p^*_n = 0, \tag{38}$$

$$\nu^*_m\left(p^*_{m,n} - P^D_m\right) = 0, \mu^*_m p^*_{m,n} = 0, \forall m, \tag{39}$$

$$\nu^*_m, \mu^*_m, \chi^*, \psi^*, \lambda^* \geq 0, \tag{40}$$

where λ^*, χ^*, ψ^*, ν^*_m's and μ^*_m's are the optimal dual points.

Based on equations (35) $\sim$ (40), we can obtain

$$p^*_{m,n} = \left[\frac{1}{\ln 2 \cdot \lambda^* \widetilde{H}^D_{m,n}} - \frac{\widetilde{H}^C_{m,n} p^*_n + \sigma_0^2}{H_{m,n}}\right]_0^{P^D_m}. \tag{41}$$

Furthermore, $\sum_{m \in \mathcal{M}_n} p^*_{m,n} \widetilde{H}^D_{m,n} - I_{th}(p^*_n) = 0$ always holds because if it is not the case, the D2D links could transmit with higher powers, or CU n could transmit with lower power without violating the rate constraint, which contradicts with the fact that $p^*_{m,n}$ and p^*_n are optimal. With simple mathematical operation, we have

$$p^*_n = \frac{\sum_{m \in \mathcal{M}_n} p^*_{m,n} \widetilde{H}^D_{m,n} + \sigma_0^2}{G_n}. \tag{42}$$

We have $p^*_n > 0$ since $R^C_{n,min} > 0$, implying $\psi^* = 0$. If $P^C_m > p^*_n$, we also have $\chi^* = 0$. By substituting $\chi^* = 0, \psi^* = 0$ into (36), we can obtain

$$\lambda^* = \frac{1}{\ln 2 \cdot G_n} \cdot \sum_{m \in \mathcal{M}_n}\left(-\frac{\widetilde{H}^C_{m,n}}{H_{m,n} p^*_{m,n} + \widetilde{H}^C_{m,n} p^*_n + \sigma_0^2} + \frac{\widetilde{H}^C_{m,n}}{\widetilde{H}^C_{m,n} p^*_n + \sigma_0^2}\right). \tag{43}$$

We develop a bisection algorithm by employing equations (41) $\sim$ (43). The power allocation algorithm is shown in Table III, where p^{ub}_n and p^{lb}_n are the upper bound and lower bound of p_n, ϵ is a small acceptable tolerance. Starting from a feasible value of p_n, i.e., $p_n = (p^{ub}_n + p^{lb}_n)/2$ in our algorithm, the corresponding $p_{m,n}$ and λ can be calculated by solving (41) and (42). Then λ' is worked out by using (43). Compare λ' with λ, if $\lambda' > \lambda$, decrease p_n; if $\lambda' < \lambda$, increase p_n. The adjustment procedure runs until $\lambda' = \lambda$ or p_n does not change. Algorithm

TABLE III
BISECTION ALGORITHM FOR (33)

Algorithm 2
Initialization: $p_n^{ub} = P_n^C$, $p_n^{lb} = p_n^{min}$.
repeat
$\quad p_n = (p_n^{ub} + p_n^{lb})/2$;
$\quad$ Calculate $p_{m,n}$ and λ by solving (41) and (42);
$\quad$ **if** $p_{m,n} = P_m^D, \forall m \in \mathcal{M}_n$
$\quad\quad \lambda = 0$;
$\quad$ **endif**
$\quad$ Calculate λ' by using (43);
$\quad$ **if** $\lambda' > \lambda$
$\quad\quad p_n^{ub} = p_n$;
$\quad$ **else**
$\quad\quad p_n^{lb} = p_n$;
$\quad$ **endif**
until $\lvert p_n^{ub} - p_n^{lb} \rvert \le \epsilon$
return p_n, $p_{m,n}$

2 can always find a stationary point of problem (33). However, problem (33) is a non-convex optimization problem, implying that Algorithm 2 cannot always find the global optimum since there might exist multiple local optima. It is worthy of noticing that, for a practical system, there usually exists a unique stationary point in the feasible region. So generally, the obtained power allocation is globally optimal for problem (33), which will be confirmed by the numerical results.

E. Computational Complexity Analysis

The computational complexity of the proposed resource sharing scheme can be counted as follows. First, we need to calculate the maximum achievable rate of any two D2D pairs reusing each subchannel by using (31), where the complexity of searching the optimal power allocation for the two D2D pairs and the CU is $O(1)$. Hence the complexity of the proposed subchannel sharing protocol is $O(M^2N)$. Second, the complexity of the subchannel assignment algorithm is $O(MN)$. Third, the complexity of the power allocation for the nth subchannel is $O(\log_2(1/\epsilon)|\mathcal{M}_n|)$ since we need to obtain the dual optimal point defined in (43) by using the bisection algorithm. So the complexity of overall power allocation is bounded by $O(\log_2(1/\epsilon)MN)$.

IV. NUMERICAL RESULTS

We conduct a series of experiments to evaluate the performance of our proposed resource sharing scheme. Simulation parameters such as path loss models, maximum transmission power, etc. are the same as these proposed in [34], and the main simulation parameters are listed in Table IV. The loss factor γ is set to 0.9 and the tolerance ϵ is set to 10^{-6}. All results are averaged by 1000 Monte Carlo simulations.

For comparison, we also implement other schemes which can be only applied to the case that each subchannel is used by at most one D2D pair. Scheme 1 and Scheme 2 are proposed in [17]. If $M > N$, Scheme 1 chooses N D2D pairs that can maximize the total achievable rate of the D2D pairs by using Hungarian method [35]; Scheme 2 randomly selects N D2D pairs to reuse all subchannels. If $M \le N$, Scheme 1 and

TABLE IV
SIMULATION PARAMETERS

Parameter	**Value**
Cellular layout	Isolated cell, 1-sector
System area	The radius of the cell is 500 m
Maximum distance of D2D	50 m
Number of subchannels	20
Maximum power of CU	20 dBm
Rate requirement of CU	0 bps/Hz $\sim$ 50 bps/Hz
Number of CUs	20
Number of D2D pairs	10 $\sim$ 70
Maximum power of D2D-Tx	-10 dBm $\sim$ 20 dBm
Pass loss (in dB)	$15.3 + 37.6 \log_{10} D$ (D in m)
Noise power on subchannel	-100 dBm

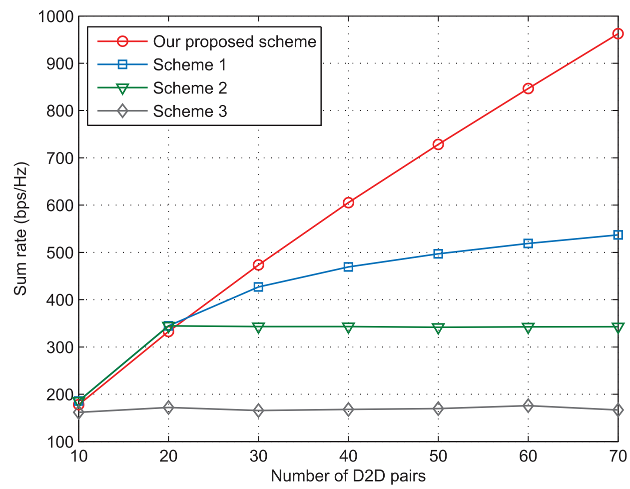

Fig. 2. The sum rate of the D2D pairs as a function of M. $R_{n,min}^C = 10$ bps/Hz, $\forall n \in \mathcal{N}$, $P_m^D = 0.1$ W, $\forall m \in \mathcal{M}$.

Scheme 2 can always find optimal subchannel and power allocation among D2D pairs and CUs to maximize the sum rate of the D2D pairs. Scheme 3 is developed in [15], where only one D2D pair that has the maximum achievable rate among all subchannels has the authority to use all subchannels.

The sum rate with different numbers of D2D pairs is illustrated in Fig. 2. We can see that the performance of our proposed resource sharing scheme is very close to Scheme 1 and Scheme 2 when $M < N$. As mentioned above, Scheme 1 and Scheme 2 can always find the optimal subchannel and power allocation. So we can conclude that our proposed scheme is very close to the optimal solution in this case. When $M > N$, our proposed scheme outperforms the other three schemes significantly. Specifically, the performance of our proposed scheme grows approximately linearly with the increase of D2D pairs, which indicates that our proposed scheme can be widely used in different scenarios in D2D communication underlaying cellular networks.

Fig. 3 and Fig. 4 illustrate the sum rate of D2D pairs with different transmission power budgets, as well as different rate requirements of the CUs, respectively. It can be seen from Fig. 3 and Fig. 4 that our proposed scheme outperforms the other three ones. It is worth noting that the capacity gain of D2D communication only decreases sightly with the increase of the rate requirement of each CU. The reason is that each CU will use its maximum transmission power to satisfy the rate requirement

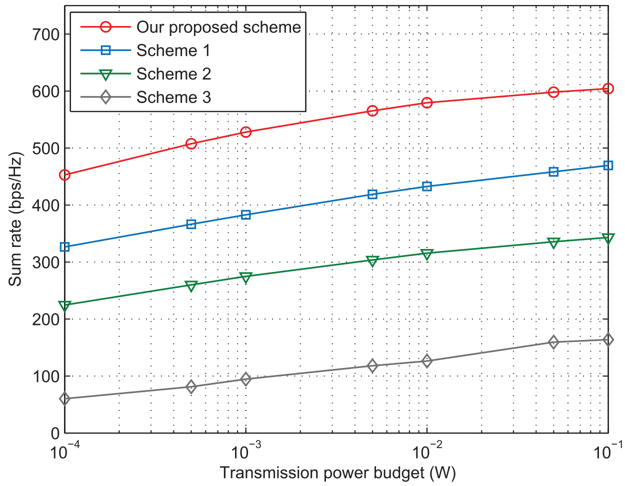

Fig. 3. The sum rate of the D2D pairs as a function of transmission power budget. $R^C_{n,min} = 10$ bps/Hz, $\forall n \in \mathcal{N}$, $M = 40$.

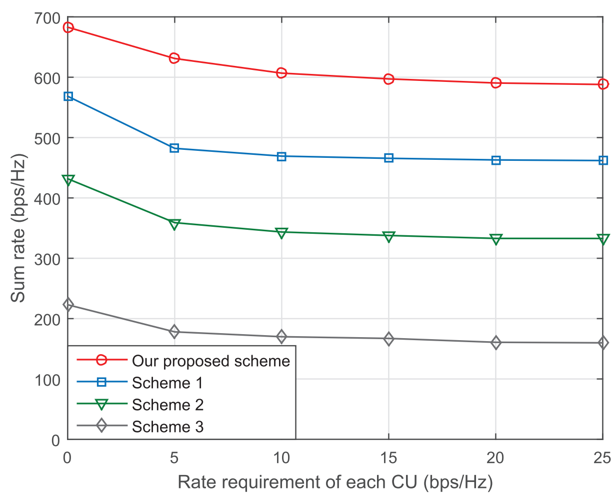

Fig. 4. The sum rate of the D2D pairs as a function of each CU's rate requirement. $M = 40$, $P^D_m = 0.1$ W, $\forall m \in \mathcal{M}$.

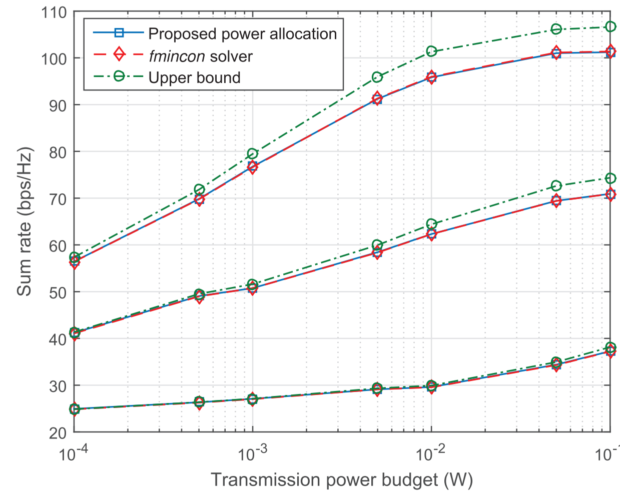

Fig. 5. Comparison of upper bound and actual achievable rate on a subchannel. $R^C_{n,min} = 10$ bps/Hz, $\forall n \in \mathcal{N}$.

on each subchannel, which means that the maximum transmission powers of the D2D pairs using this subchannel are also bounded, resulting that only the D2D pairs which generate low interference to the BS can use the subchannel.

Then, we evaluate the performance of our proposed subchannel sharing protocol for two D2D pairs. Fig. 5 illustrates the average achievable rate on one subchannel with different transmission power budgets and different number of D2D pairs. The results of the locally optimal solutions, which are obtained by in-built *fmincon* solver of *Matlab*, are given for comparison. We also take the objective value of (33) as an upper bound, which corresponds to the case that the mutual interference among the D2D pairs is ignored. As it can be observed from Fig. 5, our proposed power allocation algorithm can always obtain the locally optimal solutions. In fact, the solutions are usually globally optimal. When $|\mathcal{M}_n| = 2$, it can be seen from that the achievable rate is very close to the upper bound, which implies that the mutual interference among the two D2D pairs is negligible. With the increase of $|\mathcal{M}_n|$, the gap between the upper bound and the achievable rate increases slightly. Even for the case of $|\mathcal{M}_n| = 6$, the gap is less than 5%. We can conclude that our proposed resource sharing scheme is effective and efficient.

We also compare our proposed resource sharing scheme with other fixed power allocation schemes that allow multiple D2D pairs to share a subchannel, such as

SSP+GSA+FPA: The subchannels are assigned to all D2D pairs by using our greedy-based subchannel allocation algorithm. All transmitters (including D2D-Tx's and CUs) transmit at their maximum powers.

GSA+FPA: The subchannels are assigned to the D2D pairs by using the proposed subchannel allocation algorithm without our subchannel sharing protocol. All transmitters transmit at their maximum powers.

RSA+FPA: The subchannels are randomly assigned to the D2D pairs and all transmitters transmit at their maximum powers.

The results are presented in Fig. 6. It can be observed that our scheme can achieve a higher capacity gain compared with these fixed power allocation schemes. The sum rate of the D2D pairs is at least 20% greater than that of any fixed power allocation scheme. Furthermore, the fixed power allocation scheme with our subchannel sharing protocol outperforms others without employing the protocol, which further verifies the efficiency of our proposed subchannel sharing protocol.

Finally, we compare our proposed resource sharing scheme with an upper bound. We take the optimal value of the relaxation of (3) as the upper bound, which is obtained by *fmincon* solver. It is worth noticing that, when the number of the subchannels is set to 20 and the number of D2D pairs varies from 10 to 70, *fmincon* cannot solve the relaxed form of (3). The number of subchannels is set to 5 and the number of the D2D pairs varies from 5 to 30. The sum rate with different numbers of D2D pairs is illustrated in Fig. 7. It can be seen from Fig. 7 that the performance of our proposed algorithm is close to the upper bound. The gap is always less than 10%. We can conclude that the solution yielded by our proposed algorithm is close to the optimum.

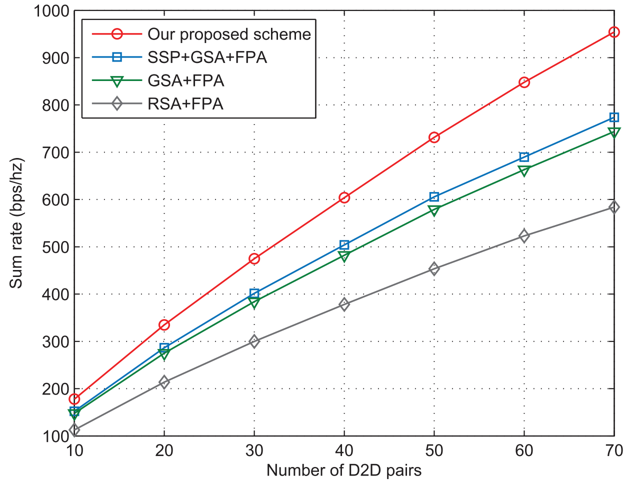

Fig. 6. A comparison of our scheme and fixed power allocation schemes. $R^C_{n,min} = 10$ bps/Hz, $\forall n \in \mathcal{N}$. $P^D_m = 0.1$ W, $\forall m \in \mathcal{M}$.

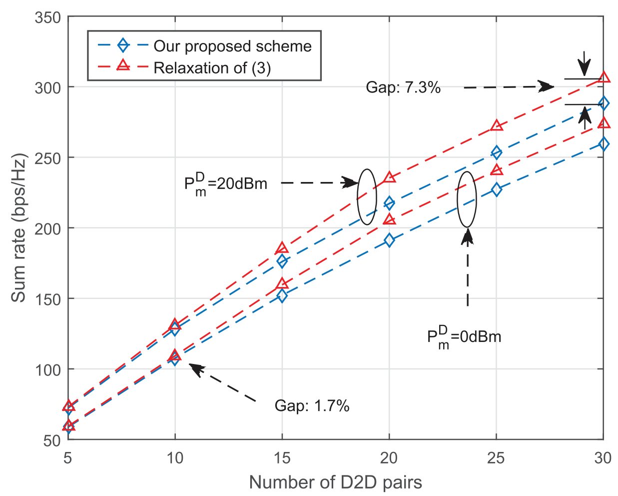

Fig. 7. A comparison of our scheme and an upper bound. $N = 5$, $R^C_{n,min} = 10$ bps/Hz, $\forall n \in \mathcal{N}$.

V. Conclusion

In this paper, we studied the resource sharing scheme for a D2D communication underlaying cellular network, where a subchannel can be shared by a CU and multiple D2D pairs. Different from most of existing models, our resource sharing scheme exploits the spatial reuse of the D2D pairs to further increase spectrum efficiency of the cellular system. We try to maximize the sum rate of the D2D pairs while satisfying the rate requirements of all CUs. First, a subchannel sharing protocol is developed for two D2D pairs that potentially use the same subchannels. Then we design a greedy-based algorithm to allocate the D2D pair to the subchannels on which the D2D pair can achieve the maximum throughput. Finally, we propose an efficient power distribution algorithm for the D2D pairs and the CUs. The numerical results validate the effectiveness and efficiency of our proposed resource sharing scheme. The system spectrum efficiency can be enhanced significantly in this way.

Appendix

Consider the case of a set $\mathcal{M}_n$ of D2D pairs reusing the nth subchannel with the CU n. Collect p_n and $p_{m,n}$'s into one vector $\bar{p}_n$, the optimization problem of maximizing the sum rate of $\mathcal{M}_n$ is

$$\begin{aligned} \max_{\bar{p}_n} \quad & \sum_{m\in\mathcal{M}_n} \log_2(1+\gamma^D_{m,n}) \\ s.t.\ C_1: \quad & \sum_{m\in\mathcal{M}_n} p_{m,n}\widetilde{H}^D_{m,n} \le I_{th}(p_n), \\ C_2: \quad & P^D_m \ge p_{m,n} \ge 0, \forall m \in \mathcal{M}_n, \\ C_3: \quad & P^C_n \ge p_n \ge 0. \end{aligned} \tag{44}$$

Denote

$$\mathcal{P}_{\mathcal{M}_n} = \{\bar{p}_n | C_1, C_2 \text{ and } C_3 \text{ in (44) are satisfied}\} \tag{45}$$

as the feasible set to (44). For positive number $\alpha > 1$ and power vector $\bar{p}_n \in \mathcal{P}_{\mathcal{M}_n}$, which satisfy $\alpha p_n \le P^C_n$ and $\alpha p_{m,n} \le P^D_m$, $\forall m \in \mathcal{M}_n$, we have

$$\begin{aligned} \log_2\left(1+\frac{\alpha p_{m,n}H_{m,n}}{\alpha I_{m,n}+\sigma_0^2}\right) &= \log_2\left(1+\frac{p_{m,n}H_{m,n}}{I_{m,n}+\frac{\sigma_0^2}{\alpha}}\right) \\ &> \log_2\left(1+\frac{p_{m,n}H_{m,n}}{I_{m,n}+\sigma_0^2}\right), \end{aligned} \tag{46}$$

$\forall m \in \mathcal{M}_n$, where

$$I_{m,n} = p_n\widetilde{H}^C_{m,n} + \sum_{m'\in\mathcal{M}_n\backslash\{m\}} p_{m',n}\widetilde{H}^n_{m',m}. \tag{47}$$

Furthermore, multiplying both sides of C_1 in (44) by α, we can obtain

$$\begin{aligned} \sum_{m\in\mathcal{M}_n} \alpha p_{m,n}\widetilde{H}^D_{m,n} &\le \alpha p_n G_n - \alpha\sigma_0^2 \\ &< \alpha p_n G_n - \sigma_0^2 \\ &= I_{th}(\alpha p_n). \end{aligned} \tag{48}$$

(46) and (48) indicate that for any given power vector $\bar{p}_n \in \mathcal{P}_{\mathcal{M}_n}$ in the interior of the feasible region, there always exists another power vector $\alpha\bar{p}_n \in \mathcal{P}_{\mathcal{M}_n}$ that can yield higher throughput on the nth subchannel, for some $\alpha > 1$. Thus, at least one D2D-Tx $m \in \mathcal{M}_n$ or the CU n will transmit at the maximum transmission power.

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References

[1] K. Doppler, M. Rinne, C. Wijting, C. Ribeiro, and K. Hugl, "Device-to-device communication as an underlay to LTE-advanced networks," *IEEE Commun. Mag.*, vol. 47, no. 12, pp. 42–49, Dec. 2009.

[2] P. Phunchongharn, E. Hossain, and D. I. Kim, "Resource allocation for device-to-device communications underlaying LTE-advanced networks," *IEEE Wireless Commun.*, vol. 20, no. 4, pp. 91–100, Aug. 2013.

[3] G. Fodor *et al.*, "Design aspects of network assisted device-to-device communications," *IEEE Commun. Mag.*, vol. 50, no. 3, pp. 170–177, Mar. 2012.

[4] X. Lin, J. Andrews, A. Ghosh, and R. Ratasuk, "An overview of 3GPP device-to-device proximity services," *IEEE Commun. Mag.*, vol. 52, no. 4, pp. 40–48, Apr. 2014.

[5] M. Yang, S. Lim, H. Park, and N. Park, "Solving the data overload: Device-to-device bearer control architecture for cellular data offloading," *IEEE Veh. Technol. Mag.*, vol. 8, no. 1, pp. 31–39, Mar. 2013.

[6] L. Wei, R. Hu, Y. Qian, and G. Wu, "Enable device-to-device communications underlaying cellular networks: Challenges and research aspects," *IEEE Commun. Mag.*, vol. 52, no. 6, pp. 90–96, Jun. 2014.

[7] K. Doppler, C.-H. Yu, C. Ribeiro, and P. Janis, "Mode selection for device-to-device communication underlaying an LTE-advanced network," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC'10)*, Apr. 2010, pp. 1–6.

[8] P. Phunchongharn, E. Hossain, and D. I. Kim, "Resource allocation for device-to-device communications underlaying LTE-advanced networks," *IEEE Wireless Commun.*, vol. 20, no. 4, pp. 91–100, Aug. 2013.

[9] A. Asadi, Q. Wang, and V. Mancuso, "A survey on device-to-device communication in cellular networks," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 4, pp. 1801–1819, Nov. 2014.

[10] C.-H. Yu, K. Doppler, C. Ribeiro, and O. Tirkkonen, "Resource sharing optimization for device-to-device communication underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 2752–2763, Aug. 2011.

[11] D. Feng *et al.*, "Mode switching for energy-efficient device-to-device communications in cellular networks," *IEEE Trans. Wireless Commun.*, 2015, doi: 10.1109/TWC.2015.2463280.

[12] Y. Pei and Y.-C. Liang, "Resource allocation for device-to-device communications overlaying two-way cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3611–3621, Jul. 2013.

[13] W. Zhao and S. Wang, "Low complexity power allocation for device-to-device communication underlaying cellular networks," in *Proc. IEEE Int. Conf. Commun. (ICC'14)*, Jun. 2014, pp. 5532–5537.

[14] H. Min, J. Lee, S. Park, and D. Hong, "Capacity enhancement using an interference limited area for device-to-device uplink underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 3995–4000, Dec. 2011.

[15] J. Wang, D. Zhu, C. Zhao, J. Li, and M. Lei, "Resource sharing of underlaying device-to-device and uplink cellular communications," *IEEE Commun. Lett.*, vol. 17, no. 6, pp. 1148–1151, Jun. 2013.

[16] M. Zulhasnine, C. Huang, and A. Srinivasan, "Efficient resource allocation for device-to-device communication underlaying LTE network," in *Proc. IEEE Int. Conf. Wireless Mobile Comput. Netw. Commun. (WiMob'10)*, Oct. 2010, pp. 368–375.

[17] D. Feng, L. Lu, Y. Wu, G. Li, G. Feng, and S. Li, "Device-to-device communications underlaying cellular networks," *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3541–3551, Aug. 2013.

[18] D. Wu, Y. Cai, R. Hu, and Y. Qian, "Dynamic distributed resource sharing for mobile D2D communications," *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5417–5429, Oct. 2015.

[19] D. Zhu, J. Wang, A. Swindlehurst, and C. Zhao, "Downlink resource reuse for device-to-device communications underlaying cellular networks," *IEEE Signal Process. Lett.*, vol. 21, no. 5, pp. 531–534, May 2014.

[20] M. Hasan, E. Hossain, and D. I. Kim, "Resource allocation under channel uncertainties for relay-aided device-to-device communication underlaying LTE-A cellular networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 2322–2338, Apr. 2014.

[21] D. Wu, J. Wang, R. Hu, Y. Cai, and L. Zhou, "Energy-efficient resource sharing for mobile device-to-device multimedia communications," *IEEE Trans. Veh. Technol.*, vol. 63, no. 5, pp. 2093–2103, Jun. 2014.

[22] W. Zhao and S. Wang, "Energy-efficient resource allocation for OFDM-based cognitive radio networks," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1398–1401, Aug. 2015.

[23] C. Xu *et al.*, "Efficiency resource allocation for device-to-device underlay communication systems: A reverse iterative combinatorial auction based approach," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 9, pp. 348–358, Sep. 2013.

[24] D. H. Lee, K. W. Choi, W. S. Jeon, and D. G. Jeong, "Two-stage semi-distributed resource management for device-to-device communication in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 4, pp. 1908–1920, Apr. 2014.

[25] Y. Li, D. Jin, J. Yuan, and Z. Han, "Coalitional games for resource allocation in the device-to-device uplink underlaying cellular networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 7, pp. 3965–3977, Jul. 2014.

[26] R. Zhang, X. Cheng, L. Yang, and B. Jiao, "Interference graph-based resource allocation (InGRA) for D2D communications underlaying cellular networks," *IEEE Trans. Veh. Technol.*, vol. 64, no. 8, pp. 3844–3850, Aug. 2015.

[27] D. Lee, K. Choi, W. Jeon, and D. Jeong, "Resource allocation scheme for device-to-device communication for maximizing spatial reuse," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC'13)*, Apr. 2013, pp. 112–117.

[28] T. D. Hoang, L. B. Le, and T. Le-Ngoc, "Resource allocation for D2D communications under proportional fairness," in *Proc. IEEE GLOBECOM*, Dec. 2014, pp. 1259–1264.

[29] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[30] S. Wang, M. Ge, and C. Wang, "Efficient resource allocation for cognitive radio networks with cooperative relays," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 11, pp. 2432–2441, Nov. 2013.

[31] S. Wang, M. Ge, and W. Zhao, "Energy-efficient resource allocation for OFDM-based cognitive radio networks," *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3181–3191, Aug. 2013.

[32] A. Gjendemsjo, D. Gesbert, G. Oien, and S. Kiani, "Optimal power allocation and scheduling for two-cell capacity maximization," in *Proc. Int. Symp. Model. Optim. Mobile Ad Hoc Wireless Netw. (WiOpt'06)*, Apr. 2006, pp. 1–6.

[33] V. Chandrasekhar and Z. Shen, "Optimal uplink power control in two-cell systems with rise-over-thermal constraints," *IEEE Commun. Lett.*, vol. 12, no. 3, pp. 173–175, Mar. 2008.

[34] 3GPP, "Evolved universal terrestrial radio access (E-UTRA); Further advancements for E-UTRA physical layer aspects (TR 36.814)," Mar. 2010.

[35] H. Kuhn, "The Hungarian method for the assignment problem," *Naval Res. Logist. Quart.*, vol. 2, no. 1, pp. 83–97, Mar. 1955.

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