Efficient Algorithm for Baseband Unit Pool Planning in Cloud Radio Access Networks

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***Abstract*—Cloud Radio Access Network (C-RAN) is deemed as a promising architecture to enhance the spectrum and energy efficiency of the cellular systems. The key feature of the C-RAN is that most of signal processing functions are conducted in a centralized baseband unit (BBU) pool. As a result, the remote radio heads (RRHs) that are relatively simple can be distributed in the large network in a cost-efficient manner. In this paper, we investigate the BBU pool placement problem in the C-RAN, where we try to minimize the deployment cost of the BBUs while considering their processing capacities, the traffic demands of RRHs and the signal synchronization between RRHs and BBUs. Our general problem formulation leads to an NP-hard integer programming optimization task. We propose an efficient local search algorithm to address it. Numerical results show that the proposed algorithm converges quickly and always yields a performance-guaranteed solution to the BBU pool planning.**

***Index Terms*—BBU pool planning, C-RAN, local search.**

I. Introduction

Mobile data traffic is kept increasing quickly in recent years, and this trend will be continue at an annual rate of 66% in the next decade [1]. As a result, mobile service providers (MSPs) have to expand the network capacity to meet the requirement of the quality of service (QoS). Massive multiple input multiple output (MIMO) [2] and heterogeneous networks (HetNets) [3] are widely known approaches to enhance the system capacity of the cellular networks. However, the performance of massive MIMO is limited by correlated scattering with the antenna spacing constraints, which also brings high deployment cost to maintain the minimum spacing. The limitation of HetNets is that the inevitable co-tier and cross-tier interference could offset most of the potential capacity gain obtained by the increased number of cells. Moreover, deploying more and more small cells may suffer significant cost and burden for the MSPs. From the research of Juniper in [4], MSPs are facing cases (2014-2015) where network cost may exceed revenues if no remedial actions are taken. So these methods for capacity expansion will become far too expensive for the MSPs to keep competitive in the future.

On the other hand, cloud radio access network (C-RAN) is a novel mobile network architecture which can address these challenges efficiently [5]. The concept of the C-RAN was first proposed in [6] and described in detail in [7]. The key feature behind the C-RAN is to separate baseband processing function from the remote radio heads (RRHs) and process the baseband signals in a centralized baseband unit (BBU) pool [8]. Different from the radio access networks in conventional cellular systems where computation resource of each RRH is isolated from each other, the C-RAN centralizes most of the computation resources into the BBU pool and dispatches them flexibly according to the demand of the system, which can significantly improve the system capacity and reduce the energy consumption [9]. Moreover, MSPs can cut down capital expenditure dramatically because of the simplified RRHs.

In the C-RAN, an RRH is connected to a virtual BBU in the BBU pool through a low latency, high bandwidth optical transport link. Generally, the centralized BBUs which are equipped with high performance processors are expensive, as well as the optical fiber links between the RRHs and the BBUs. Moreover, the distance between the RRHs and the BBU pool cannot be too long because the synchronization signaling sent by the BBU control center should be sent to the RRHs simultaneously. It is reasonable even necessary to deploy multiple BBU pools in the C-RAN to provide performance guaranteed service in a large area, e.g., the metropolises. As a result, planning these BBU pools in a cost-efficient way is crucial for such a C-RAN architecture. In this paper, we study the BBU pool planning problem in the C-RAN, where we formulate a general model to illustrate the optimization task and develop efficient algorithm to address it. The main contributions of this work are summarized as follows:

- We give a general optimization framework for the C-RAN with multiple BBU pools, which covers practical scenarios in current cellular systems.
- We develop efficient algorithm to tackle the formulated integer programming problem that is NP-hard. Numerical results verify the effectiveness of our proposal.

The rest of the paper is organized as follows. In section II, we present system model and formulate the problem. The proposed algorithm is introduced in Section III. Numerical results are given in Section IV. Finally, we conclude the paper in Section V.

II. System Model and Problem Formulation

Consider a C-RAN with M RRHs. Each RRH is connected to a virtual BBU in one of the BBU pools. Denote $\mathcal{M} = \{1, 2, \ldots, M\}$ as the set of RRHs and $\mathcal{N} = \{1, 2, \ldots, N\}$ as the set of candidate BBU pools. Optical fiber cable is used to provide data transmission between the RRHs and the BBU Pools. The architecture of our considered C-RAN is shown in

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TABLE I

LIST OF TERMINOLOGIES AND SYMBOL NOTATIONS

$c_{m,n}$	Link cost between the RRH m and the BBU pool n
L	Maximum length of optical link between RRHs and the BBU pool
d_m	Traffic demand of the RRH m
f_n	Cost of the BBU pool n
$\mathcal{M}$	Set of RRHs
M	Number of RRHs
$\mathcal{N}$	Set of candidate BBU pools
N	Number of candidate BBU pools
w_n	Capacity of the BBU pool n
$x_{m,n}$	Binary variable indicates whether the RRH m is assigned to the BBU pool n or not
y_n	Binary variable indicates whether the BBU pool n is selected or not
$dist_{m,n}$	Length of optical link between the RRH m and the BBU pool n

Fig. 1. To make the rest of this paper easy to follow, frequently used symbols terminologies and symbol notations are listed in Table I.

$x_{m,n}$ is the binary variable indicates whether the RRH m is assigned to the BBU pool n or not.

$$x_{m,n} = \begin{cases} 1, & \text{RRH } m \text{ is connected to} \\ & \text{BBU pool } n; \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

The length of optical link between the RRH m and the BBU pool n is denoted as $dist_{m,n}$. Analysis shows that the distance between the BBU pool and the RRHs connected to it should not exceed the maximum distance to guarantee the latencies below a threshold that can maintain synchronization [10]. So the signal synchronization constraint for RRHs can be written as follows:

$$x_{m,n} = 0, if\ dist_{m,n} \geq L, \forall m \in \mathcal{M}, n \in \mathcal{N}. \quad (2)$$

Variable y_n indicates whether the BBU pool n is selected or not.

$$y_n = \begin{cases} 1, & \text{BBU pool n is selected;} \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

The cost function includes the cost of deploying BBU pool denoted as C_b,

$$C_b = \sum_{n \in \mathcal{N}} y_n f_n, \forall n \in \mathcal{N}. \quad (4)$$

The cost of optical link between the RRHs and the BBU pools denoted as C_l,

$$C_l = \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} x_{m,n} c_{m,n}, \forall m \in \mathcal{M}, n \in \mathcal{N}. \quad (5)$$

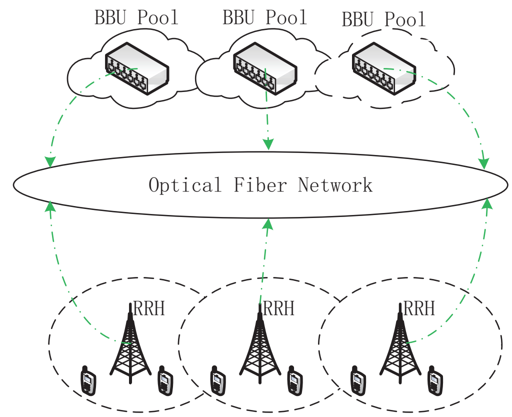

Fig. 1. Architecture of C-RAN.

The cost of installing the nth BBU pool is f_n. Linking cost between the RRH m and the BBU pool n is $c_{m,n}$.

Our main optimization task is to select a subset of the candidate sites to deploy the BBU pool to satisfy the traffic demand of all RRHs with the minimum cost. We need to determine the positions of the BBU pools and the connections between the RRHs and the BBU pools. Mathematically, the optimization problem can be formulated as follows:

$$\begin{aligned} \min_{y_n, x_{m,n}} \quad & \sum_{n \in \mathcal{N}} y_n f_n + \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} x_{m,n} c_{m,n} \\ s.t. \quad C_1: \quad & \sum_{m \in \mathcal{M}} x_{m,n} d_m \leq w_n y_n, \forall n \in \mathcal{N}, \\ C_2: \quad & x_{m,n} \leq y_n, \forall m \in \mathcal{M}, n \in \mathcal{N}, \\ C_3: \quad & x_{m,n} = 0, if\ dist_{m,n} \geq L, \forall m \in \mathcal{M}, n \in \mathcal{N}, \\ C_4: \quad & \sum_{n \in \mathcal{N}} x_{m,n} = 1, \forall m \in \mathcal{M}, \\ C_5: \quad & y_n \in \{0,1\}, \forall n \in \mathcal{N}, \\ C_6: \quad & x_{m,n} \in \{0,1\}, \forall m \in \mathcal{M}, n \in \mathcal{N}. \end{aligned} \quad (6)$$

C_1 ensures that the data sent by the RRHs can be processed by the BBU pool. C_2 means that if the RRH m is assigned to the BBU pool n then n must be open. C_3 indicates the length of optical link between the BBU pool and RRHs for signal synchronization can not exceed L. C_4 makes sure that there is only one link between a given RRH and the BBU pool. Obviously, Eq.(6) is an integer linear program problem which is NP-hard. We develop a local search based algorithm to tackle it.

III. PROPOSED ALGORITHM

If the subset $\mathcal{S} \subseteq \mathcal{N}$ for BBU pool installing is given, Eq.(6) can be transformed into the following integer linear program problem ILP ($\mathcal{S}$):

$$
\begin{aligned}
\min_{x_{m,n}} \quad & \sum_{n\in\mathcal{S}} f_n + \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{S}} x_{m,n}c_{m,n} \\
s.t. \quad C_1: \quad & \sum_{m\in\mathcal{M}} x_{m,n}d_m \le w_n, \forall n \in \mathcal{S}, \\
C_2: \quad & x_{m,n} = 0, if\ dist_{m,n} \ge L, \forall m \in \mathcal{M}, n \in \mathcal{S}, \\
C_3: \quad & \sum_{n\in\mathcal{S}} x_{m,n} = 1, \forall m \in \mathcal{M}, \\
C_4: \quad & x_{m,n} \in \{0,1\}, \forall m \in \mathcal{M}, n \in \mathcal{S}.
\end{aligned}
\tag{7}
$$

The relaxed form of (7) can be written as follows:

$$
\begin{aligned}
\min_{x_{m,n}} \quad & \sum_{n\in\mathcal{S}} f_n + \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{S}} x_{m,n}c_{m,n} \\
s.t. \quad & C_1 \sim C_3 \text{ in (7)}, \\
& x_{m,n} \ge 0, \forall m \in \mathcal{M}, n \in \mathcal{S}.
\end{aligned}
\tag{8}
$$

Eq.(8) can be solved by standard algorithm or solver such as CVX [11]. If it is feasible, the solution to (8) can be found in polynomial time. Then we can obtain the solution to (7) using the rounding procedure as follows:

$$
x_{m,n} = \begin{cases} 1 & n = \arg\max_{m^*\in\mathcal{M}} x_{m^*,n}, \\ 0 & \text{otherwise}, \end{cases} \quad \forall n \in \mathcal{N}. \tag{9}
$$

$x_{m^*,n}$ is the solution to (8). The minimum cost of deploying a subset $\mathcal{S}$ can be obtained from Eq.(7), denoted as $C(\mathcal{S})$. Therefore, the BBU pool planning is simplified to find a subset $\mathcal{S} \subseteq \mathcal{N}$ to minimize the total cost. We introduce a local search algorithm to find the subset $\mathcal{S}$, which consists of three types of local improvement steps:

Initialization$(\mathcal{S})$**:** Give a set of $\mathcal{S} \subseteq \mathcal{N}$ as an initial solution of problem (6). To make Eq.(8) feasible, we set $\mathcal{S} = \mathcal{N}$.

Step 1: add(n): Open a BBU pool $n \in \mathcal{N} \setminus \mathcal{S}$ and reassign all the demand of RRHs to $\mathcal{S} \cup \{n\}$ by computing $x_{m,n}$ and $C(\mathcal{S} \cup \{n\})$ using the integer linear program ILP$(\mathcal{S} \cup \{n\})$. The cost of this step is defined as $C(\mathcal{S} \cup \{n\}) - C(\mathcal{S})$, which can be computed in polynomial time for each $n \in \mathcal{N} \setminus \mathcal{S}$. If $C(\mathcal{S} \cup \{n\}) - C(\mathcal{S}) < 0$, we add the BBU pool n to $\mathcal{S}$, i.e. $\mathcal{S} \leftarrow \mathcal{S} \cup \{n\}$. The first step of proposed local search algorithm is summarized in Table II.

Step 2: open$(n, \mathcal{T})$: Open a BBU Pool $n \in \mathcal{N}$ and close a set of BBU pools $\mathcal{T} \subseteq \mathcal{S} \setminus \{n\}$. Obviously, the possible combinations of $\mathcal{T}$ increase exponentially. A reasonable way to address this barrier is as follows. The open operation in [12, 13] do not compute the exact cost of the new solution but only an estimated cost which overestimates the exact cost. That is, all the traffic demands served by $\mathcal{T}$ are only reassigned to n. However, such a simplified operation cannot be applied to our formulated problem due to signal synchronization requirements. We proposed a modified open step as follows.

TABLE II
STEP 1 OF LOCAL SEARCH ALGORITHM

Step 1: **Add**(n)
repeat
for $n \in \mathcal{N} \setminus \mathcal{S}$
Calculate $C(\mathcal{S} \cup \{n\})$;
if $C(\mathcal{S} \cup \{n\}) < C(\mathcal{S})$;
$\mathcal{S} \leftarrow \mathcal{S} \cup \{n\}$;
end if
end for
until Total cost cannot be decreased

Theorem 1. *If there exists a set of BBU pools $\mathcal{T} \subseteq \mathcal{S}$ that satisfy*

$$
C(\mathcal{S} \cup \{n\} \setminus \mathcal{T}) < C(\mathcal{S} \cup \{n\}) \tag{10}
$$

there must exists a BBU pool $t \in \mathcal{T}$ that satisfies

$$
C(\mathcal{S} \cup \{n\} \setminus \{t\}) < C(\mathcal{S} \cup \{n\}) \tag{11}
$$

Proof: The proof is presented in Appendix. ■

Based on Theorem 1, it is not necessary to search all possible combinations in the set $\mathcal{T}$ because if there exists a subset of BBU pools that can decrease the total cost of BBU pool planning, there must exist a BBU pool that has lower cost. So the key idea of proposed open step is to find the BBU pool $t \in \mathcal{S}$ that satisfies (11). If no BBU pool satisfies (11), the operation cannot improve the current solution; otherwise, close the BBU pool t and repeatedly search the remaining BBU pools in $\mathcal{S}$ until no BBU pool satisfies (11). Obviously, the proposed open step can be terminated in polynomial time. The second step of proposed local search algorithm is summarized in Table III.

Step 3: close$(n, \mathcal{T})$: In this step, a BBU $n \in \mathcal{S}$ is closed and a subset of BBU pools $\mathcal{T} \subseteq \mathcal{N} \setminus \{n\}$ is opened. Similar to the proposed open step, if there exit a subset $\mathcal{T} \subseteq \mathcal{N} \setminus \{n\}$ that can decrease the total cost of the BBU pools, there must exist a BBU pool $t \in \mathcal{T}$ that can also lower the cost. The close step is as follows: Find the BBU pool $t \in \mathcal{N} \setminus \{n\}$ that can decrease the total cost of BBU pool planning; if no BBU pool exists, the operation cannot improve the current solution; otherwise, open the BBU pool t and repeatedly search the remaining BBU pools in $\mathcal{N} \setminus \mathcal{S}$ until no operation can decrease the total cost of the BBU pools. The third step is summarized in Table IV.

After the algorithm terminates, the output $\mathcal{S}$ is the local optimal and can be taken as the solution to the problem.

IV. NUMERICAL RESULTS

Some preliminary numerical results are given in this section. The service region is $20 \times 20\ km^2$. The RRHs and the BBU pools are uniform distribution. For the BBU pool n, its

TABLE III
STEP 2 OF LOCAL SEARCH ALGORITHM

Step 2: **Open**$(n, \mathcal{T})$

repeat
  Open operation
    for $n \in \mathcal{N} \setminus \mathcal{S}$
      if $C(\mathcal{S} \cup \{n\}) < C(\mathcal{S})$;
        $\mathcal{S} \leftarrow \mathcal{S} \cup \{n\}$;
      end if
    end for
  Close operation
    for $t \in \mathcal{S}$
      if $C(\mathcal{S} \cup \{n\} \setminus \{t\}) < C(\mathcal{S} \cup \{n\})$;
        $\mathcal{S} \leftarrow \mathcal{S} \setminus \{t\}$;
      end if
    end for
until Total cost cannot be decreased

TABLE IV
STEP 3 OF LOCAL SEARCH ALGORITHM

Step 3: **Close**$(n, \mathcal{T})$

repeat
  Close operation
    for $n \in \mathcal{S}$
      if $C(\mathcal{S} \setminus \{n\}) < C(\mathcal{S})$;
        $\mathcal{S} \leftarrow \mathcal{S} \setminus \{n\}$;
      end if
    end for
  Open operation
    for $t \in \mathcal{S} \setminus \{n\}$
      if $C(\mathcal{S} \setminus \{n\} \cup \{t\}) < C(\mathcal{S} \setminus \{n\})$;
        $\mathcal{S} \leftarrow \mathcal{S} \cup \{t\}$;
      end if
    end for
until Total cost cannot be decreased

capacity w_n is distributed uniformly within (80, 100). For the RRH m, its traffic demand d_m is distributed uniformly within (10, 20). The maximum length of optical link between RRHs and the BBU pool is set to 10 km. The installation cost of each BBU pool is equal to 20000. The cost of optical fiber between the RRH m and the BBU pool n is set to $1000/km$.

Fig. 2 illustrates the total cost during each iteration with different number of BBU pools. From Fig. 2 we can conclude that our proposed algorithm converges rapidly. It requires 25 iterations for the case that N = 20 and 46 iterations for the case that N = 40, respectively. Obviously, the total cost for the case that N = 40 is lower since there is more combination

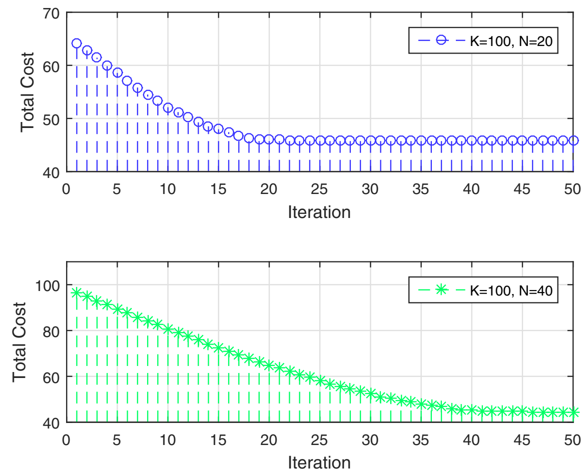

Fig. 2. Total cost during each iteration with different number of BBU pools.

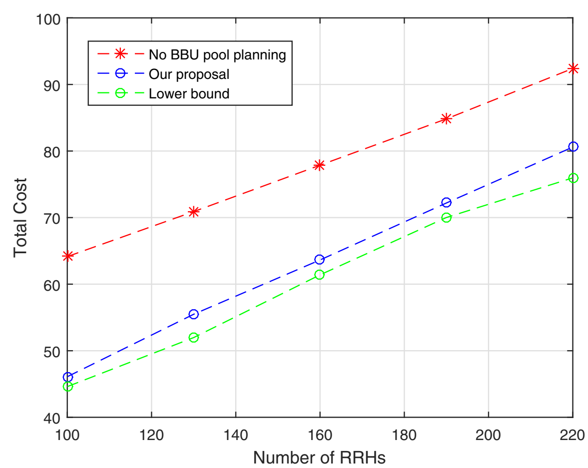

Fig. 3. Objective value of (6) as a function of different numbers of RRHs, N = 20.

set of BBU pools. As seen from Fig. 2, we can preliminarily conclude that our proposal can reduce the total cost of BBU pool planning. About 27% and 53% cost can be reduced by our proposed algorithm for the cases that N = 20 and N = 40, respectively.

Fig.3 shows the total cost as the function of the number of RRHs. The number of candidate BBU pools is 20. the numbers of RRHs M varies from 100 to 220. For comparison, the optimal solution to the relaxation of (6) is given as a lower bound. From Fig. 3, we can observe that our proposed algorithm is close to the lower bound. The gap is about 6% for the case M = 220. Note that the BBU pools are fractionally opened for the lower bound, so the lower bound is not tight. For the case that M = 100, the gap is less than 4%. We can conclude that our proposed algorithm is close to the optimum. Again, it shows our proposal can reduce the total cost of BBU pool planning in the C-RAN.

V. Conclusion

In this paper, we investigated the BBU pool planning problem in the C-RAN while considering the traffic demand and the capacity of BBU pools, as well as the signal synchronization among the RRHs. The general problem formulation yields an NP-hard optimization task. We propose a local search algorithm to solve it efficiently, where three local improvement operations are introduced to find out the locally optimal solution quickly. Numerical results show our proposed scheme can reduce the deployment cost of the BBU pools planning in the C-RAN significantly.

Appendix

If all BBU Pools in $\mathcal{T}$ do not satisfy (11), i.e.

$$C(\mathcal{S}\cup\{n\}\setminus\{t\}) \geq C(\mathcal{S}\cup\{n\}), \forall t\in\mathcal{T}, \tag{12}$$

based on the definition of $C(\mathcal{S})$, we can obtain

$$-C_b(t)+C_l(\mathcal{S}\cup\{n\}\setminus\{t\}) \geq C_l(\mathcal{S}\cup\{n\}), \forall t\in\mathcal{T}. \tag{13}$$

Inequality(13) is equivalent to

$$\sum_{t\in\mathcal{T}}[-C_b(t)+C_l(\mathcal{S}\cup\{n\}\setminus\{t\})] \geq \sum_{t\in\mathcal{T}} C_l(\mathcal{S}\cup\{n\}). \tag{14}$$

Since $\sum_{t\in\mathcal{T}} C_b(t) = C_b(\mathcal{T})$, we can obtain

$$-C_b(\mathcal{T})+\sum_{t\in\mathcal{T}}[C_l(\mathcal{S}\cup\{n\}\setminus\{t\})-C_l(\mathcal{S}\cup\{n\}] \geq 0. \tag{15}$$

Notice that $C_l(\mathcal{S}\cup\{n\}-C_l(\mathcal{S}\cup\{n\}\setminus\{t\})$ denotes the linking cost that can be saved by close t, indicating that

$$\begin{aligned} & \sum_{t\in\mathcal{T}}[C_l(\mathcal{S}\cup\{n\}-C_l(\mathcal{S}\cup\{n\}\setminus\{t\})] \\ \geq\ & C_l(\mathcal{S}\cup\{n\})-C_l(\mathcal{S}\cup\{n\}\setminus\mathcal{T}) \end{aligned} \tag{16}$$

Inequality (16) always holds because the linking cost saved by the BBU pool $t\in\mathcal{T}$ when $\mathcal{T}$ is active is always no bigger than the saved linking cost by exclusively opening the BBU pool t. Then, we have

$$\begin{aligned} & -C_b(\mathcal{T})+C_l(\mathcal{S}\cup\{n\}\setminus\mathcal{T})-C_l(\mathcal{S}\cup\{n\}) \\ \geq\ & -C_b(\mathcal{T})+\sum_{t\in\mathcal{T}}[C_l(\mathcal{S}\cup\{n\}\setminus\mathcal{T})-C_l(\mathcal{S}\cup\{n\})] \\ \geq\ & 0. \end{aligned} \tag{17}$$

With simple mathematical operations, we have

$$\begin{aligned} C(\mathcal{S}\cup\{n\}\setminus\mathcal{T}) &= C_b(\mathcal{S}\cup\{n\})-C_b(\mathcal{T}) \\ &\quad +C_l(\mathcal{S}\cup\{n\}\setminus\mathcal{T}) \\ &\geq C_b(\mathcal{S}\cup\{n\})+C_l(\mathcal{S}\cup\{n\}) \\ &= C(\mathcal{S}\cup\{n\}). \end{aligned} \tag{18}$$

Inequation (18) violates the condition(10). Therefore, there must exist a BBU pool $t\in\mathcal{T}$ that satisfies (11).

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