Energy-Efficient Resource Allocation in Cognitive Radio Systems

Weijia Shi & Shaowei Wang
School of Electronic Science & Engineering, Nanjing University, Nanjing 210093, China
E-mail: shiwj@smail.nju.edu.cn, wangsw@nju.edu.cn

Abstract—**In this paper, we investigate the energy consumption issue of Cognitive Radio (CR) systems. We aim to maximize the energy efficiency of the considered CR system with practical constraints, such as the power budget of the CR system, the interference thresholds of the primary users, the minimal throughput requirements and the proportional fairness of the CR users. Since the formulated mixed integer programming task is generally hard to tackle, we relax the original problem and convert it into a quasiconvex one. A bisection-based algorithm is employed to work out the optimal solution in an iterative manner. In each iteration, we develop a fast barrier method to reduce the computational complexity by exploiting the problem's structure. Simulation results show that our proposed method can obtain solutions close to the upper bound with reasonable complexity.**

I. INTRODUCTION

Due to the scarcity of radio spectrum and the inefficiency of the regularized spectrum usage manner, some insightful spectrum utilization schemes have been introduced to improve spectrum usage efficiency [1]. As a highly promising technique, Cognitive Radio (CR) attracts more and more attentions in recent years [2], which allows Secondary Users (SUs) to sense radio spectrum environment and dynamically adjust transmission parameters to access the licensed spectrum, as long as the interference to Primary Users (PUs) can be kept under their tolerances, such as interference temperature [3]. Orthogonal Frequency Division Multiplexing (OFDM) has been widely recognized as a fascinating air interface for CR systems due to its flexibility of allocating radio resource among SUs, which is the prerequisite for the CR system to acquire high performance [4].

Resource Allocation (RA) is one of the most important problems in OFDM-based wireless networks. An optimized RA scheme can maximize the throughput of an OFDM system, minimize the transmission power, or support more users with given Quality-of-Service (QoS) guarantee. For OFDM-based CR networks, there are many research results on how to improve the system throughput. In [5], RA for OFDM-based CR network is formulated as a multidimensional knapsack problem. A greedy heuristic algorithm is proposed, which can produce solution close to the optimal. However, the computational cost would be very high for multiple SUs case. In [6], optimal and suboptimal power loading algorithms are presented for the single SU case. Downlink sum capacity is maximized under the constraint of PUs' interference thresholds. In [7], an efficient algorithm is proposed to allocate bits among all OFDM subchannels in CR systems. The proposed algorithm can obtain the optimal solution with low computational complexity in most cases. In [8], a fast algorithm is developed to tackle the optimal power allocation problem in an OFDM-based CR network. In [9], a low complexity algorithm is proposed to maximize the throughput of a CR system while guaranteeing proportional fairness among SUs.

In contrast to the flourish on capacity enhancing, little attention has been paid to energy efficiency of the CR systems. Nowadays, excessive energy consumption becomes a critical issue because of the consequent environment problems and operational cost [9] [10]. Green communication, which emphasizes on incorporating energy awareness in communication systems, is becoming more and more important [11]. Different from the throughput-oriented RA targets, energy efficient RA aims at maximizing the energy efficiency of a wireless system. In [12], weighted energy efficiency is maximized under prescribed users' QoS requirements. In [13], a non-cooperative game is developed for energy efficient power optimization. In [14], the tradeoff between energy efficiency and spectral efficiency is investigated for the downlink of a multiuser distributed antenna system.

In this paper, we study the energy efficient RA for the OFDM-based CR systems. We try to maximize the energy efficiency of the considered system, while satisfying the throughput requirements of the SUs. Besides, the interference introduced to the PUs is kept below a tolerable threshold to prohibit the unacceptable performance degradation of the PUs. Proportional fairness among users is also considered. The formulated optimization task is computationally intractable since it is a mixed integer programming problem. To make it tractable, we relax the problem by using time-sharing method to convert it into a quasiconvex optimization one. A bisection-based algorithm is developed to work out the optimal solution. We also exploit the structure of the problem and derive a fast barrier method to reduce the computational complexity.

The rest of this paper is organized as follows. In Section II, we illustrate system model and formulate optimization task. In Section III, we proposed a fast barrier-based bisection method to solve the quasiconvex optimization problem. In Section IV, we give an integer subchannel assignment scheme and an optimal power distribution. Simulation results are given in Section V, as well as discussions. We conclude the paper in Section VI.

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II. System Model and Problem Formulation

A. System Model

Consider the downlink of an OFDM-based CR system with K SUs, denoted by $\mathcal{K} = \{1, 2, \ldots, K\}$, coexisting with L active PUs served a licensed system. The available bandwidth W is divided into N OFDM subchannels in the CR system, denoted as $\mathcal{N} = \{1, 2, \ldots, N\}$. The bandwidth of the nth subchannel spans from $f_0 + (n-1)W/N$ to $f_0 + nW/N$, where f_0 is the starting frequency. The lth PU's nominal band ranges from f_l to f_l+B_l, where f_l and B_l are the starting frequency and the lth PU's occupied bandwidth, respectively.

In this paper we assume that perfect channel-state information is available at the transceivers of the SUs and the PUs. The interference introduced to the lth PU by SUs' access on the nth subchannel with unit transmission power can be represented as follows [15]

$$I_{n,l}^{SP} = \int_{f_l - f_0 - (n-1/2)W/N}^{f_l + B_l - f_0 - (n-1/2)W/N} g_{n,l}^{SP} \phi_n^{SU}(f) df, \quad (1)$$

where $g_{n,l}^{SP}$ is the power gain from the SU's transmitter to the lth PU's receiver on the nth subchannel. $\phi_n^{SU}(f)$ is the Power Spectrum Density (PSD) of the OFDM subchannel used by an SU, which can be expressed as $\phi_n^{SU}(f) = T_s(\frac{\sin \pi f T_s}{\pi f T_s})^2$, where T_s is OFDM symbol duration. On the other hand, the interference generated by the lth PU into the nth subchannel used by the kth SU is

$$I_{k,n,l}^{PS} = \int_{f_0 + (n-1)W/N - f_l - B_l/2}^{f_0 + nW/N - f_l - B_l/2} g_{k,n,l}^{PS} \phi_l^{PU}(f) df, \quad (2)$$

where $g_{k,n,l}^{PS}$ is the power gain from the lth PU's transmitter to the kth SU's receiver on the nth subchannel and $\phi_l^{PU}(f)$ is the PSD of the lth PU's signal. Note that we do not assume that the PUs also adopt OFDM modulation.

Define the Signal-to-Noise Ratio (SNR) of the kth SU on the nth subchannel as

$$H_{k,n} = \frac{g_{k,n}^{SS}}{\Gamma(N_0 B + \sum_{l=1}^{L} I_{k,n,l}^{PS})}, \quad (3)$$

where $g_{k,n}^{SS}$ is the power gain of the kth SU on the nth subchannel, N_0 is the PSD of additive white Gaussian noise, Γ is the SNR gap and can be represented as $\Gamma = -\frac{\ln(5BER)}{1.5}$ for an uncoded MQAM with a specified BER [16]. The transmission rate of the nth subchannel used by the kth SU is

$$r_{k,n} = \rho_{k,n} \log(1 + p_{k,n} H_{k,n}), \quad (4)$$

where $p_{k,n}$ is the kth SU's transmission power on the nth subchannel, $\rho_{k,n}$ can only be either 1 or 0, indicating whether the nth subchannel is used by the kth SU or not. The data rate of the kth SU, denoted as R_k, can be represented as

$$R_k = \sum_{n=1}^{N} r_{k,n}, \quad (5)$$

We define the energy-efficiency η_{EE} as the ratio of the system throughput over the total power consumption

$$\eta_{EE} = \frac{\sum_{k=1}^{K} R_k}{\sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} p_{k,n} + P_c}, \quad (6)$$

where P_c is the circuit power consumption which can be a constant [17].

B. Problem Formulation

The optimization objective is to maximize the energy-efficiency of the CR system which operate in a power-limited situation while keeping the level of interference to the PUs not exceeding the specified thresholds. Besides, the CR system requires a minimal transmission rate and proportional fairness among users. Our optimization problem thus can be formulated as follows,

$$\begin{aligned} &\max_{r_{k,n}, \rho_{k,n}} \eta_{EE} \\ s.t. \quad & \text{C1}: \textstyle\sum_{n=1}^{N} \sum_{k=1}^{K} r_{k,n} \geq R_{min} \\ & \text{C2}: \textstyle\sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} p_{k,n} \leq P_t \\ & \text{C3}: \textstyle\sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} p_{k,n} I_{n,l}^{SP} \leq I_l^{th}, \forall l \\ & \text{C4}: r_{k,n} \geq 0, \forall k, n \\ & \text{C5}: \rho_{k,n} \in \{0, 1\}, \forall k, n \\ & \text{C6}: \textstyle\sum_{k=1}^{K} \rho_{k,n} = 1, \forall n \\ & \text{C7}: R_1 : R_2 : \cdots : R_K = \gamma_1 : \gamma_2 : \cdots : \gamma_K, \end{aligned} \quad (7)$$

where P_t is the power limit of the access point in the CR system and I_l^{th} is the interference power threshold of the lth PU. C1 is the throughput requirement of the CR system. C2 and C3 are the power limitation and the interference constraint, respectively. C5 means that an OFDM subchannel can be used by only one SU. C5 and C6 indicate that a subchannel is not shared among SUs. C7 is the proportional rate constraints of the SUs.

III. The Quasiconvex Optimization

Since both binary variables $\rho_{k,n}$'s and real variables $r_{k,n}$'s are involved in the (7), it defines a mixed integer programming problem. An intuitive way to tackle such kind of problems is the time-sharing method, which relaxes integer variables into continuous ones. Redefine the variable $\rho_{k,n} \in [0,1]$ as the faction of the nth subchannel allocated to the kth SU, the transmission power and the transmission rate of the nth subchannel used by the kth SU can be represented as $\rho_{k,n} p_{k,n}$ and $\rho_{k,n} \log(1 + p_{k,n} H_{k,n})$, respectively. The (7) can be converted into the following form,

$$\begin{aligned} &\max_{r_{k,n}, \rho_{k,n}} \eta_{EE} = \frac{\sum_{n=1}^{N} \sum_{k=1}^{K} r_{k,n}}{\sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} p_{k,n} + P_c} \\ s.t. \quad & 0 < \rho_{k,n} \leq 1, \forall k, n \\ & C1 \sim C4,\ C6 \sim C7 \text{ in the (7)} \end{aligned} \quad (8)$$

Theorem 1: The function η_{EE} is strictly quasiconcave in $\mathbf{x}$, where $\mathbf{x} = (r_{1,1}, \rho_{1,1}, r_{1,2}, \cdots, \rho_{K,N})$.

Proof: Denote the superlevel sets of $\eta_{EE}(\mathbf{x})$ as

$$S_\alpha = \{\mathbf{x} \in \textbf{dom}\ \eta_{EE} \mid \eta_{EE} \geq \alpha\}, \tag{9}$$

for $\alpha \in \mathcal{R}$, $\eta_{EE}(\mathbf{x})$ is strictly quasiconcave in $\mathbf{x}$ if and only if S_α is strictly convex for any real number α [18]. When $\alpha < 0$, no points exist in $\eta_{EE} < \alpha$ and $S_\alpha = \{\mathbf{x} \in \textbf{dom}\ \eta_{EE}\}$. When $\alpha > 0$, $S_\alpha = \{\mathbf{x} \in \textbf{dom}\ \eta_{EE} \mid \sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n} - \alpha(\sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}} + P_c) \geq 0\}$. Obviously, S_α is strictly convex in $\mathbf{x}$. ■

Furthermore, the (8) is a quasiconvex optimization problem because the inequality constraint functions in the (8) are all convex while the equality constraint functions are affine [18].

A. Quasiconvex optimization via convex feasibility problems

A general approach to the quasiconvex optimization problems relies on the representation of the sublevel sets of a quasiconvex function via a family of convex inequalities. Denote $f(\mathbf{x}) = -\eta_{EE}(\mathbf{x})$ and let $\varphi_\tau(\mathbf{x})$ be a family of convex functions that satisfy

$$f(\mathbf{x}) \leq \tau \Longleftrightarrow \varphi_\tau(\mathbf{x}) \leq 0, \tag{10}$$

we have

$$\varphi_\tau(\mathbf{x}) = -\sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n} - \tau(\sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}} + P_c), \tag{11}$$

where $\tau \leq 0$, φ_τ is convex for each τ and decreasing in τ.

Denote f^* as the optimal solution of the quasiconvex problem. If the following feasibility problem

$$\begin{array}{ll} find & \mathbf{x} \\ s.t. & \varphi_\tau(\mathbf{x}) \leq 0 \\ & C1 \sim C6 \ \ in \ \ (8) \end{array} \tag{12}$$

has feasible solutions, we have $f^* \leq \tau$; otherwise $f^* \geq \tau$. The (12) can be solved by a simple bisection algorithm. In each iteration of the bisection method, we need to solve a feasibility problem with a given parameter τ. According to [16], we introduce a crucial indicator parameter z and formulate the following minimization problem to check the feasibility of the (12)

$$\begin{aligned} &\min_{\mathbf{x},z} e^z \\ s.t.\quad & \text{C1}: \varphi_\tau(\mathbf{x}) \leq z \\ & \text{C2}: \textstyle\sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n} \geq R_{min} \\ & \text{C3}: \textstyle\sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}} \leq P_t + z \\ & \text{C4}: \textstyle\sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}} I_{n,l}^{SP} \leq I_l^{th} + z, \forall l \\ & \text{C5}: r_{k,n} \geq 0, \forall k,n \\ & \text{C6}: 0 < \rho_{k,n} \leq 1, \forall k,n \\ & \text{C7}: \textstyle\sum_{k=1}^{K} \rho_{k,n} = 1, \forall n \\ & \text{C8}: \textstyle\sum_{n=1}^{N} r_{k,n} = \beta_k \sum_{n=1}^{N} r_{1,n}, \end{aligned} \tag{13}$$

where $\beta_k = \frac{\gamma_k}{\gamma_1}$, $k = 2, 3, \cdots K$. The variable z can be interpreted as an upper bound of the maximum infeasibility of the inequalities as seen from the (13).

B. Fast barrier method

Obviously, the (13) defines a convex optimization problem because the objective function and the constraints are all concave or convex. Generally, barrier method is a standard technique to solve convex optimization problems. For the barrier method, original problem is converted into a sequence of unconstrained minimization problems by defining a logarithmic barrier function with parameter t which decides the accuracy of the approximation to the original problem. As t increases, the approximation will be more and more close to the optimal solution.

First, we reformulate the problem into a set of unconstrained optimization problems by making all inequality constraints implicit in an objective function. The logarithmic barrier function is

$$\begin{aligned} \phi(\mathbf{x}, z) = \quad & -\log(z - \varphi_\tau(\mathbf{x})) - \log(\sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n} - R_{min}) \\ & - \sum_{l=1}^{L} \log(I_l^{th} - \sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}} I_{n,l}^{SP} + z) \\ & - \log(P_t - \sum_{n=1}^{N}\sum_{k=1}^{K} \rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}} + z) \\ & - \sum_{n=1}^{N}\sum_{k=1}^{K} (\log r_{k,n} + \log \rho_{k,n} + \log(1 - \rho_{k,n})). \end{aligned} \tag{14}$$

Thus, the optimal solution to the (13) can be approximated by solving the following minimization problem with a certain parameter t

$$\begin{array}{ll} & \min \psi_t(\mathbf{x}, z) = te^z + \phi(\mathbf{x}, z) \\ s.t. & A\mathbf{x} = \mathbf{b} \end{array} \tag{15}$$

where A is an $(N + K - 1) \times (2KN + 1)$ matrix and $\mathbf{b} \in$

$\mathcal{R}^{\mathcal{N}+\mathcal{K}+\mathbf{1}}$ with

$$A_{i,j}=\begin{cases}1, & j=2(k-1)N+2i,\ i=1,\cdots,N,\ \forall k\\ \beta_k, & j=2n-1, i=N+1,\cdots,N+K-1,\\ & k=2,\cdots,K,\ \forall n\\ -1, & j=2(k-1)N+2(n-1),\\ & i=N+1,\cdots,N+K-1,\\ & k=2,\cdots,K,\ \forall n\\ 0, & \text{otherwise},\end{cases} \tag{16}$$

$$\mathbf{b}_i=\begin{cases}1, & i=1,2,\cdots,N\\ 0, & i=N+1,N+2,\cdots,N+K-1.\end{cases} \tag{17}$$

The (15) has only equality constraints and can be solved efficiently by Newton method, where Newton step $\Delta\mathbf{x}_{nt}$ and the associated dual variables ν are given by the following Karush-Kuhn-Tucker (KKT) systems:

$$\begin{bmatrix}\nabla^2\psi_t(\mathbf{x},z) & A^T\\ A & \mathbf{0}_n\end{bmatrix}\begin{bmatrix}\Delta\mathbf{x}_{nt}\\ \nu\end{bmatrix}=\begin{bmatrix}-\nabla\psi_t(\mathbf{x},z)\\ \mathbf{0}_\nu\end{bmatrix}, \tag{18}$$

where $\Delta\mathbf{x}_{nt}\in\mathcal{R}^{2\mathcal{K}\mathcal{N}+1}$, $\mathbf{0}_n\in\mathcal{R}^{(\mathcal{N}+\mathcal{K}-1)\times(\mathcal{N}+\mathcal{K}-1)}$, $\mathbf{0}_\nu\in\mathcal{R}^{\mathcal{N}+\mathcal{K}-1}$.

The complexity of the barrier method mainly lies in the computation of Newton step that needs matrix inversion. In order to reduce the computational cost, we exploit the structure of the (15) and develop a fast algorithm to compute the Newton step with lower complexity. Denote

$$\begin{aligned}g_0&=z-\varphi_\tau(\mathbf{x}),\\ f_0&=P_t-\sum_{n=1}^{N}\sum_{k=1}^{K}\rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}}+z,\\ f_1&=\sum_{n=1}^{N}\sum_{k=1}^{K}r_{k,n}-R_{min},\\ g_l&=I_l^{th}-\sum_{n=1}^{N}\sum_{k=1}^{K}\rho_{k,n}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}-1}{H_{k,n}}I_{n,l}^{SP}+z.\end{aligned} \tag{19}$$

The Hessian of $\psi_t(\mathbf{x},z)$ is

$$\begin{aligned}\nabla^2\psi_t(\mathbf{x},z)&=\begin{bmatrix}D_{1,1} & & & \\ & \ddots & & \\ & & D_{k,n} & \\ & & & Z\end{bmatrix}\\ &+\frac{\nabla f_0\nabla f_0^T}{f_0^2}+\frac{\nabla g_0\nabla g_0^T}{g_0^2}+\sum_{l=1}^{L}\frac{\nabla g_l\nabla g_l^T}{g_l^2}\\ &=D+\sum_{i=1}^{L+2}F_iF_i^T.\end{aligned} \tag{20}$$

where $D=\text{diag}(D_{1,1},D_{1,2},\cdots,D_{k,n},Z)$ with

$$\begin{aligned}D_{k,n}&=\begin{bmatrix}D_{k,n}^1 & D_{k,n}^2\\ D_{k,n}^2 & D_{k,n}^3\end{bmatrix},\\ Z&=te^z,\end{aligned} \tag{21}$$

where

$$\begin{aligned}D_{k,n}^1=&\ (\tfrac{1}{f_0}-\tfrac{\tau}{g_0})\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}}{H_{k,n}\rho_{k,n}}+\sum_{l=1}^{L}\frac{I_{n,l}^{SP}}{g_l}\frac{e^{\frac{r_{k,n}}{\rho_{k,n}}}}{H_{k,n}\rho_{k,n}}\\ &+\tfrac{1}{r_{k,n}^2},\\ D_{k,n}^2=&\ -(\tfrac{1}{f_0}-\tfrac{\tau}{g_0})\frac{r_{k,n}e^{\frac{r_{k,n}}{\rho_{k,n}}}}{H_{k,n}\rho_{k,n}^2}-\sum_{l=1}^{L}\frac{I_{n,l}^{SP}}{g_l}\frac{r_{k,n}e^{\frac{r_{k,n}}{\rho_{k,n}}}}{H_{k,n}\rho_{k,n}^2},\\ D_{k,n}^3=&\ (\tfrac{1}{f_0}-\tfrac{\tau}{g_0})\frac{r_{k,n}^2e^{\frac{r_{k,n}}{\rho_{k,n}}}}{H_{k,n}\rho_{k,n}^3}+\sum_{l=1}^{L}\frac{I_{n,l}^{SP}}{g_l}\frac{r_{k,n}^2e^{\frac{r_{k,n}}{\rho_{k,n}}}}{H_{k,n}\rho_{k,n}^3}\\ &+\tfrac{1}{\rho_{k,n}^2}+\tfrac{1}{(1-\rho_{k,n})^2},\end{aligned} \tag{22}$$

and F_i with

$$F_i=\begin{cases}\frac{\nabla f_0}{f_0}, & i=1\\ \frac{\nabla f_1}{f_1}, & i=2\\ \frac{\nabla g_0}{g_0}, & i=3\\ \frac{\nabla g_l}{g_l}, & l=1,\cdots,L, i=l+3.\end{cases} \tag{23}$$

It is easy to prove that the matrix D is positive definite, and it follows that the Hessian matrix $\nabla^2\psi_t(\mathbf{x},z)$ is invertible. However, if we compute the inversion of the KKT matrix directly, it has a complexity of $O((2KN+N+K)^3)$, which is too high for application in practical wireless systems.

Theorem 2: The equation (11) can be solved with the complexity of $O(M^2KN)$, where $M=L+3$.

The proof is given in Appendix. In practical CR systems, $M\ll N$ generally holds, so the complexity of the algorithm is significantly decreased in this way.

IV. Integer Subchannel Allocation and Power Allocation

For practical wireless systems, each subchannel can be assigned to only one SU and the allocation indexes $\rho_{k,n}$'s are constrained to be 0 or 1. Considering the optimal solution of the relaxation form (8), the fractional $\rho_{k,n}$ can be regarded as the metric to determine the exact assignment of subchannels. According to [19], when $K\ll N$, only a few subcarriers are shared among users as $\rho_{k,n}$ is mostly either 1 or 0 in the optimal solution to the (8). Hence, we allocate the nth subchannel to the kth SU with the maximum $\rho_{k,n}$,

$$\rho_{k,n}^*=\begin{cases}1, & k=\text{argmax}\rho_{k,n}(k)\\ 0, & \text{otherwise}.\end{cases} \tag{24}$$

Given a subchannel assignment, the power distribution problem among all subchannels is following:

$$\begin{aligned}\max_{p_{k,n}}\ &\eta_{EE}=\frac{\sum_{k=1}^{K}\sum_{n\in\Omega_k}r_{k,n}}{\sum_{k=1}^{K}\sum_{n\in\Omega_k}p_{k,n}+P_c}\\ s.t.\quad &C1:\sum_{k=1}^{K}\sum_{n\in\Omega_k}r_{k,n}\geq R_{min}\\ &C2:\sum_{k=1}^{K}\sum_{n\in\Omega_k}p_{k,n}\leq P_t\\ &C3:\sum_{k=1}^{K}\sum_{n\in\Omega_k}p_{k,n}I_{n,l}^{SP}\leq I_l^{th},\forall l\\ &C4:p_{k,n}\geq 0,\forall k,n\\ &C5:R_1:R_2:\cdots:R_K=\gamma_1:\gamma_2:\cdots:\gamma_K.\end{aligned} \tag{25}$$

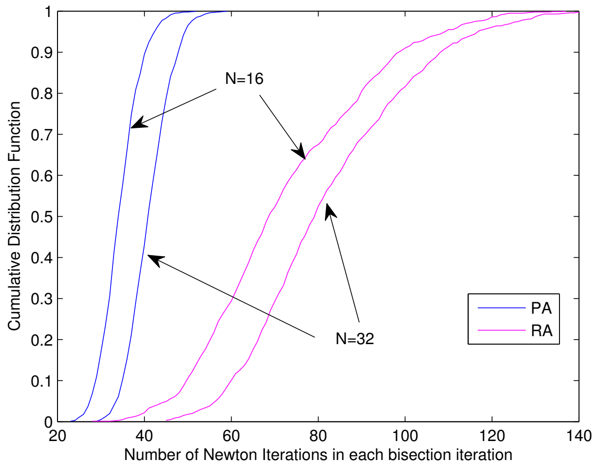

Fig. 1. CDF of the number of Newton iterations in each bisection iteration

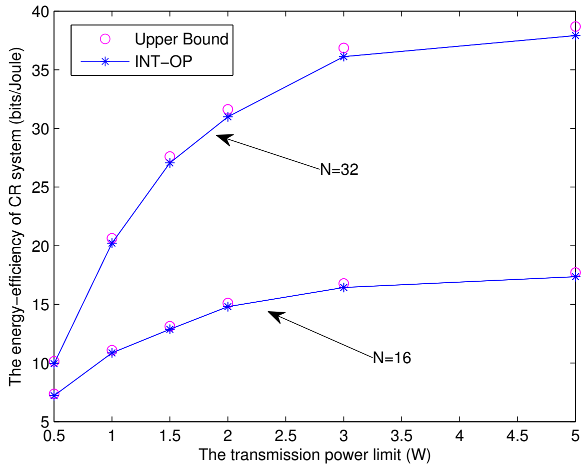

Fig. 2. The energy-efficiency of the CR system versus the transmission power limit for different numbers of subchannels.

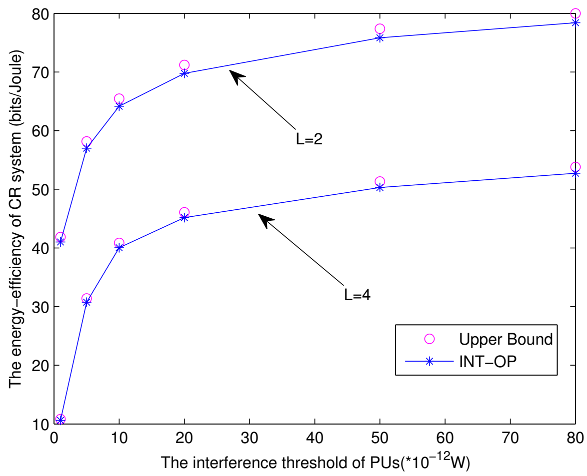

Fig. 3. The energy-efficiency of the CR system versus the interference threshold for different numbers of PUs.

The (25) is also a quasiconvex optimization problem and can be solved by employing the method developed in Section III. We omit the details for room limitation.

V. Simulation Results

Consider a multiuser OFDM-based CR system, where all users randomly locate in an area of 3×3km, and each SU's receiver is distributed in a circle within 0.5km from its transmitter. The path loss exponent is 4, the variance of the shadowing effect is 10dB, and the multipath fading is assumed to be Rayleigh. The noise power on a subchannel is set to 10^{-13} W. The frequency bands occupied by PUs are generated randomly with the maximum number of OFDM subchannels $2W/3L$.

First, we investigate the convergence of the proposed algorithms. As discussed in Section III, the computational load mainly lies in the computation of Newton step. Fig.1 gives the cumulative distribution function (CDF) of Newton iterations in each bisection iteration for solving the relaxed problem (RA) and the optimal power allocation (PA) with different settings of N. As seen in Fig.1, the number of Newton iterations is not large and varies in a narrow range, indicating our proposed algorithm is very efficient.

Fig.2 shows the energy-efficiency of the CR system as a function of the transmission power limit. The numbers of SUs and PUs are 4 and 2, respectively. The static circuit power is set to 0.25W. The minimal rate requirement of the CR system is 10 bits/symbol and the interference threshold of each PU is 5×10^{-12}W. As discussed in Section III, the optimal solution of the relaxation form serves as an upper bound of the original problem, which is not practical for the original problem because the subchannel assignment should be integer in practical systems. We also give the integer subchannel assignment with optimal power allocation (INT-OP) as discussed in Section IV. As seen in Fig.2, the performance of the INT-OP is always quite close to the Upper Bound. We can observe that the energy-efficiency of the CR system grows with the increase of the transmission power budget until the radio resource is sufficient to satisfy all rate requirements. Besides, more subchannels can enhance the energy-efficiency, which results from the channel diversity in wireless environment.

Fig.3 shows the energy-efficiency of CR system as a function of interference threshold for different numbers of PUs [7]. There are 4 SUs and the minimal rate requirement of the CR system is 10 bits/symbol. The transmission power limit is 1W and the static circuit power is set to 0.25W. The number of subchannels is 32. The energy-efficiency increases with the growth of the interference threshold. Additionally, we can observe that more PUs can lower energy-efficiency. The reason is that for either lower interference threshold or more PUs scenarios, more subchannels will be interference limited and fail to maintain the rate requirements. Again, we can see that the INT-OP also always performs close to the Upper Bound.

VI. Conclusion

In this paper, we studied the energy-efficient resource allocation in OFDM-based CR networks. We give the relax form of the formulated mixed integer programming problem by using the time-sharing method, converting it into a quasiconvex one. A bisection algorithm is derived to work out the optimal solution, followed by a fast barrier method which can reduce the complexity dramatically. Simulation results validate that our proposed RA scheme can improve the energy efficiency of the CR system and the proposed algorithm converges quickly.

Appendix

A. Proof of Theorem 2

Proof: Rewrite the KKT system (18) as follows,

$$\Lambda_0 \mu = G_0, \tag{26}$$

where $\mu = \begin{bmatrix} \Delta \mathbf{x}_{nt} \\ \nu \end{bmatrix}$ and $G_0 = \begin{bmatrix} -\nabla \psi_t(\mathbf{x}, z) \\ \mathbf{0}_\nu \end{bmatrix}$. According to the (20), Λ_0 can be written as

$$\Lambda_0 = \begin{bmatrix} D & A^T \\ A & \mathbf{0}_n \end{bmatrix} + \sum_{i=1}^{M} G_i G_i^T, \tag{27}$$

where $G_i = \begin{bmatrix} F_i \\ \mathbf{0}_\nu \end{bmatrix}$,$i = 1, 2, \cdots, M$, where $M = L + 3$. Λ_0 can be decomposed into M equations,

$$\Lambda_i = \Lambda_{i+1} + G_{i+1} G_{i+1}^T, i = 0, 1, \cdots, M - 1, \tag{28}$$

By exploiting the structure of Λ_i's, we give an M-step procedure to compute the Newton step.

First, use the (28) to decompose Λ_0, $\Lambda_0 = \Lambda_1 + G_1 G_1^T$. Denote two intermediate variables as the solutions of the following two sets of linear equations, $\Lambda_1 v_1^1 = G_0$ and $\Lambda_1 v_2^1 = G_1$. Then μ can be obtained by $\mu = v_1^1 - \frac{G_1 v_1^1}{1+G_1 v_2^1} v_2^1$. So we can figure out μ if knowing the v_1^1 and the v_2^1.

Then we can decompose Λ_1 with $\Lambda_1 = \Lambda_2 + G_2 G_2^T$. Then the two variables introduced in step 1 can be updated by solving the following three sets of linear equations, $\Lambda_2 v_i^2 = G_{i-1}$,$i = 1, 2, 3$, where v_1^2, v_2^2 and v_3^2 are other intermediate variables.

For the mth step, decompose Λ_{m-1} with $\Lambda_m = \Lambda_m + G_m G_m^T$. We can update the m variables introduced in Step $m - 1$ by $v_i^{m-1} = v_i^m - \frac{G_m^T v_i^m}{1+G_m v_{m+1}^m} v_{m+1}^m$,$i = 1, 2, \cdots, m$, which is obtained by solving the following $m + 1$ sets of linear equations, $\Lambda_m v_i^m = G_{i-1}$,$i = 1, 2, \cdots, m + 1$.

Continue the procedure to the Mth step, and there are $M+1$ matrix systems $\Lambda_M v_i^M = G_{i-1}$,$i = 1, 2, \cdots, M+1$. Form the derivation process, we can find that the m variables v_i^{m-1},$i = 1, 2, \cdots, m$ at the $(m-1)$th Step can be obtained by the $m+1$ variables v_i^m,$i = 1, 2, \cdots, m + 1$ at the mth Step. Thus, if we figure out the $M + 1$ variables v_i^M,$i = 1, 2, \cdots, M + 1$, μ will be indirectly obtained. The complexity of solving the $M + 1$ matrix systems is $O(KN)$. Obviously, a reverse M-step iteration is necessary when we solve the $M + 1$ matrix systems at the Mth Step. The total computational cost for the fast barrier method is $O(M^2 KN)$. ■

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