Approximation Algorithms for Cell Planning in Heterogeneous Networks

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***Abstract*—Small cells are introduced to cellular systems to enhance coverage and improve capacity. Densely deploying small cells can not only offload the traffic of macrocells but also provide an energy- and cost-efficient way to meet the sharp increase in traffic demands in mobile networks. However, such a cell deployment paradigm also leads to heterogeneous network (HetNet) infrastructure and raises new challenges for cell planning. In this paper, we study the cell planning issue in the HetNet. Our optimization task is to select a subset of candidate sites to deploy macro or small cells to minimize the total cost of ownership (TCO) or the energy consumption of the cellular system while satisfying practical constraints. We introduce approximation algorithms to cope with two different cell-planning cases, which are both NP-hard. First, we discuss the macrocell-only case. Our proposed algorithm achieves an approximation ratio of $O(\log R)$ in this scenario, where R is the maximum achievable capacity of macrocells. Then, we introduce an $O(\log \widetilde{R})$-approximation algorithm to the small-cell scenario, where $\widetilde{R}$ is the maximum achievable capacity of a macrocell with small cells overlaid on it. Numerical results indicate that the HetNet can significantly reduce the TCO and the energy consumption of the cellular system.**

***Index Terms*—Approximation algorithm, cell planning, heterogeneous network (HetNet).**

I. Introduction

With the explosive growth of traffic demands in cellular networks, service providers are obliged to increase the system capacity significantly. Due to the high installation and site acquisition cost, energy- and cost-efficient cell planning appears to be of utmost importance for the service providers. Generally, cell planning refers to deploying a number of base stations (BSs, also referred to as cells) at candidate sites and configuring their parameters to provide required coverage with guaranteed quality of service at the least cost.

In second-generation (2G) cellular networks, cell planning usually consists of two stages. First, a set of macrocells is selected from a candidate site list, each configured with a series of radio parameters, such as transmission power, antenna type, azimuth, and tilt, to provide radio coverage for the service area [1]–[3]. The planning objective at this stage is to minimize the total deployment cost while ensuring that the signal strength is high enough for the users at the edge of each cell. Second, radio frequency is grouped, and adjacent cells are assigned to different frequency to leave the mutual interference among them as low as possible so that the signal-to-interference-plus-noise ratio (SINR) of the received signal is over a threshold [4]–[6].

In third-generation (3G) cellular networks, frequency grouping is no longer necessary since each cell uses the whole radio spectrum. It inevitably results in that a user may suffer strong interference from adjacent cells. Cell planning always focuses on how to select a set of cells from candidate sites and configure their radio parameters, particularly transmission power, to keep the SINRs of users above a predefined threshold. Generally, cell planning for the 3G cellular systems is formulated as a capacitated facility location problem that is NP-hard [7]. Heuristic methods, such as tabu search, simulated annealing, and genetic algorithms, can be employed to address this problem [8], [9].

In the fourth-generation network and beyond, e.g., Third-Generation Partnership Project Long-Term Evolution Advanced (LTE-A), a heterogeneous network (HetNet) that generally consists of macrocells and small cells, such as picocells, relays, and femtocells, is deemed as an energy- and cost-efficient way to keep up with the increasing traffic demands of users [10]–[13]. Femtocells are usually deployed by users in an unplanned way to enhance indoor coverage. Picocells are operator-installed and usually adopt the same backhaul and access features as the macrocells, except for serving much smaller areas compared with the macrocells. Picocells are usually deployed at hotspots or coverage holes. The capital expenditure (CAPEX) and operating expenditure (OPEX) of small cells are much lower than that of the macrocells. Moreover, due to its much lower transmission power and smaller size, the small cell only requires flexible site acquisition, which is attractive for service providers. By overlaying low-power and low-cost small cells in the coverage holes, a HetNet can enhance coverage and improve system capacity by offloading the traffic of macrocells.

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On the other hand, one of the most important issues facing the next-generation cellular system is the system energy consumption, which always increases as the rapid growth of traffic demands [14], [15]. It is worth noting that the power consumption of BSs accounts for about 72% of the total power consumption, and the data transmission only accounts for about 15% according to the investigation results given in [16]. In other words, the power consumption of BSs is the majority of the energy consumption of the cellular systems. Considering that the power consumption of small cells is much lower than that of the macrocells, the HetNet demonstrates great potential to improve the energy efficiency of the cellular network as discussed in [17]. An energy- and cost-efficient cell planning strategy is of utmost importance for the HetNet, which has been verified by our preliminary results [18], [19]. However, planning macrocells and small cells in an almost optimal way has not been studied extensively in the literature. Most of the related works on the cell planning for HetNets only focus on how to deploy the BSs to satisfy the traffic requirements under ideal network models [20]–[22]. In [20], cell planning with varying spatial and temporal user densities is studied, where the optimization objective is to find the optimal BS locations to satisfy the cell capacity constraint per subarea, as well as the coverage requirement. The number of BSs is estimated, and a meta-heuristic algorithm is developed to find the optimal BS locations at the first stage, followed by a procedure to eliminate redundant BSs. In [21], the total number of required BSs is estimated first. Then, the locations and the coverage of all cells are determined. Finally, small cells are deployed to fill the coverage holes. In [22], the optimization target is to reduce the total deployment cost and enhance the cell coverage, leading to an intractable multiobjective integer programming problem, which is addressed by a two-level search algorithm.

It is worth noting that mutual interference among cells and spectral efficiency are not considered fully in [18]–[22]. We jointly consider power and bandwidth allocation, traffic requirement, and spectral efficiency in this paper. We propose a novel HetNet planning model that involves macrocells and operator-installed small cells, where the transmission power and bandwidth budgets are at different levels. We try to select a subset of candidate sites to deploy macrocells and small cells at the minimum total cost of ownership (TCO) or with the least energy consumption to provide throughput and spectral efficiency guarantee for all demand nodes (DNs). We investigate how to allocate power and bandwidth to maximize the sum rate of a given set of cells, based on which we design approximation algorithms for different scenarios to provide worst-case performance guaranteed cell planning schemes. First, an approximation algorithm is introduced to solve the macro-only cell planning case, which also indicates how to allocate bandwidth to avoid the co-tier interference among macrocells. The proposed algorithm can achieve an $O(\log R)$-approximation ratio, where R is the maximum achievable transmission rate of the macrocells. Then, we develop an $O(\log \widetilde{R})$-approximation algorithm for the small-cell scenario, where $\widetilde{R}$ is the maximum achievable capacity of the macrocell with small cells overlaid on it. Theoretical analysis and numerical results validate the effectiveness of our proposed cell planning schemes.

TABLE I
NOTATIONS

Symbol	Semantics
$\mathcal{N}_m$	Set of candidate sites for deploying marco cells
$\mathcal{N}_s$	Set of candidate sites for deploying small cells
$\mathcal{M}_n$	Set of small cells covered by macro cell n
$\mathcal{N}$	Set of all candidate sites
N	Number of all cells
c_n	Cost of cell n
c_n^{CAPEX}	CAPEX of cell n
c_n^{OPEX}	OPEX of cell n
$\bar{c}_n^{O\&M}$	Average O&M cost of cell n per year
$\bar{c}^E$	Cost of unit energy consumption
$\bar{E}_n$	Average energy consumption of cell n per year
z_n	Selection variable of cell n
$\mathcal{I}_m$	Set of interference groups of macro cells
$\mathcal{I}_s$	Set of interference groups of small cells
$\mathcal{I}$	Set of interference groups of co-tier cells
P_n^{max}	Maximum transmission power of cell n
B_m	Total available bandwidth of macro cell
B_s	Total available bandwidth of small cell
$\mathcal{K}$	Set of DNs
K	Number of DNs
R_k^{min}	Rate demand of DN k
$b_{k,n}$	Bandwidth allocated to DN k by cell n
$p_{k,n}$	Power allocated to DN k by cell n
$h_{k,n}$	Channel gain between cell n and DN k
$r_{k,n}$	Achievable rate of DN k covered by cell n
$SE_{k,n}$	Spectral efficiency of DN k covered by cell n
SE_k^{min}	Spectral efficiency requirement of DN k
Γ	SNR gap
N_0	PSD of AWGN
$I_{k,n}$	Interference with unit bandwidth

The remainder of this paper is organized as follows. In Section II, we illustrate network model and formulate the optimization task. In Section III, we address the power and bandwidth allocation problem for a given set of cells. In Section IV, approximation algorithms for the cell planning problems are proposed in detail. In Section V, numerical results are given with discussions. Conclusions are drawn in Section VI.

II. Problem Formulation

A. Network Model

Some frequently used notations are listed in Table I.

Consider a HetNet where macrocells and small cells are involved.[1] The sets of candidate sites of macrocells and small cells are denoted $\mathcal{N}_m$ and $\mathcal{N}_s$, respectively. Denote $\mathcal{N} = \mathcal{N}_m \cup \mathcal{N}_s = \{1, 2, \ldots, N\}$ as the set of all candidate sites to deploy cells. Macrocells provide wide basic coverage and support high-mobility users to reduce handover frequency of the cellular system. Small cells usually locate in the coverage of a macrocell to serve users with high traffic demands. Denote $\mathcal{M}_n$ as the set of small cells that can be covered by macrocell $n \in \mathcal{N}_m$. We assume that $|\mathcal{M}_n|$ is a constant integer. $P_n^{\max}$ is the transmission power budget of cell n. The maximum

[1]There are also small cells installed by users in an unplanned manner, which are usually referred to as femtocells. Such kind of cells is not considered in this paper. We consider the small cells deployed only by service providers.

transmission power of a macrocell (e.g., 46 dBm) is usually much higher than that of a small cell (e.g., 30 dBm).

Since the available spectrum below 3 GHz is limited, higher frequency bands can be used by small cells to enjoy more available spectrum [23], e.g., the LTE-Hi project targets frequency band from 3.4 to 3.6 GHz [24]. By exploiting wider bandwidth in higher frequency, small cells can provide higher performance in hotspots and indoor environments. In this paper, we assume that macrocells and small cells use different frequency bands to avoid cross-tier interference among them. The available bandwidths of the macrocells and the small cells are B_m and B_s, respectively.

Reducing TCO is of great interest to service providers, which always includes CAPEX and OPEX. Denote

$$c_n = c_n^{\text{CAPEX}} + c_n^{\text{OPEX}} \tag{1}$$

as the cost of cell n, where c_n^{CAPEX} and c_n^{OPEX} are the CAPEX and OPEX of the cell, respectively. c_n^{CAPEX} is the total cost of site acquisition, equipment installation, and civil works [16]. It is fixed and independent of wireless functionality. For cell n, c_n^{OPEX} mainly involves site rent, electricity, and operation and maintenance (O&M). Thus, OPEX spent on cell n can be expressed as follows:

$$c_n^{\text{OPEX}} = y\left(\bar{c}_n^{O\&M} + \bar{c}^E \bar{E}_n\right) \tag{2}$$

where y is operating years, $\bar{c}_n^{O\&M}$ is the cost of O&M of cell n per year, $\bar{c}^E$ is the cost of unit energy consumption, and $\bar{E}_n$ is the average energy consumption of the cell n per year.[2] Compared with a macrocell, a small cell is much more economic in site acquisition, equipment installation, O&M, and energy consumption.

We use a DN to represent the spatial distribution of traffic demand as proposed in [2] and [3]. A DN represents average traffic demand in a small area. The set of DNs is denoted $\mathcal{K} = \{1, 2, \ldots, K\}$. DN $k \in \mathcal{K}$ requires a minimal rate $R_k^{\min}$, where $R_k^{\min} > 0$. Denote $h_{k,n}$ as the channel gain between cell n and DN k, which is a function of the distance between the cell and the DN with a predefined path-loss model. Denote $b_{k,n}$ and $p_{k,n}$ as the bandwidth and power of the cell n allocated to the DN k, respectively, the achievable rate of the DN k served by the cell n can be calculated as

$$r_{k,n} = b_{k,n} \log_2 \left[1 + \frac{p_{k,n}|h_{k,n}|^2}{\Gamma b_{k,n}(N_0 + I_{k,n})}\right] \tag{3}$$

where Γ is the SNR gap and related to a given bit error rate (BER) for a specific modulation/demodulation scheme, e.g., $\Gamma = -\ln(5\text{BER})/1.6$ for an uncoded multilevel quadrature amplitude modulation system [25]. N_0 is the power spectral density of additive white Gaussian noise. $I_{k,n}$ is the interference introduced by adjacent cells with unit bandwidth.

[2]Since the power consumption of a cell site accounts for the majority of total power consumption, we assume that the average energy consumption of each cell per year, i.e., $\bar{E}_n$, is fixed and irrelevant to the transmission power.

B. Interference Model

In a practical HetNet, co-tier and cross-tier interference are major factors that influence system capacity, which should be managed properly in cell planning. In 2G cellular networks, e.g., the GSM systems, radio frequency is grouped, and adjacent cells are assigned to different frequency to leave mutual interference among the cells as low as possible. In 3G networks, code-division multiple access is adopted, where the whole radio spectrum is shared by all cells. Each user is assigned to a special spreading sequence to avoid cochannel interference. Different from conventional cellular networks, frequency grouping is no longer required in the HetNet since the whole radio spectrum is shared by co-tier cells. Furthermore, orthogonal frequency-division multiplexing (OFDM) is employed in the HetNet for adaptive resource allocation to exploit diversity gains among users to improve the system throughput and energy-efficiency [26]. Hence, mutual interference among cells varies significantly due to the mobility of users and the changes of channel gain. Power and bandwidth should be dynamically allocated to users to achieve a high system capacity and decrease intercell interference.

As mentioned earlier, small cells can use higher frequency bands to further improve user throughput in hotspots and indoor environments. Therefore, the cross-tier interference between the small cells and the macrocells can be eliminated. Even if macrocells and small cells use the same frequency band, resulting to heavy cross-tier interference among them, our proposed cell planning schemes can also be adopted with necessary modifications. For the LTE-A networks, macrocells are interconnected with each other by X2 interface; therefore, coordinated-multipoint transmission/reception and intercell interference coordination can be employed to reduce intercell interference [27], [28]. Due to the low transmission power of small cell and the large propagation loss of high frequency bands, the co-tier interference among small cells is usually slight if the distances between them are far enough. Moreover, clustering can be used to manage co-tier interference by coordinating the transmissions of the cells [29], e.g., if a cell is in close proximity of another, the two cells can be assigned to different spectrum. Such a clustering strategy can still be deemed as a penalty of co-tier interference in cell planning since the available bandwidth is reduced for each cell. Even though they are selected simultaneously, the co-tier interference can be safely managed by assigning the cells to different frequency bands.

Denote $\mathcal{I}_m$ and $\mathcal{I}_s$ as the set of interference groups of macrocells and small cells, respectively, which can be predetermined. Let $\mathcal{I} = \mathcal{I}_m \cup \mathcal{I}_p$. For each subset $\mathcal{N}'$ in $\mathcal{I}$, the cells in $\mathcal{N}'$ use different spectra, and we have the following constraints:

$$\begin{aligned} \sum_{n\in\mathcal{N}_1}\sum_{k\in\mathcal{K}} b_{k,n} \le B_m, \qquad \mathcal{N}_1 \in \mathcal{I}_m \\ \sum_{n\in\mathcal{N}_2}\sum_{k\in\mathcal{K}} b_{k,n} \le B_s, \qquad \mathcal{N}_2 \in \mathcal{I}_s. \end{aligned} \tag{4}$$

We also need the following assumptions: If $\mathcal{N}_1$ and $\mathcal{N}_2$ are two different subsets of $\mathcal{I}$, then $\mathcal{N}_1 \cap \mathcal{N}_2 = \emptyset$; if $n_1, n_2 \in \mathcal{N}'$, where $\mathcal{N}'$ is a subset of $\mathcal{I}$, then $\mathcal{M}_{n_1} = \mathcal{M}_{n_2}$. It is reasonable due to the wide coverage of macrocells. The latter ensures that the macrocells in group $\mathcal{N}'$ cover the same set as the small cells. We use $\mathcal{M}_{\mathcal{N}'}$ to denote the set of small cells that can be covered by the group $\mathcal{N}'$ of macrocells.

Based on the analysis above, the co-tier interference and cross-tier interference can be carefully managed, $I_{k,n}$ is thus approximately zero. Equation (3) can be rewritten as

$$r_{k,n} = b_{k,n} \log_2 \left(1 + \frac{p_{k,n} H_{k,n}}{b_{k,n}}\right) \quad (5)$$

where $H_{k,n} = |h_{k,n}|^2/\Gamma N_0$. Note that $r_{k,n}$ is a concave function of $p_{k,n}$ and $b_{k,n}$ in (5).

C. Network Constraints

Define Boolean variables z_n as the deployment indicators of cells, i.e.,

$$z_n = \begin{cases} 1, & \text{cell } n \text{ is selected} \\ 0, & \text{otherwise} \end{cases} \quad \forall n \in \mathcal{N}.$$

For a practical cellular system, there are the following constraints.

Power Budget: Each cell is power limited, i.e.,

$$\sum_{k\in\mathcal{K}} p_{k,n} \le z_n P_n^{\max} \qquad \forall n \in \mathcal{N}. \quad (6)$$

Bandwidth Budget: The available bandwidth of each cell is also limited, i.e.,

$$\begin{aligned} &\sum_{k\in\mathcal{K}} b_{k,n_1} \le z_{n_1} B_m && \forall n_1 \in \mathcal{N}_m \\ &\sum_{k\in\mathcal{K}} b_{k,n_2} \le z_{n_2} B_s && \forall n_2 \in \mathcal{N}_s. \end{aligned} \quad (7)$$

Traffic Requirements: The selected cells should satisfy all rate requirements of the DNs

$$R_k = \sum_{n\in\mathcal{N}} r_{k,n} \ge R_k^{\min} \qquad \forall k \in \mathcal{K}. \quad (8)$$

Note that a DN is an abstraction of the average traffic demand in a small area. In practical networks, a DN can be served by multiple cells.

Spectral Efficiency Guarantee: Spectral efficiency is an important issue in cell planning from the viewpoint of the cellular system. Without considering spectral efficiency, a DN far away from a cell may consume most of the radio resource of the cell. Define the spectral efficiency function $\mathrm{SE}_{k,n}$ for $k\in\mathcal{K}, n\in\mathcal{N}$ as

$$\mathrm{SE}_{k,n} = \frac{r_{k,n}}{b_{k,n}} = \log_2 \left(1 + \frac{p_{k,n} H_{k,n}}{b_{k,n}}\right). \quad (9)$$

A minimum spectral efficiency is required if cell n allocates a part of its bandwidth to DN k. If $b_{k,n} > 0$, it requires that

$$\mathrm{SE}_{k,n} \ge \mathrm{SE}_k^{\min}. \quad (10)$$

According to (9) and (10), we have the following affine constraint:

$$p_{k,n} \ge \Delta_{k,n} b_{k,n} \quad (11)$$

where $\Delta_{k,n} = (2^{\mathrm{SE}_k^{\min}} - 1)/H_{k,n}$. Note that we usually take the cell-edge user spectral efficiency requirement as the minimum spectral efficiency.

D. Problem Formulation

Our optimization objective is to select a subset of $\mathcal{N}$ with the minimum TCO to satisfy all rate requirements of uses in $\mathcal{K}$, while satisfying a series of network constraints. First, the total allocated power and bandwidth of each cell cannot exceed its maximum transmission power and available bandwidth. Second, the rate requirements of all DNs should be satisfied by the selected cells. Finally, the spectral efficiency on the allocated bandwidth cannot be lower than a given threshold. The optimization problem can be mathematically formulated as follows:

$$\begin{aligned} \min_{z_n, p_{k,n}, b_{k,n}} \quad & \sum_{n\in\mathcal{N}} c_n z_n \\ \text{s.t. } C_1: \quad & \sum_{k\in\mathcal{K}} b_{k,n_1} \le z_{n_1} B_m \quad \forall n_1 \in \mathcal{N}_m \\ C_2: \quad & \sum_{n\in\mathcal{N}_1} \sum_{k\in\mathcal{K}} b_{k,n} \le B_m \quad \forall \mathcal{N}_1 \in \mathcal{I}_m \\ C_3: \quad & \sum_{k\in\mathcal{K}} b_{k,n_2} \le z_{n_2} B_s \quad \forall n_2 \in \mathcal{N}_s \\ C_4: \quad & \sum_{n\in\mathcal{N}_2} \sum_{k\in\mathcal{K}} b_{k,n} \le B_s \quad \forall \mathcal{N}_2 \in \mathcal{I}_s \\ C_5: \quad & \sum_{k\in\mathcal{K}} p_{k,n} \le z_n P_n^{\max} \quad \forall n \in \mathcal{N} \\ C_6: \quad & R_k \ge R_k^{\min} \quad \forall k \in \mathcal{K} \\ C_7: \quad & p_{k,n} \ge \Delta_{k,n} b_{k,n} \quad \forall k \in \mathcal{K}, n \in \mathcal{N} \\ C_8: \quad & \max_{n: n\in\mathcal{N}_m, n'\in\mathcal{M}_n} z_n \ge z_{n'} \quad \forall n' \in \mathcal{N}_s \\ C_9: \quad & b_{k,n} \ge 0, p_{k,n} \ge 0 \quad \forall k \in \mathcal{K}, n \in \mathcal{N} \\ C_{10}: \quad & z_n \in \{0,1\} \quad \forall n \in \mathcal{N}. \end{aligned} \quad (12)$$

Note that (12) defines a general form of cell planning for the HetNet. If our goal is to minimize the average system energy consumption, we can set $c_n^{\mathrm{CAPEX}} = 0 \, \forall n \in \mathcal{N}$ and $\bar{c}^{O\&M} = 0$. Then, the objective becomes

$$\min_{z_n, p_{k,n}, b_{k,n}} \sum_{n\in\mathcal{N}} \bar{E}_n z_n.$$

In the remainder of this paper, we mainly focus on the minimization of TCO. The same methods can be employed to minimize the system energy consumption by replacing c_n by $\bar{E}_n$.

III. Bandwidth and Power Allocation for a Given Subset of Cells

Notice that (12) is similar to a set cover problem with hard capacities [30] without considering small cells. Additionally, a generalized method to cope with the cell planning problem is proposed in [31]. However, the power and bandwidth are not considered continuous variables to satisfy traffic demands. The resource consumption for a cell to serve a DN is fixed, which is only related to the traffic demand of the DN. It means that the resource consumption is independent of the channel gain between the DN and the cell. Recall that the method proposed in [18] cannot be applied since there exist cells that need to use different spectrum to avoid co-tier interference.

Before introducing our approximation algorithms, we need to obtain the maximum satisfied sum rate of a given set of cells where the achievable rate of each DN is kept below its rate requirement. With this information, we can generalize the results of [18], [30], and [31] to solve (12).

The bandwidth and power allocation problem for a given set of cells is defined as follows: Given a set of macrocells $\mathcal{N}'_m$ and a set of small cells $\mathcal{N}'_s$, we try to work out the achievable rate that can be sustained by those cells while keeping the bandwidth and power constraints of the cells satisfied. Let $\mathcal{N}' = \mathcal{N}'_m \cup \mathcal{N}'_s$ be the set of given cells and $N' = |\mathcal{N}'|$ be the number of cells in $\mathcal{N}'$. Mathematically, the optimization problem is as follows:

$$
\begin{aligned}
\max_{b_{k,n}, p_{k,n}} \quad & \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} r_{k,n} \\
\text{s.t. } C_1: \quad & 0 \le b_{k,n}, 0 \le p_{k,n}, \Delta_{k,n} b_{k,n} \le p_{k,n} \\
& \forall\, k \in \mathcal{K}, n \in \mathcal{N}' \\
C_2: \quad & \sum_{k \in \mathcal{K}} b_{k,n_1} \le B_m \quad \forall\, n_1 \in \mathcal{N}'_m \\
C_3: \quad & \sum_{k \in \mathcal{K}} b_{k,n_2} \le B_s \quad \forall\, n_2 \in \mathcal{N}'_s \\
C_4: \quad & \sum_{n_1 \in \mathcal{N}'_m \cap \mathcal{N}_1} \sum_{k \in \mathcal{K}} b_{k,n_1} \le B_m \quad \forall \mathcal{N}_1 \in \mathcal{I}_m \\
C_5: \quad & \sum_{n_2 \in \mathcal{N}'_s \cap \mathcal{N}_2} \sum_{k \in \mathcal{K}} b_{k,n_2} \le B_s \quad \forall \mathcal{N}_2 \in \mathcal{I}_s \\
C_6: \quad & \sum_{k \in \mathcal{K}} p_{k,n} \le P_n^{\max} \quad \forall\, n \in \mathcal{N}' \\
C_7: \quad & \sum_{n \in \mathcal{N}'} r_{k,n} \le R_k^{\min} \quad \forall\, k \in \mathcal{K}.
\end{aligned} \tag{13}
$$

It is worth noting that the achievable rate of each DN is required below the minimum rate requirement in (13). With the constraints in C_7, $\mathcal{N}'$ is a feasible solution to (12) if and only if the optimal value of the objective function of (13) is $\sum_{k \in \mathcal{K}} R_k^{\min}$. More importantly, we can design approximation algorithm to solve (12) based on the optimal solution to (13). However, one requirement of a convex optimization problem is that all inequality constraint functions are convex [32]. As mentioned earlier, C_7 is concave for both $b_{k,n}$ and $p_{k,n}$, resulting in that (13) is not a convex/concave optimization problem. We use a general transformation to yield an equivalent problem whose feasible set is convex. Since

$$
p_{k,n} = \frac{b_{k,n}}{H_{k,n}} \cdot \left(2^{\frac{r_{k,n}}{b_{k,n}}} - 1 \right)
$$

the equivalent problem of (13) can be written as

$$
\begin{aligned}
\max_{b_{k,n}, r_{k,n}} \quad & \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} r_{k,n} \\
\text{s.t.} \quad & 0 \le b_{k,n}, 0 \le r_{k,n} \quad \forall\, k \in \mathcal{K}, n \in \mathcal{N}' \\
& \mathrm{SE}_k^{\min} b_{k,n} \le r_{k,n} \quad \forall\, k \in \mathcal{K}, n \in \mathcal{N}' \\
& C_2 \sim C_7 \text{ in } (13).
\end{aligned} \tag{14}
$$

The objective function and the constraints C_2, C_3, C_4, C_5, C_7 are affine for both $b_{k,n}, r_{k,n}$ in (14). Moreover, C_6 is convex since $p_{k,n}$ is convex for both $b_{k,n}$ and $r_{k,n}$. Thus, (14) defines a convex optimization problem [32].

The barrier method is a standard technique to solve constrained convex optimization problems. It makes all inequality constraints implicit in the optimization objective. The main disadvantage of the barrier method is that the cost of storing Hessian is high, as well as the load of computing Newton step. To reduce the storage complexity and computational complexity, we employ a fast barrier method by exploiting the special structure of (14), as suggested in [33]–[35].

For $\mathcal{N}_1, \mathcal{N}_2$ that $\mathcal{N}_1 \in \mathcal{I}_m$, $\mathcal{N}_2 \in \mathcal{I}_s$ and $\mathcal{N}'_m \cap \mathcal{N}_1 \ne \emptyset$, $\mathcal{N}'_s \cap \mathcal{N}_2 \ne \emptyset$, denote

$$
\begin{aligned}
b_{\mathcal{N}_1} &= B_m - \sum_{n_1 \in \mathcal{N}'_m \cap \mathcal{N}_1} \sum_{k \in \mathcal{K}} b_{k,n_1} \\
b_{\mathcal{N}_2} &= B_s - \sum_{n_2 \in \mathcal{N}'_s \cap \mathcal{N}_2} \sum_{k \in \mathcal{K}} b_{k,n_2}.
\end{aligned}
$$

Denote

$$
\begin{aligned}
b_{n_1} &= B_m - \sum_{k \in \mathcal{K}} b_{k,n_1} \quad \forall\, n_1 \in \mathcal{N}'_m \\
b_{n_2} &= B_s - \sum_{k \in \mathcal{K}} b_{k,n_2} \quad \forall\, n_2 \in \mathcal{N}'_s \\
p_n &= P_n^{\max} - \sum_{k \in \mathcal{K}} p_{k,n} \quad \forall\, n \in \mathcal{N}' \\
r_k &= R_k^{\min} - \sum_{n \in \mathcal{N}'} r_{k,n} \quad \forall\, k \in \mathcal{K}.
\end{aligned}
$$

Finally, for each $k \in \mathcal{K}, n \in \mathcal{N}'$, denote

$$
s_{k,n} = r_{k,n} - b_{k,n} \mathrm{SE}_k^{\min}.
$$

First, we collect all variables into one vector $\boldsymbol{x} \in \mathbf{R}^{2KN'}$, $\boldsymbol{x} = (r_{1,1}, b_{1,1}, r_{1,2}, b_{1,2}, \ldots, r_{K,N'}, b_{K,N'})$. Then, we convert all inequality constraints into a logarithmic barrier function $\phi(\boldsymbol{x})$

$$
\begin{aligned}
\phi(\boldsymbol{x}) = &- \sum_{n \in \mathcal{N}'} \log b_n - \sum_{n \in \mathcal{N}'} \log p_n - \sum_{k \in \mathcal{K}} \log r_k \\
&- \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} \log r_{k,n} - \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} \log b_{k,n} \\
&- \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} \log s_{k,n} - \sum_{\mathcal{N}_1} \log b_{\mathcal{N}_1} - \sum_{\mathcal{N}_2} \log b_{\mathcal{N}_2}.
\end{aligned}
$$

The optimization problem can be converted into a sequence of minimization problems by introducing a logarithmic barrier function with a parameter t. For (14), its optimal solution can be approximated by solving the following unconstrained convex problem:

$$\min_{\boldsymbol{x}} \quad \psi_t(\boldsymbol{x}) = -t \sum_{n\in\mathcal{N}'} \sum_{k\in\mathcal{K}} r_{k,n} + \phi(\boldsymbol{x}).$$

As t increases, such an approximation becomes closer to the optimal solution to (14). Generally, the Newton method is preferred to solve the unconstrained convex problem mentioned earlier because of its quadratic convergence property [32]. For a given parameter t, Newton step $\triangle\boldsymbol{x}$ can be computed by solving the following equation:

$$\nabla^2\psi_t(\boldsymbol{x})\triangle\boldsymbol{x} = -\nabla\psi_t(\boldsymbol{x}) \tag{15}$$

where $\nabla^2\psi_t(\boldsymbol{x})$ and $\nabla\psi_t(\boldsymbol{x})$ are the Hessian and the gradient of $\psi_t(\boldsymbol{x})$, respectively. The Hessian of $\psi_t(\boldsymbol{x})$ can be written as

$$\begin{aligned}\nabla^2\psi_t(\boldsymbol{x}) = \mathrm{diag}(D_{1,1},\ldots,D_{K,N'}) + \sum_{k\in\mathcal{K}} \frac{\nabla r_k \nabla r_k^T}{r_k^2} \\ + \sum_{n\in\mathcal{N}'} \frac{\nabla p_n \nabla p_n^T}{p_n^2} + \sum_{n\in\mathcal{N}'} \frac{\nabla b_n \nabla b_n^T}{b_n^2} \\ + \sum_{\mathcal{N}_1} \frac{\nabla b_{\mathcal{N}_1} \nabla b_{\mathcal{N}_1}^T}{b_{\mathcal{N}_1}^2} + \sum_{\mathcal{N}_2} \frac{\nabla b_{\mathcal{N}_2} \nabla b_{\mathcal{N}_2}^T}{b_{\mathcal{N}_2}^2}\end{aligned}$$

where $D_{k,n}$ is presented in

$$\begin{aligned} D_{k,n} = \begin{bmatrix} \frac{1}{r_{k,n}^2} + \frac{1}{p_n}\frac{\partial^2 p_{k,n}}{\partial r_{k,n}^2} & \frac{1}{p_n}\frac{\partial^2 p_{k,n}}{\partial r_{k,n}\partial b_{k,n}} \\ \frac{1}{p_n}\frac{\partial^2 p_{k,n}}{\partial b_{k,n}\partial r_{k,n}} & \frac{1}{b_{k,n}^2} + \frac{1}{p_n}\frac{\partial^2 p_{k,n}}{\partial b_{k,n}^2} \end{bmatrix} \\ + \begin{bmatrix} \frac{1}{s_{k,n}^2} & -\frac{\mathrm{SE}_k^{\min}}{s_{k,n}^2} \\ -\frac{\mathrm{SE}_k^{\min}}{s_{k,n}^2} & \left(\frac{\mathrm{SE}_k^{\min}}{s_{k,n}}\right)^2 \end{bmatrix}. \end{aligned} \tag{16}$$

Due to the special structure of the Hessian, we can solve (15) by using the method proposed in [33] and [34] efficiently. Based on the analyses of [33] and [34], the computational complexity is bounded by $O(UKN')$, where $U = \max\{K^2, N^2\}$. More importantly, the storage complexity is bounded by $O(KN')$ intuitively, making that the method that can be applied to large-scale wireless system. The outline of the barrier method is summarized in Table II, where $\mathbf{0} \in \mathbf{R}^{2KN'}$ whose elements are all zeros. ϵ_b and ϵ_n are the tolerances of the barrier method and the Newton method, respectively. α and β are two constants utilized in backtracking line search with $\alpha \in (0, 0.5)$ and $\beta \in (0,1)$. The step size of the backtracking line search is s with $s > 0$. t and μ are parameters associated with a tradeoff between outer iterations and inner iterations. Based on the optimal solution to (14), we can design approximation algorithms to address the cell planning problem defined by (12).

TABLE II
BARRIER METHOD FOR BANDWIDTH AND POWER ALLOCATION

Algorithm 1

Initialization: $\boldsymbol{x} = \mathbf{0}$
while $KN'/t > \epsilon_b$
  while True
    Compute $\triangle\boldsymbol{x}$ by (15) and $\lambda^2 = \nabla\psi_t(\boldsymbol{x})\triangle\boldsymbol{x}$;
    Set $s = 1$;
    if $\lambda^2/2 \le \epsilon_n$
      break;
    end if
    while $\psi_t(\boldsymbol{x} + s\triangle\boldsymbol{x}) > \psi_t(\boldsymbol{x}) - \alpha s\lambda^2$
      $s = \beta s$;
    end while
    Update $\boldsymbol{x} = \boldsymbol{x} + s\triangle\boldsymbol{x}$;
  end while
  $t = \mu t$;
end while
return $\boldsymbol{x}$

IV. APPROXIMATION ALGORITHMS FOR CELL PLANNING

Here, we first focus on the macro-only cell planning problem, where small cells are not involved. We propose an $O(\log R)$-approximation algorithm to address the problem. Then, an $O(\log \widetilde{R})$-approximation algorithm for the cell planning with small cells, which is based on the theoretical analyses of the algorithm for macro-only cell planning, is also developed.

A. $O(\log R)$-*Approximation Algorithm for Macro-only Cell Planning*

Given a set $\mathcal{N}_1 \subseteq \mathcal{N}$ of cells, let $w(\mathcal{N}_1)$ be the optimal value of the objective function of (14). Denote $c(\mathcal{N}_1)$ as the total cost of $\mathcal{N}_1$. For each $\mathcal{N}_2 \subseteq \mathcal{N}$, we define

$$\begin{aligned} w_{\mathcal{N}_1}(\mathcal{N}_2) &= w(\mathcal{N}_1 \cup \mathcal{N}_2) - w(\mathcal{N}_1) \\ W_{\mathcal{N}_1}(\mathcal{N}_2) &= \frac{w_{\mathcal{N}_1}(\mathcal{N}_2)}{c(\mathcal{N}_2)}. \end{aligned}$$

The key idea of our proposed algorithm lies in a greedy strategy: We try to add a set of cells G_l at the lth iteration to satisfy

$$\begin{aligned} W_{\mathcal{N}_{l-1}}(G_l) &\ge \max_{n\in\mathcal{N}_m\setminus\mathcal{N}_{l-1}} W_{\mathcal{N}_{l-1}}(\{n\}) \\ W_{\mathcal{N}_{l-1}}(G_l) &\ge \max_{\mathcal{N}'\in\mathcal{I}_m} \max_{G\subseteq\mathcal{N}'} W_{\mathcal{N}_{l-1}}(G) \end{aligned} \tag{17}$$

where $\mathcal{N}_l$ is the set of selected cells at the lth iteration. If we can find out G_l that satisfies (17), all possible combinations of macrocells in an interference group should be searched exhaustively at the lth iteration, which yields an unacceptable computation burden. Fortunately, we can only search each unselected cell to find G_l by using the following theorem.

Lemma 1: Given a subset $\mathcal{N}'$ in $\mathcal{I}_m$, if cell $n^* \in \mathcal{N}'$ can maximize $W_{\mathcal{N}_{l-1}}(\{n\})$ at the lth iteration, it always holds that

$$W_{\mathcal{N}_{l-1}}(\{n^*\}) \ge W_{\mathcal{N}_{l-1}}(G) \quad \forall G \subseteq \mathcal{N}'.$$

Proof: Given positive numbers $a_1,\ldots,a_n$ and $b_1,\ldots,b_n$, we have the following fact:

Fact 1.

$$\max_{i=1,\ldots,n} \frac{a_i}{b_i} \ge \frac{\sum_{i\in I} a_i}{\sum_{i\in I} b_i} \qquad \forall I \subseteq \{1,2,\ldots,n\}.$$

TABLE III
$O(\log R)$-APPROXIMATION ALGORITHM FOR MACRO-ONLY CELL PLANNING

Algorithm 2

1: *Initialization:* $\mathcal{N}_l \leftarrow \emptyset$, $l = 0$.
2: **while** $w(\mathcal{N}_l) < \sum_{k\in\mathcal{K}} R_k^{min}$ **do**
3: $l = l + 1, W = 0$;
4: **for all** $n \in \mathcal{N}_m \backslash \mathcal{N}_{l-1}$ **do**
5: Calculate $W_n = W_{\mathcal{N}_{l-1}}(\{n\})$;
6: **if** $W_n > W$
7: $n_l = n$, $W = W_n$;
8: **end if**
9: **end for**
10: $\mathcal{N}_l \leftarrow \mathcal{N}_{l-1} \cup \{n_l\}$;
11: **end while**
12: **return** $\mathcal{N}_l$

By using Fact 1, we have

$$\max_{n\in\mathcal{N}'} W_{\mathcal{N}_{l-1}}(\{n\}) \geq \frac{\sum_{n\in G} w_{\mathcal{N}_{l-1}}(\{n\})}{\sum_{n\in G} c_n} \quad \forall G \subseteq \mathcal{N}'.$$

Intuitively, $\sum_{n\in G} w_{\mathcal{N}_{l-1}}(\{n\})$ is always not less than $w_{\mathcal{N}_{l-1}}(G)$. Thus, it can be concluded that

$$\max_{n\in\mathcal{N}'} W_{\mathcal{N}_{l-1}}(\{n\}) \geq \frac{w_{\mathcal{N}_{l-1}}(G)}{\sum_{n\in G} c_n} \geq W_{\mathcal{N}_{l-1}}(G) \quad \forall G \subseteq \mathcal{N}'.$$

■

According to Lemma 1, we always select a candidate cell to increase the system capacity with the least cost. That is, the cell with the maximum average capacity per unit cost is selected with priority. We test all remaining candidate cells at each iteration and select a candidate cell n_l to increase the system capacity with the least cost. The cell always satisfies (17). Such a procedure repeats until the rate requirements of all DNs are satisfied. Then, the set of selected cells can be output as a feasible solution. The outline of our proposed approximation algorithm for macro-only cell planning is described in Table III, where $\mathcal{X}\backslash\mathcal{Y} = \{x \in \mathcal{X} | x \notin \mathcal{Y}\}$. Note that the proposed algorithm can always provide coverage if a feasible solution to the planning problem exists because the worst case is that all cells are selected.

B. Analysis of Algorithm 2

To analyze the approximation ratio of the proposed approximation algorithm (Algorithm 2), we need to describe the optimal solution to (12) at first. Given a cell planning instance, let $z_n^*, b_{k,n}^*, p_{k,n}^*$ be an optimal solution to (12), and let $r_{k,n}^* = b_{k,n}^* \log_2(1 + p_{k,n}^* H_{k,n}/b_{k,n}^*)$ is the achievable transmission rate. Without loss of generality, we assume $\sum_{n\in\mathcal{N}} r_{k,n}^* = R_k^{\min}$. Denote $\mathcal{N}_m^* = \{n \in \mathcal{N}_m | z_n^* = 1\}$ as the set of selected macrocells that exist in the optimal solution.

For each $n \in \mathcal{N}^*$, denote R_n^* as the total transmission rate of cell n

$$R_n^* = \sum_{k\in\mathcal{K}} r_{k,n}^*.$$

Similar to a single-cell scenario, denote $R_{\mathcal{N}'}^*$ as the total transmission rate of group $\mathcal{N}' \in \mathcal{I}_m$

$$R_{\mathcal{N}'}^* = \sum_{n\in\mathcal{N}'} z_n^* \sum_{k\in\mathcal{K}} r_{k,n}^*.$$

Each macrocell $n \in \mathcal{N}_m^*$ sustains a rate R_n^* at the employment cost of c_n. Each group $\mathcal{N}' \in \mathcal{I}_m$ sustains a rate $R_{\mathcal{N}'}^*$ at the cost of $c(\mathcal{N}')$.

Theorem 1: Algorithm 2 achieves an approximation factor of $O(\log R)$ for the macro-only cell planning problem defined by (12), where

$$R = \max\left\{\max_{\mathcal{N}'\in\mathcal{I}_m} w(\mathcal{N}'), \max_{n\in\mathcal{N}_m} w(\{n\})\right\}.$$

Proof: Let $z_n', b_{k,n}', p_{k,n}'$ be the solution of Algorithm 2. Denote L as the number of iterations and $\mathcal{N}_L = \{n \in \mathcal{N}_m | z_n' = 1\}$ as the corresponding set of selected cells by Algorithm 2.

We divide $\mathcal{N}_m^*$ into two different subsets: $\mathcal{N}_1^*$ and $\mathcal{N}_2^*$. $\mathcal{N}_1^*$ is the set of macrocells, which use whole bandwidth (also means that there is no interference to other cells in $\mathcal{N}_m^*$). $\mathcal{N}_2^*$ is the set of macrocells that have to use a part of bandwidth to avoid mutual interference.

After the lth iteration, the cells in $\mathcal{N}_l$ have been selected. If all the cells in $\mathcal{N}_m^* \cup \mathcal{N}_l$ are selected, the total traffic demand is satisfied. We suppose that the cells in $\mathcal{N}_l$ satisfy $w(\mathcal{N}_l)$, whereas the cells $\mathcal{N}_m^* \backslash \mathcal{N}_l$ provide the remaining traffic demand. Suppose $a_l(n)$ is the remaining traffic demand at the lth iteration, $n \in \mathcal{N}_1^* \backslash \mathcal{N}_l$. If there exist at least two cells in $\mathcal{N}' \in \mathcal{I}$ that are also in $\mathcal{N}_2^*$, we assume that $a_l(\tilde{\mathcal{N}})$ is the remaining traffic demand at the lth iteration, where $\tilde{\mathcal{N}} = \mathcal{N}' \cap \mathcal{N}_2^*$. Intuitively, if the procedure selects cell $n \in \mathcal{N}_1^*$ at the lth iteration, we have $a_{l+1}(n) = 0$. Similarly, if the procedure has selected all cells in $\tilde{\mathcal{N}}$ after the lth iteration, we also have $a_{l+1}(\tilde{\mathcal{N}}) = 0$.

First, we consider the macrocell n in $\mathcal{N}_1^*$. As mentioned earlier, if we add cell n to $\mathcal{N}_l$, the rate requirement $a_l(n)$ can be obviously satisfied. Therefore, we have

$$a_l(n) \leq w_{\mathcal{N}_l}(\{n\}). \tag{18}$$

Based on (17) and (18), at the lth iteration, it holds that

$$\begin{aligned} W_{\mathcal{N}_{l-1}}(\{n_l\}) &\geq \max_{n'\in\mathcal{N}_m} W_{\mathcal{N}_{l-1}}(\{n'\}) \\ &\geq W_{\mathcal{N}_{l-1}}(\{n\}) \\ &\geq a_{l-1}(n)/c_n. \end{aligned} \tag{19}$$

Then, we charge the cost of each cell in $\mathcal{N}_1^*$ at the lth iteration. Since $W_{\mathcal{N}_{l-1}}(\{n_l\}) = w_{\mathcal{N}_{l-1}}(\{n_l\})/c_{n_l}$, the cost of cell n in $\mathcal{N}_1^*$ at the lth iteration is

$$c_l(n) = \frac{a_{l-1}(n) - a_l(n)}{W_{\mathcal{N}_{l-1}}(\{n_l\})}.$$

The total cost of cell n is

$$\begin{aligned} \sum_{l=1}^{L} c_l(n) &= \sum_{l=1}^{L} \frac{a_{l-1}(n) - a_l(n)}{W_{\mathcal{N}_{l-1}}(\{n_l\})} \\ &\leq c_n \cdot \sum_{l=1}^{L} \frac{a_{l-1}(n) - a_l(n)}{a_{l-1}(n)} \\ &\leq c_n \cdot H(R_n^*) \\ &= c_n \cdot O(\log R_n^*) \\ &\leq c_n \cdot O(\log R) \end{aligned} \tag{20}$$

where $H(r)$ is the rth harmonic number.

Next, we consider the macrocells in $\mathcal{N}_2^*$. Similar to the former proof, we have

$$a_l(\tilde{\mathcal{N}}) \leq w_{\mathcal{N}_l}(\tilde{\mathcal{N}}) \tag{21}$$

since $\tilde{\mathcal{N}}$ can at least cover $\tilde{\mathcal{N}}$ demands in the optimum. Similar to (19), by using (17) and (21), it can be concluded that

$$W_{\mathcal{N}_{l-1}}(\{n_l\}) \geq \frac{a_{l-1}(\tilde{\mathcal{N}})}{c(\tilde{\mathcal{N}})}. \tag{22}$$

We also charge the cost of $\tilde{\mathcal{N}}$ at the lth iteration, where the cost of $\tilde{\mathcal{N}}$ at the lth iteration is

$$c_l(\tilde{\mathcal{N}}) = \frac{a_{l-1}(\tilde{\mathcal{N}}) - a_l(\tilde{\mathcal{N}})}{W_{\mathcal{N}_{l-1}}(\{n_l\})}.$$

The total cost of cell $\mathcal{N}'$ is

$$\begin{aligned}\sum_{l=1}^{L} c_l(\tilde{\mathcal{N}}) &= \sum_{l=1}^{L} \frac{a_{l-1}(\tilde{\mathcal{N}}) - a_l(\tilde{\mathcal{N}})}{W_{\mathcal{N}_{l-1}}(\{n_l\})} \\ &\leq c(\tilde{\mathcal{N}}) \cdot \sum_{l=1}^{L} \frac{a_{l-1}(\tilde{\mathcal{N}}) - a_l(\tilde{\mathcal{N}})}{a_{l-1}(\tilde{\mathcal{N}})} \\ &\leq c(\tilde{\mathcal{N}}) \cdot H\left(R_{\mathcal{N}'}^*\right) \\ &= c(\tilde{\mathcal{N}}) \cdot O\left(\log R_{\mathcal{N}'}^*\right) \\ &\leq c(\tilde{\mathcal{N}}) \cdot O(\log R).\end{aligned} \tag{23}$$

Therefore, we can conclude that Algorithm 2 achieves an approximation factor of $O(\log R)$ for macro-only cell planning problem. ■

Note that the first inequality of (23) gives a much tighter bound than the third inequality. Although we cannot get the information of the optimal solution, we can optimistically conclude that the solutions obtained by Algorithm 2 are better than the provable worst-case theoretical bound.

C. $O(\log \widetilde{R})$-*Approximation Algorithm for Cell Planning with Small Cells*

Based on the theoretical analysis on the macro-only cell planning problem, we can obtain that (18) and (21) always hold in different situations. Therefore, the key technology of designing the approximation algorithm for the cell planning with small cells is how to find the cell that satisfies (19) and (22). The difference between the macro-only case and that with small cells is that a small cell should be installed only to cover the macrocell that has been installed, which makes the problem a little more difficult to solve. We can design an approximation algorithm for the cell planning with small cells following the same idea mentioned earlier. The key issue of the algorithm is how to select cells at each iteration.

Denote $\mathcal{N}_m^*$ as the set of selected macrocells existing in the optimal solution. $\mathcal{N}_s^*$ is the set of selected small cells. Similarly, we divide $\mathcal{N}_m^*$ into two different subsets: $\mathcal{N}_1^*$ and $\mathcal{N}_2^*$. $\mathcal{N}_1^*$ is the set of macrocells that use whole bandwidth, and $\mathcal{N}_2^*$ is the set of macrocells that use a part of bandwidth. Denote $\mathcal{M}_n^*$ as the set of selected small cells that can be covered by the macrocell $n \in \mathcal{N}_1^*$. We treat macrocell n and $\mathcal{M}_n^*$ as a unity. Define $\mathcal{N}_n^* = \{n\} \cup \mathcal{M}_n^*$. Suppose that $a_l(n)$ is the remaining rate that should be covered by $\mathcal{N}_n^*$ at the lth iteration. Intuitively, if we select $\mathcal{N}_n^*$ at the lth iteration, $a_l(n)$ can be satisfied. We have

$$a_l(n) \leq w_{\mathcal{N}_l}\left(\mathcal{N}_n^*\right). \tag{24}$$

For each group $\mathcal{N}' \in \mathcal{I}_m$, where $|\mathcal{N}' \cap \mathcal{N}_2^*| \geq 2$, define $\widetilde{\mathcal{N}'} = \mathcal{N}' \cap \mathcal{N}_2^*$ and $\widetilde{\mathcal{M}_{\mathcal{N}'}^*} = \mathcal{M}_{\mathcal{N}'} \cap \mathcal{N}_s^*$. We also suppose that $a_l(\widetilde{\mathcal{N}'})$ is the remaining rate that should be covered by $\widetilde{\mathcal{N}'}$ and $\widetilde{\mathcal{M}_{\mathcal{N}'}^*}$ at the lth iteration. It can be concluded that

$$a_l(\mathcal{N}') \leq w_{\mathcal{N}_l}\left(\widetilde{\mathcal{N}'} \cup \widetilde{\mathcal{M}_{\mathcal{N}'}^*}\right). \tag{25}$$

Therefore, we try to add a set of cells G_l at the lth iteration to satisfy

$$\begin{aligned}W_{\mathcal{N}_{l-1}}(G_l) &\geq \max_{n \in \mathcal{N}_m \backslash \mathcal{N}_{l-1}} \max_{G \subseteq \mathcal{M}_n} W_{\mathcal{N}_{l-1}}(\{n\} \cup G) \\ W_{\mathcal{N}_{l-1}}(G_l) &\geq \max_{\mathcal{N}' \in \mathcal{I}_m} \max_{\substack{G_1 \subseteq \mathcal{N}' \\ G_2 \subseteq \mathcal{M}_{\mathcal{N}'}}} W_{\mathcal{N}_{l-1}}(G_1 \cup G_2).\end{aligned} \tag{26}$$

The outline of our proposed approximation algorithm for cell planning with small cells is described in Table IV. We also select a candidate cell (or cells) to increase the system capacity with the least cost. That is, the cell with the maximum average capacity per unit deployment cost is selected with priority. Different from Algorithm 2, we also need to search all possible combinations of small cells of which can be covered by each unselected macrocell at each iteration. Based on the following theorem, for each interference group, we only search one macrocell and all possible combinations of small cells.

Lemma 2: Given a subset $\mathcal{N}'$ in $\mathcal{I}_m$, if the subset of macrocells $G_1 \subseteq \mathcal{N}'$ with small cells $G_2 \subseteq \mathcal{M}_{\mathcal{N}'}$ can maximize $W = W_{\mathcal{N}_{l-1}}(G_1 \cup G_2)$ at the lth iteration, then it holds $|G_1| = 1$.

TABLE IV
$O(\log \widetilde{R})$-APPROXIMATION ALGORITHM FOR CELL PLANNING WITH SMALL CELLS

Algorithm 3

Initialization: $\mathcal{N}_l \leftarrow \emptyset$, $l = 0$.
while $w(\mathcal{N}_l) < \sum_{k \in \mathcal{K}} R_k^{min}$ **do**
  $l = l + 1, W = 0$;
  for all $n \in \mathcal{N}_m \backslash \mathcal{N}_{l-1}$ **do**
    for all $G \in \mathcal{M}_n$ **do**
      Calculate $W_n = W_{\mathcal{N}_{l-1}}(\{n\} \cup G)$;
      if $W_n > W$
        $G_l = \{n\} \cup G$, $W = W_n$;
      end if
    end for
  end for
  for all $n \in \mathcal{N}_m \cap \mathcal{N}_{l-1}$ **do**
    for all $n' \in \mathcal{N}_s \backslash \mathcal{N}_{l-1}$ **do**
      Calculate $W_{n'} = W_{\mathcal{N}_{l-1}}(\{n'\})$;
      if $W_{n'} > W$
        $G_l = \{n'\}$, $W = W_n$;
      end if
    end for
  end for
  $\mathcal{N}_l \leftarrow \mathcal{N}_{l-1} \cup G_l$;
end while
return $\mathcal{N}_l$

Proof: For the proof, see the Appendix. ■

Lemma 3: If macrocell n has been selected before the lth iteration and $G^* = \arg\max_{G\subseteq\mathcal{M}_n\backslash N_{l-1}} W_{\mathcal{N}_{l-1}}(G)$ at the lth iteration, then it holds $|G^*| = 1$.

Proof: By using Fact 1, for all $G \subseteq \mathcal{M}_n\backslash N_{l-1}$, we have

$$\begin{aligned}\max_{n\in\mathcal{M}_n\backslash N_{l-1}} W_{\mathcal{N}_{l-1}}(\{n\}) &\geq \frac{\sum_{n\in G} w_{\mathcal{N}_{l-1}}(\{n\})}{\sum_{n\in G} c_n} \\ &\geq \frac{w_{\mathcal{N}_{l-1}}(G)}{\sum_{n\in G} c_n} \\ &\geq W_{\mathcal{N}_{l-1}}(G).\end{aligned}$$

■

According to Lemma 3, we only need to search each unselected small cell that can be covered by the selected macrocells at each iteration.

For each $n \in \mathcal{N}_m^*$, redefine R_n^* as the total transmission rate of cell n, where

$$R_n^* = \sum_{n'\in\mathcal{N}_n^*}\sum_{k\in\mathcal{K}} r_{k,n}^*.$$

Similar to single macrocell, redefine $R_{\mathcal{N}'}^*$ as the total transmission rate of group $\mathcal{N}' \in \mathcal{I}_m$, i.e.,

$$R_{\mathcal{N}'}^* = \sum_{n\in\mathcal{N}'\cap\mathcal{N}_m^*}\sum_{k\in\mathcal{K}} r_{k,n}^* + \sum_{n'\in\mathcal{M}_{\mathcal{N}'}^*}\sum_{k\in\mathcal{K}} r_{k,n}^*.$$

Each macrocell $n \in \mathcal{N}_m^*$ sustains a rate R_n^* at the employment cost of c_n. Each group $\mathcal{N}' \in \mathcal{I}_m$ sustains a rate $R_{\mathcal{N}'}^*$ at the cost of $c(\mathcal{N}')$.

Theorem 2: Algorithm 3 achieves an approximation factor of $O(\log\widetilde{R})$ for cell planning problem with small cells, where

$$\widetilde{R} = \max\left\{\max_{\mathcal{N}'\in\mathcal{I}_m} w(\mathcal{N}'\cup\mathcal{M}_{\mathcal{N}'}), \max_{n\in\mathcal{N}_m} w(\{n\}\cup\mathcal{M}_n)\right\}.$$

Proof: We consider the macrocell $n \in \mathcal{N}$ that does not belong to any $\mathcal{N}' \subseteq \mathcal{I}$. As discussed in Lemma 2, we can similarly prove that $a_l(n) \leq w_{\mathcal{N}_l}(\mathcal{N}_n^*)$ and $a_l(n) \leq w_{\mathcal{N}_l\backslash\{n\}\cup\mathcal{M}_n}(\mathcal{N}_n^*)$ always hold. Therefore, we have

$$W_{\mathcal{N}_{l-1}}(G_l) \geq \frac{a_{l-1}(n)}{c(\mathcal{N}_n^*)}.$$

The total cost of $\mathcal{N}_n^*$ is

$$\begin{aligned}\sum_{l=1}^{L} c_l(n) &= \sum_{l=1}^{L}\frac{a_{l-1}(n)-a_l(n)}{W_{\mathcal{N}_{l-1}}(G_l)} \\ &\leq c(\mathcal{N}_n^*)\cdot H(R_n^*) \\ &\leq c(\mathcal{N}_n^*)\cdot O(\log\widetilde{R}).\end{aligned} \tag{27}$$

Next, we consider the group $\mathcal{N}' \in \mathcal{I}$ that has at least two macrocells in the optimum. Similar to the proof given earlier, we have

$$W_{\mathcal{N}_{l-1}}(G_l) \geq \frac{a_{l-1}(\widetilde{\mathcal{N}'})}{c\left(\widetilde{\mathcal{N}'}\cup\widetilde{\mathcal{M}_{\mathcal{N}'}^*}\right)}.$$

The total cost of $\mathcal{N}'$ and $\mathcal{M}_{\mathcal{N}'}^*$ is

$$\begin{aligned}\sum_{l=1}^{L} c_l(\mathcal{N}') &= \sum_{l=1}^{L}\frac{a_{l-1}(\widetilde{\mathcal{N}'})-a_l(\widetilde{\mathcal{N}'})}{W_{\mathcal{N}_{l-1}}(G_l)} \\ &\leq c\left(\widetilde{\mathcal{N}'}\cup\widetilde{\mathcal{M}_{\mathcal{N}'}^*}\right)\cdot H(R_{\mathcal{N}'}^*) \\ &\leq c\left(\widetilde{\mathcal{N}'}\cup\widetilde{\mathcal{M}_{\mathcal{N}'}^*}\right)\cdot O(\log\widetilde{R}).\end{aligned} \tag{28}$$

In summary, Algorithm 3 achieves an approximation ratio of $O(\log\widetilde{R})$ for the cell planning with small cells. ■

D. Complexity Analysis and Further Discussion

The complexity of Algorithm 2 can be counted as follows. We need to calculate the maximum capacity gain brought by each remaining macrocell at each iteration. Therefore, the computational complexity of each iteration is $O(UKN_m^2)$. The overall complexity of Algorithm 2 is bounded by $O(LUKN_m^2)$. The storage complexity is bounded by $O(KN_m)$ as mentioned in Section III.

Different from Algorithm 2, we need to calculate the maximum capacity gain brought by the remaining macrocell n and the subset of $\mathcal{M}_n$ by exhaustive search at each iteration. Since we have assumed that $|\mathcal{M}_n|$ is limited to a constant in the considered model, the computational complexity of each iteration is bounded by $O(UKN^2)$. The overall complexity of Algorithm 3 is also bounded by $O(LUKN^2)$. Again, the storage complexity is bounded by $O(KN)$. Note that we have to search all possible combinations of small cells to work out the optimal subset $G^* \subseteq \mathcal{M}_n$ that can maximize $W_{\mathcal{N}_{l-1}}(\{n\}\cup G)$, which costs huge computational load. To reduce computational load, we can adopt some heuristic approaches without deteriorating the performance. For instance, we can set $G = \mathcal{M}_n$ in the first place and calculate $W_{\mathcal{N}_{l-1}}(\{n\}\cup G)$. Then, we remove the small cell $n' \in G$ that minimizes $W_{\mathcal{N}_{l-1}}(\{n\}\cup G\backslash\{n'\})$ from the subset G. Such procedure stops when $W_{\mathcal{N}_{l-1}}(\{n\}\cup G\backslash\{n'\}) < W_{\mathcal{N}_{l-1}}(\{n\}\cup G)$. However, the worst-case theoretical bound of Algorithm 3 can no longer be guaranteed.

V. Numerical Results

We give numerical results to evaluate the performance of our proposed algorithms. The system parameters, such as the path-loss model, maximum transmission power, system bandwidth, etc., are based on the specifications proposed in [36]. All results are averaged over 200 Monte Carlo simulations. The service area is 3×3 km^2. There are 20 candidate sites for macrocells and 100 candidate sites for small cells. Each candidate site is uniformly distributed in the deployment area. The maximum transmission power of a macrocell is 46 dBm. If the distance between two macrocells is less than 500 m, they need to use different spectrum. For a small cell, the transmission power is limited by 30 dBm. If the distance between a macrocell and a picocell is less than 800 m, the picocell can be covered by the macrocell. There are 100 DNs that are distributed uniformly in the deployment area. The required spectral efficiency $\mathrm{SE}_k^{\min}$ is 1 bit/Hz, and the rate requirement $R_k^{\min}$ is 1 Mb/s for each DN. Path loss (in decibels) from the macrocell to a DN

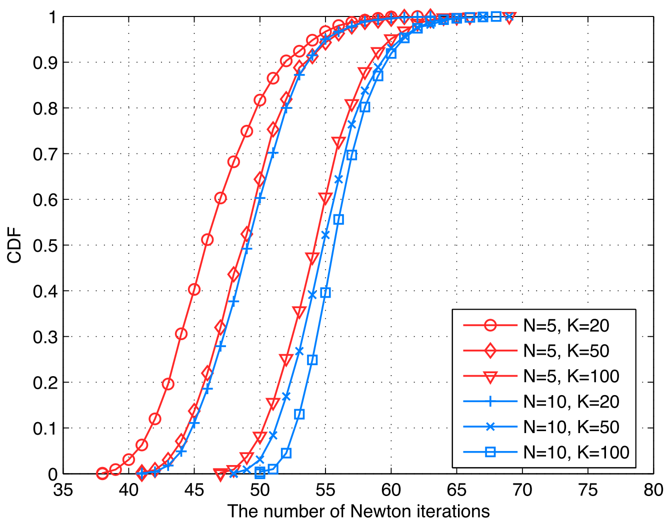

Fig. 1. CDF of the number of Newton iterations required for convergence. $t^{(0)} = 0.1$, $\epsilon_b = \epsilon_n = 10^{-3}$, $\mu = 10$, $\alpha = 0.01$, and $\beta = 0.1$.

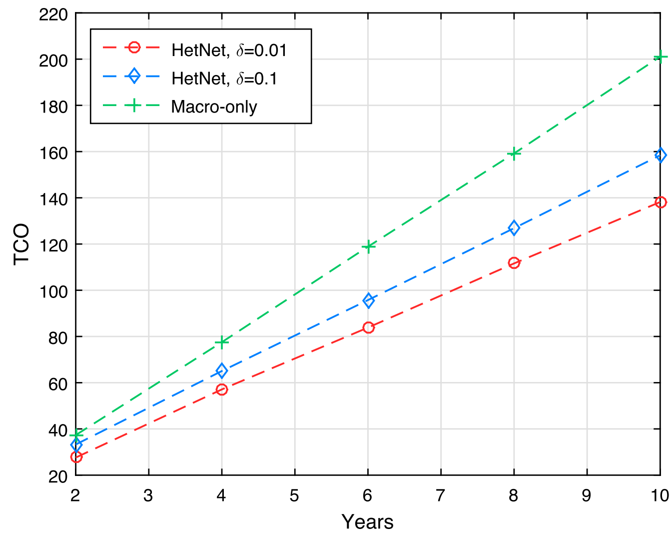

Fig. 2. Average TCO as a function of years. $|\mathcal{N}_m| = 20$, $|\mathcal{N}_s| = 100$, $K = 100$, $\text{SE}_k^{\min} = 1$ bit/Hz, and $R_k^{\min} = 1$ Mbps for all k, $P_n^{\max} = 40$ W $\forall n \in \mathcal{N}_m$, $P_n^{\max} = 1$ W $\forall n \in \mathcal{N}_s$, and $B_m = B_s = 100$ MHz.

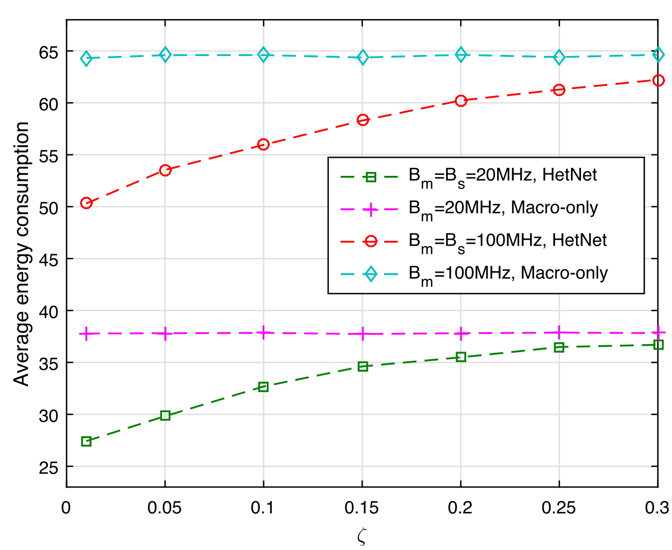

Fig. 3. Average energy consumption as a function of ζ. $|\mathcal{N}_m| = 20$, $|\mathcal{N}_s| = 100$, $K = 100$, $\text{SE}_k^{\min} = 1$ bit/Hz and $R_k^{\min} = 1$ Mbps for all k, $P_n^{\max} = 40$ W $\forall n \in \mathcal{N}_m$, $P_n^{\max} = 1$ W $\forall n \in \mathcal{N}_s$.

is calculated as $128.1 + 37.6 \log_{10}(D)$, whereas the path loss from a small cell to a DN is $140.7 + 36.7 \log_{10}(D)$, where D is the distance between transmitter and receiver (in kilometers), and the standard deviation of lognormal shadowing is 10 dB. The noise power spectral density is -184 dBm/Hz and Γ is set to 7.6288 (BER $= 10^{-6}$).

First, we investigate the convergence of our proposed algorithm. Fig. 1 gives the cumulative distribution function (cdf) of the number of Newton iterations. Some key parameters are set as follows: $t^{(0)} = 0.1$, $\epsilon_b = \epsilon_n = 10^{-3}$, $\mu = 10$, $\alpha = 0.01$, and $\beta = 0.1$. As shown in Fig. 1, 95% of Newton iterations are less than 60 for all cases. Moreover, the number of Newton iterations only increases slightly as the increasing of DNs and cells. We can conclude that our proposed barrier method works effectively and efficiently even for large-scale wireless systems.

Then, we study the average TCO versus operating years. For each macrocell, the CAPEX is distributed uniformly in an interval of (8, 12), and the OPEX per year is distributed uniformly in an interval of (1.6, 2.5). For each small cell, the CAPEX and OPEX per year are distributed uniformly in intervals of $(8\delta, 12\delta)$ and $(1.6\delta,\ 2.5\delta)$, respectively, where δ is the average cost ratio of a small cell to a macrocell. The available bandwidth of each cell is 100 MHz. We can observe in Fig. 2 that the average TCO of HetNet is lower than that of the macro-only scheme. Our proposal can save about 20% and 30% TCO for the cases of $\delta = 0.1$ and $\delta = 0.01$. Intuitively, the TCO rapidly increases with longer operating years, as shown in Fig. 2. In the end, cell planning with small cells is a cost-efficient way to keep up with the increasing traffic demands.

We also investigate the changes of the average energy consumption for different ζ, where ζ is the average energy consumption ratio of a small cell to a macrocell. The energy consumption of a macrocell is set to 10. As shown in Fig. 3, the cell planning scheme with small cells is the most energy-efficient scheme to satisfy the rate requirements of all users, even for the case that the energy consumption of small cell is relatively high compared with a macrocell (Note that ζ is usually far less than 0.1 in a HetNet). We can observe in Fig. 3 that the average energy consumption of the HetNet is lower than that of the macro-only scheme for different available bandwidth. Intuitively, more small cells can be selected when ζ decreases. As a result, spatial spectrum reuse efficiency is improved, which can satisfy more DNs at a relative low energy. When $\zeta \geq 0.2$, the performance gap between HetNet scheme and others becomes moderate. It is reasonable because a large ζ means that the energy consumption of a small cell certainly increases to obtain capacity gain benefitting from spatial spectrum efficiency. We can conservatively conclude that our proposal is energy and cost efficient for next-generation wireless systems.

VI. Conclusion

In this paper, we have studied the cell planning problem in the heterogeneous cellular network. Given a candidate set of macrocells and small cells, the cell planning task for the HetNet is to select a subset of cells with the minimum TCO to fulfill the traffic demands of all DNs. We first tried to maximize the sum rate of a given subset of cells while keeping the achievable rate of each DN below its rate requirement. We used a general transformation to yield an equivalent convex problem and employed a fast barrier method to address it efficiently. We then proposed an $O(\log R)$-approximation algorithm to solve macro-only cell planning. Based on the analysis on the approximation algorithm, we further developed an $O(\log \widetilde{R})$-approximation algorithm for HetNet cell planning. Numerical results are given and validate the capacity effectiveness of the HetNet and the efficiency of our proposed approximation algorithm, throwing some insights on how to plan HetNets. However, although we discussed the interference management issue briefly, such an important topic should be extensively investigated in future work.

APPENDIX
PROOF OF LEMMA 2

Consider the case of $G_2 \neq \emptyset$, we can initiatively obtain

$$W_{\mathcal{N}_{l-1}\cup G_1}(G_2) \geq W_{\mathcal{N}_{l-1}}(G_1)$$

since other G_2 must be an empty set. Therefore, we have

$$W_{\mathcal{N}_{l-1}\cup G_2}(G_1) \leq W_{\mathcal{N}_{l-1}}(G_2).$$

Since G_1 and G_2 can maximize $W_{\mathcal{N}_{l-1}}(G_1 \cup G_2)$, we have

$$W_{\mathcal{N}_{l-1}\cup G_2}(\{n\}) \leq W_{\mathcal{N}_{l-1}\cup G_2}(G_1) \quad \forall\, n \in G_1.$$

We can conclude that $W_{\mathcal{N}_{l-1}\cup G_2}(\{n\}) \leq W_{\mathcal{N}_{l-1}}(G_2)$, $\forall\, n \in G_1$. In addition, we also have

$$\begin{aligned} W_{\mathcal{N}_{l-1}}(G_1 \cup G_2) &= \frac{w_{\mathcal{N}_{l-1}}(G_2) + w_{\mathcal{N}_{l-1}\cup G_2}(G_1)}{c(G_2) + c(G_1)} \\ &\leq \frac{w_{\mathcal{N}_{l-1}}(G_2) + \sum_{n\in G_1} w_{\mathcal{N}_{l-1}\cup G_2}(\{n\})}{c(G_2) + c(G_1)} \\ &\leq \max_{n\in G_1} \frac{w_{\mathcal{N}_{l-1}}(G_2) + w_{\mathcal{N}_{l-1}\cup G_2}(\{n\})}{c(G_2) + c_n}. \end{aligned}$$

If the number of cells of G_1 is larger than 1, we can select any one to increase $W_{\mathcal{N}_{l-1}}(G_1 \cup G_2)$. Therefore, we have $|G_1| = 1$.

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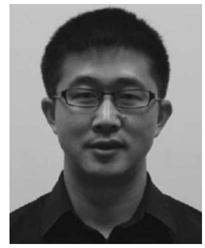

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