

# Cell Planning for Heterogeneous Cellular Networks

Wentao Zhao & Shaowei Wang

School of Electrical Science & Engineering Nanjing University, Nanjing 210093, China

E-mail: zhangwt@smail.nju.edu.cn, wangsw@nju.edu.cn

**Abstract**—Low-power base stations (BSs), such as pico BSs, femto BSs, and relay nodes, are introduced to the heterogeneous cellular networks to enhance coverage and improve system capacity. Compare with macro BS, low-power BS has much lower transmission power, smaller physical size and lower cost. Deploying low-power BSs within the coverage of macro BSs is considered as a cost-efficient way to meet the sharp increase of wireless applications, leading to a new radio network planning paradigm for the next generation cellular networks. In this paper, we study the minimum cost cell planning problem in such heterogeneous networks, where planning task is how to select a subset of possible BS sites, including macro BSs, pico BSs and relay nodes, to minimize the total deployment cost while satisfying all rate requirements of demand nodes. We develop an approximation algorithm to tackle the formulated NP-hard problem, which guarantees an approximation ratio of  $O(\log R)$  to the optimal solution, where  $R$  is the maximal achievable capacity of BSs.

## I. INTRODUCTION

Cell planning is an important issue in deploying cellular networks. For a cellular network, the major design target is to select the locations of BSs and configure their parameters to obtain the best coverage with the least cost. For the second generation cellular systems, cell planning is usually consists of two stages. First, network designer selects a subset of the candidate sites to employ macro BSs, satisfying a given system capacity requirement with cost as low as possible [1–5]. Second, radio frequency is grouped to minimize the interference among adjacent BSs, guaranteeing the QoS of users [2–7]. For the third generation cellular systems, frequency grouping is no longer required because all cells use the same spectrum. The objective of cell planning is to select the BS sites with considering the interference among cells [8–13]. Power control is important for the third generation cellular network in order to decrease interference between adjacent cells.

In the standardization process of the next generation cellular networks, such as 3GPP Long Term Evolution-Advanced (LTE-A), heterogeneous networks are received significant attention and deemed as a cost-efficient way to satisfy the increasing data demand [14–18]. A heterogeneous network consists of macro BSs and low-power BSs, such as pico BSs, femto BSs, and relays. These BSs may coexist in the same geographical area, potentially sharing the same spectrum. Pico BSs are operator-installed BSs with the same backhaul and access features as macro BSs. Relay nodes have the similar sizes of footprints as pico BSs. The backhaul link between a macro BS and its relay is wireless, so no landline is required. Compared to a macro BS, the cost of a low-power BS, including installation cost and maintenance operation cost, is

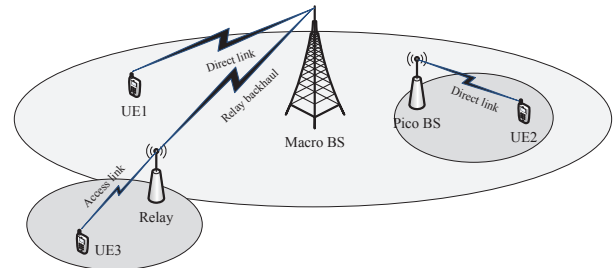


Fig. 1. A heterogeneous cellular network example.

much cheaper. Moreover, due to its lower transmission power and smaller physical size, low-power BSs can offer flexible site acquisitions.

However, cell planning for a cellular system with pico BSs and relays is not studied extensively in the literature. Most of previous research is limited to the deployment of macro BSs, or considering the bandwidth and power allocation to meet traffic requirements. In this paper, we study the minimum cost cell planning in a heterogeneous network, where macro BSs, pico BSs and relay nodes are involved. When a macro BS is open, its subsidiary relay nodes can also be selected. When a relay node is selected for opening, it consumes a part of radio resource of the donor macro BS. The cell planning problem is to select a subset of BSs with minimum cost to supply each demand node the required capacity. To solve the formulated NP-hard problem, we propose an  $O(\log R)$  approximation algorithm, where  $R$  is the maximal achievable transmission rate of BSs.

The remainder of this paper is following. In Section II, we illustrate the minimum cell planning problem and give its mathematical model. In Section III, an approximation algorithm is proposed in detail. In Section IV, numerical results are reported with discussions. Conclusion is drawn in Section V.

## II. PROBLEM FORMULATION

### A. System Model

Consider an area to be served by a heterogeneous cellular network shown in Fig.1, which involves one macro BS, one pico BS, one relay node and three user equipments (UEs). UE1 and UE2 are connected with the macro BS and pico BS, respectively. UE3 is connected with the relay node. The radio link between the macro BS and the relay node is referred to as a backhaul link. UE3 locates far away from the macro BS and accesses the macro BS with the cooperation of relay.

Some frequently used notations are listed in Table I. The sets of possible sites of macro BSs, pico BSs and relay nodes are denoted as  $\mathcal{N}_m$ ,  $\mathcal{N}_p$  and  $\mathcal{N}_r$ , respectively. Denote  $\mathcal{N} = \mathcal{N}_m \cup \mathcal{N}_p \cup \mathcal{N}_r = \{1, 2, \dots, N\}$ . For each macro BS  $n \in \mathcal{N}_m$ , denote  $\mathcal{M}_n$  as the set of relay nodes which can be connected to the macro BS. We assume that  $|\mathcal{M}_n|$  is limited by a constant number and  $\mathcal{M}_n \cap \mathcal{M}_{n'} = \emptyset$ , where  $n, n' \in \mathcal{N}_m$  and  $n \neq n'$ . When a new relay node is selected for opening, the relay node will consume resources from donor macro BS. The resource consumption for a macro BS to serve its relay is predefined, which is  $\gamma_n$  for relay node  $n \in \mathcal{N}_r$  in this work.

For each BS  $n \in \mathcal{N}$ , let  $c_n$  be the cost of BS  $n$ . The total transmission power of BS  $n$  is limited to  $P_{n,max}$ . Normally, the maximum transmission power of macro BS (e.g., 46 dBm) is much higher than that of low-power BS (e.g., 30 dBm). As a result, coverage area of low-power BS is usually much smaller than that of macro BS. The total bandwidth is  $B$  (e.g., 100MHz), which is available in macro BSs, pico BSs and relay nodes.

The set of demand nodes<sup>1</sup> is denoted as  $\mathcal{K} = \{1, 2, \dots, K\}$ . Each demand node  $k \in \mathcal{K}$  has a constant transmission rate requirement of  $R_{k,min}$ . Without loss of generality, we assume that  $R_{k,min} > 0$  for each demand node. For each pair of demand node  $k \in \mathcal{K}$  and BS  $n \in \mathcal{N}$ , denote  $h_{k,n}$  as the average channel gain between BS  $n$  and demand node  $k$ .  $h_{k,n}$  can be estimated as a function of distance between them by the pass-loss propagation model, which encompasses the antenna gain, path loss and shadowing. Denote  $b_{k,n}$  and  $p_{k,n}$  as the bandwidth and power of BS  $n$  allocated to demand node  $k$ , respectively. In this work, we only consider the achievable rate of BS  $n$  to demand node  $k$ , which can be calculated as

$$r_{k,n} = b_{k,n} \log_2 \left( 1 + \frac{p_{k,n} |h_{k,n}|^2}{\Gamma N_0 b_{k,n}} \right), \quad (1)$$

where  $\Gamma$  is the SNR gap. Generally,  $\Gamma$  is a constant that is related to a given bit-error-rate (BER) for a specific modulation/demodulation scheme, for example  $\Gamma = -\ln(5\text{BER})/1.6$  for an uncoded multilevel quadrature amplitude modulation (MQAM) modulation system [20].  $N_0$  is the power spectral density of additive white Gaussian noise (AWGN).

### B. Minimum Cost Cell Planning

For heterogeneous cellular networks cell planning, we try to select a subset of  $\mathcal{N}$  with the minimal cost to satisfy all rate requirements of  $\mathcal{K}$ , while the allocated power and bandwidth of each selected BS cannot exceed a given threshold. Each demand node can be satisfied by more than one BS as mentioned above.

Let  $z_n$  be the selection variable of BS  $n$ , and

$$z_n = \begin{cases} 1 & \text{BS } n \text{ is selected for opening,} \\ 0 & \text{otherwise,} \end{cases} \quad \forall n \in \mathcal{N}. \quad (2)$$

<sup>1</sup>Demand node is an abstraction of average traffic demands in a small area, which leads to the fact that a demand node which locates in an overlapped area can be satisfied by multiple BSs simultaneously [19].

TABLE I  
NOTATIONS

Symbol	Semantics
$\mathcal{N}_m$	The sets of possible sites of macro BSs
$\mathcal{N}_p$	The sets of possible sites of pico BSs
$\mathcal{N}_r$	The sets of possible sites of relay nodes
$\mathcal{M}_n$	The sets of relay nodes that can be connected to BS $n$
$\mathcal{N}$	The sets of possible sites of all BSs
$N$	The number of all BSs
$c_n$	The cost of BS $n$
$z_n$	The selection variable of BS $n$
$P_{n,max}$	The maximum transmission power of BS $n$
$B$	The total bandwidth
$\mathcal{K}$	The sets of demand nodes
$K$	The number of demand nodes
$R_{k,min}$	The required rate of demand node $k$
$b_{k,n}$	The bandwidth of BS $n$ allocated to demand node $k$
$p_{k,n}$	The power of BS $n$ allocated to demand node $k$
$h_{k,n}$	The channel gain between BS $n$ to demand node $k$
$r_{k,n}$	The achievable rate of BS $n$ to demand node $k$
$\Gamma$	The SNR gap
$N_0$	The power spectral density of AWGN
$\gamma_n$	The resource consumption by relay node $n$

Only if macro BS  $n$  is selected, relay node  $n' \in \mathcal{M}_n$  can be selected for opening. It holds that

$$z_{n'} \leq z_n, \forall n \in \mathcal{N}_m, \forall n' \in \mathcal{M}_n. \quad (3)$$

Then, the optimization problem can be formulated as,

$$\begin{aligned} \min_{z_n, p_{k,n}, b_{k,n}} \quad & \sum_{n \in \mathcal{N}} c_n z_n \\ \text{s.t.} \quad & C_1 : \sum_{k \in \mathcal{K}} b_{k,n} \leq z_n B, \forall n \in \mathcal{N}_p \cup \mathcal{N}_r, \\ & C_2 : \sum_{k \in \mathcal{K}} b_{k,n} + \gamma_n \sum_{\substack{n' \in \mathcal{M}_n \\ k \in \mathcal{K}}} b_{k,n'} \leq z_n B, \forall n \in \mathcal{N}_m, \\ & C_3 : \sum_{k \in \mathcal{K}} p_{k,n} \leq z_n P_{n,max}, \forall n \in \mathcal{N}_p \cup \mathcal{N}_r, \\ & C_4 : \sum_{k \in \mathcal{K}} p_{k,n} + \gamma_n \sum_{\substack{n' \in \mathcal{M}_n \\ k \in \mathcal{K}}} p_{k,n'} \leq z_n P_{n,max}, \\ & \quad \forall n \in \mathcal{N}_m, \\ & C_5 : \sum_{n \in \mathcal{N}} r_{k,n} = R_{k,min}, \forall k \in \mathcal{K}, \\ & C_6 : b_{k,n} \geq 0, p_{k,n} \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N}, \\ & C_7 : \text{Eq.(1)} \sim \text{Eq.(3)}. \end{aligned} \quad (4)$$

$C_1 \sim C_4$  mean that the allocated bandwidth and power of the selected BS  $n$  are limited to  $B$  and  $P_{n,max}$ , respectively.  $C_2$  and  $C_4$  indicate that each selected relay node consumes a part of resources from donor macro BS.  $C_5$  is the minimum rate constraint.

### III. $O(\log R)$ -APPROXIMATION ALGORITHM

Before introducing the approximation algorithm for Eq.(4), we need to obtain the maximum total transmission rates (the demand nodes may not be fully satisfied) that can be satisfied by a given set of BSs. By using this information, we can adopt a greedy algorithm which generalizes the result of [21] to solve Eq.(4).

### A. Bandwidth and Power Allocation

The bandwidth and power allocation problem can be formulated as follows: Given a set of macro BSs  $\mathcal{N}'_m$ , a set of pico BSs  $\mathcal{N}'_p$  and a set of relay nodes  $\mathcal{N}'_r$ , how to maximize the achievable sum of rates that can be satisfied by these BSs, while the satisfied rates of each demand node  $k$  cannot exceed  $R_{k,min}$ . Without loss of generality, we assume  $\mathcal{N}'_r \setminus \bigcup_{n \in \mathcal{N}'_m} \mathcal{M}_n = \emptyset$ , where  $X \setminus Y = \{x | x \in X, x \notin Y\}$ . Note that each demand node is not strictly fully satisfied at this time.

Let  $\mathcal{N}' = \mathcal{N}'_m \cup \mathcal{N}'_p \cup \mathcal{N}'_r$ . Then, the optimization problem can be mathematically formulated as,

$$\begin{aligned}
& \max_{b_{k,n}, p_{k,n}} \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} r_{k,n} \\
s.t. \quad & C_1: b_{k,n} \geq 0, p_{k,n} \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N}', \\
& C_2: \sum_{k \in \mathcal{K}} b_{k,n} \leq B, \forall n \in \mathcal{N}'_p \cup \mathcal{N}'_r, \\
& C_3: \sum_{k \in \mathcal{K}} b_{k,n} + \gamma_n \sum_{\substack{n' \in \mathcal{M}_n \cap \mathcal{N}'_r \\ k \in \mathcal{K}}} b_{k,n'} \leq B, \forall n \in \mathcal{N}'_m, \\
& C_4: \sum_{k \in \mathcal{K}} p_{k,n} \leq P_{n,max}, \forall n \in \mathcal{N}'_p \cup \mathcal{N}'_r, \\
& C_5: \sum_{k \in \mathcal{K}} p_{k,n} + \gamma_n \sum_{\substack{n' \in \mathcal{M}_n \cap \mathcal{N}'_r \\ k \in \mathcal{K}}} p_{k,n'} \leq P_{n,max}, \\
& \quad \forall n \in \mathcal{N}'_m, \\
& C_6: \sum_{n \in \mathcal{N}'} r_{k,n} \leq R_{k,min}, \forall k.
\end{aligned} \tag{5}$$

Obviously,  $\mathcal{N}'$  is a feasible solution of Eq.(4) when the optimal value of above problem is  $\sum_{k \in \mathcal{K}} R_{k,min}$ . However, the inequality constraint C6 in Eq.(5) is concave, making the set of solutions to Eq.(5) not convex [22]. We give a general transformation that yields an equivalent problem. Obviously,

$$p_{k,n} = \frac{b_{k,n}}{H_{k,n}} \cdot \left( 2^{r_{k,n}/b_{k,n}} - 1 \right),$$

where  $H_{k,n} = h_{k,n}^2 / \Gamma N_0$ . The equivalent problem of Eq.(5) can be described as

$$\begin{aligned}
& \max_{b_{k,n}, r_{k,n}} \sum_{n \in \mathcal{N}'} \sum_{k \in \mathcal{K}} r_{k,n} \\
s.t. \quad & C_1: b_{k,n} \geq 0, r_{k,n} \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N}', \\
& C_2 \text{ to } C_6 \text{ in Eq.(5)}.
\end{aligned} \tag{6}$$

It is easy to prove that Eq.(6) defines a convex problem, which can be solved by standard convex optimization techniques. Then the optimal value of Eq.(6) can be used to solve the minimum cost cell planning problem by a greedy approach.

### B. Proposed Algorithm

Given a set  $\mathcal{N}_1 \subseteq \mathcal{N}$  of BSs, let  $w(\mathcal{N}_1)$  be the optimal value of Eq.(6). Denote  $c(\mathcal{N}_1)$  as the total installation cost of BSs in  $\mathcal{N}_1$ . For each  $\mathcal{N}_2 \subseteq \mathcal{N}$ , we define

$$w_{\mathcal{N}_1}(\mathcal{N}_2) = w(\mathcal{N}_1 \cup \mathcal{N}_2) - w(\mathcal{N}_1),$$

and

$$W_{\mathcal{N}_1}(\mathcal{N}_2) = \frac{w_{\mathcal{N}_1}(\mathcal{N}_2)}{c(\mathcal{N}_2)}.$$

TABLE II  
ALGORITHM 1:  $O(\log R)$ -APPROXIMATION ALGORITHM

---

```

1:  $\mathcal{N}_l \leftarrow \emptyset, \tilde{\mathcal{N}} \leftarrow \mathcal{N}, \mathcal{N}'_m \leftarrow \emptyset, l = 0.$ 
2: while  $w(\mathcal{N}_l) < \sum_{k \in \mathcal{K}} R_{k,min}$  do
3:    $l = l + 1, W = 0;$ 
4:   for all  $n \in \tilde{\mathcal{N}} \setminus \mathcal{N}'_r$  do
5:     find  $i \subseteq \mathcal{M}_n$  that maximizes  $W_n = W_{\mathcal{N}_l}(\{n\} \cup i);$ 
6:     if  $W_n > W$ 
7:        $n_l = \{n\}, i_l = i, W = W_n;$ 
8:     end if
9:   end for
10:  for all  $n \in \mathcal{N}'_m$  do
11:    find  $i \subseteq \mathcal{M}_n \setminus \mathcal{N}_l, |i| = 1$  that maximizes  $W'_n = W_{\mathcal{N}_l}(i);$ 
12:    if  $W'_n > W$ 
13:       $n_l = \emptyset, i_l = i, W = W'_n;$ 
14:    end if
15:  end for
16:   $\mathcal{N}_l \leftarrow \mathcal{N}_l \cup n_l \cup i_l, \tilde{\mathcal{N}} \leftarrow \tilde{\mathcal{N}} \setminus (n_l \cup i_l), \mathcal{N}'_m \leftarrow \mathcal{N}'_m \cup n_l;$ 
17: end while
18: return  $\mathcal{N}_l$ 

```

---

The outline of the approximation algorithm for Eq.(4) is described in Table II. The algorithm of finding  $G_i \subseteq \mathcal{M}_n$  to maximize  $W_n = W_{\mathcal{N}_l}(\{n\} \cup G_i)$  is given in Table III.

Let  $\mathcal{N}_l$  be the set of BSs selected at the end of  $l$ th iteration of Algorithm 1. Without loss of generality, let  $n_l$  and  $i_l$  be the set of selected macro BS or pico BS ( $n_l$  may be an emptyset or a set of a single BS by Algorithm 1) and the set of selected relay node ( $i_l$  also may be an emptyset or a set relay nodes) that add to the solution at the  $l$ th iteration, respectively. Also, let  $G_l = n_l \cup i_l$  be the set of selected BSs that add to the solution at the  $l$ th iteration.

Before proving the approximation ratio, we need the following facts and lemma.

**Fact 1.** Given positive numbers  $a_1, \dots, a_n$  and  $b_1, \dots, b_n$ , then

$$\max_{i=1, \dots, n} \frac{a_i}{b_i} \geq \frac{\sum_{i=1}^n a_i}{\sum_{i=1}^n b_i}.$$

**Fact 2.** Given positive numbers  $a_0, a_1$  and  $b_0, b_1$ , if

$$\frac{a_0}{b_0} \geq \frac{a_0 + a_1}{b_0 + b_1},$$

it always holds

$$\frac{a_0}{b_0} \geq \frac{a_1}{b_1},$$

and vice versa.

**Fact 3.** Given positive numbers  $a_0, a_1, a_2$  and  $b_0, b_1, b_2$ , if

$$\frac{a_0 + a_1}{b_0 + b_1} \geq \frac{a_2}{b_2}, \frac{a_1}{b_1} \geq \frac{a_0}{b_0},$$

it always holds

$$\frac{a_0 + a_1}{b_0 + b_1} \geq \frac{a_0 + a_2}{b_0 + b_2}.$$

**Lemma 1.** At the  $l$ th iteration of Algorithm 1, it always holds

$$W_{\mathcal{N}_{l-1}}(G_l) \geq W_{\mathcal{N}_{l-1}}(\{n\} \cup G), \forall n \in \mathcal{N}_m \setminus \mathcal{N}_{l-1}, G \subseteq \mathcal{M}_n. \tag{7}$$

TABLE III  
ALGORITHM 2: RELAY SELECTION

---

```

1:  $W' = W_{\mathcal{N}_i}(\{n\}), G_i = \emptyset;$ 
2:  $M_n^* = \{G^* | W_{\mathcal{N}_i \cup \{n\}}(G^*) > W', G^* \subseteq \mathcal{M}_n, |G^*| = 1\};$ 
3: if  $M_n^* \neq \emptyset$ 
4:    $\gamma = 1, M'_n = M_n^*;$ 
5:   while  $M'_n \neq \emptyset$  and  $\gamma \leq |\mathcal{M}_n|$ 
6:     find  $G \subseteq M'_n$  that maximizes  $W_{\mathcal{N}_i}(\{n\} \cup G);$ 
7:     if  $W_{\mathcal{N}_i}(\{n\} \cup G) > W'$ 
8:        $W' = W_{\mathcal{N}_i}(\{n\} \cup G), G_i = G;$ 
9:        $M_n^* = \{G^* | W_{\mathcal{N}_i \cup \{n\}}(G^*) > W', G^* \subseteq \mathcal{M}_n, |G^*| = 1\};$ 
10:    end if
11:     $M'_n = \{G' \cup G^* | W_{\mathcal{N}_i \cup \{n\}}(G') > W', G' \in M'_n, G^* \in M_n^*,$ 
       $G' \cap G^* = \emptyset\};$ 
12:     $\gamma = \gamma + 1;$ 
13:  end while
14: end if
15: return  $W', G_i$ 

```

---

*Proof:* First, we need to prove that Algorithm 2 can always find  $G_i \subseteq \mathcal{M}_n$  to maximize  $W_{\mathcal{N}_i}(\{n\} \cup G_i)$ . If

$$W_{\mathcal{N}_{i-1}}(\{n\}) \geq W_{\mathcal{N}_{i-1} \cup \{n\}}(G), \forall G \subseteq \mathcal{M}_n,$$

we can get

$$\begin{aligned} \frac{w_{\mathcal{N}_{i-1}}(\{n\})}{c_n} &\geq \max_{|G|=1, G \subseteq \mathcal{M}'_n} \frac{w_{\mathcal{N}_{i-1} \cup \{n\}}(G)}{c(G)} \\ &\geq \frac{\sum_{|G|=1, G \subseteq \mathcal{M}'_n} w_{\mathcal{N}_{i-1} \cup \{n\}}(G)}{\sum_{|G|=1, G \subseteq \mathcal{M}'_n} c(G)} \\ &\geq \frac{w_{\mathcal{N}_{i-1} \cup \{n\}}(\mathcal{M}'_n)}{c(\mathcal{M}'_n)}, \forall \mathcal{M}'_n \subseteq \mathcal{M}_n, \end{aligned}$$

where follows by  $\sum_{|G|=1, G \subseteq \mathcal{M}'_n} w_{\mathcal{N}_{i-1} \cup \{n\}}(G) \geq w_{\mathcal{N}_{i-1} \cup \{n\}}(\mathcal{M}'_n)$  and Fact 1. According to Fact 2, we can obtain

$$W_{\mathcal{N}_{i-1}}(\{n\}) \geq W_{\mathcal{N}_{i-1}}(\{n\} \cup \mathcal{M}'_n), \forall \mathcal{M}'_n \subseteq \mathcal{M}_n.$$

On the contrary, we can ignore relay node  $n'$  which satisfies  $W_{\mathcal{N}_{i-1} \cup \{n\}}(\{n'\}) \leq W_{\mathcal{N}_{i-1}}(\{n\})$  since we have

$$W_{\mathcal{N}_{i-1}}(\{n\}) \geq W_{\mathcal{N}_{i-1}}(\{n\} \cup \{n'\}),$$

which follows Fact 2. Similarly, we also ignore all unattractive combinations of relay nodes. Using Fact 3, we can obtain that it holds  $W_{\mathcal{N}_i}(\{n\} \cup \mathcal{M}'_n) \leq W'$  if  $W_{\mathcal{N}_i \cup \{n\}}(\mathcal{M}'_n) \leq W'$ . Hence, when  $M'_n = \emptyset$  as shown in line 5 of Algorithm 2, all attractive combinations have been searched.

Second, for unselected relay nodes of macro BS  $n \in \mathcal{N}_m \cap \mathcal{N}_i$ , we ignore all the combination of relay nodes  $G' \subseteq \mathcal{M}_n \setminus \mathcal{N}_i$  since

$$\begin{aligned} \max_{n' \in G'} W_{\mathcal{N}_{i-1}}(\{n'\}) &\geq \frac{\sum_{n' \in G'} f_{\mathcal{N}_{i-1}}(\{n'\})}{\sum_{n' \in G'} c_{n'}} \\ &\geq \frac{\sum_{n' \in G'} f_{\mathcal{N}_{i-1}}(G')}{\sum_{n' \in G'} c_{n'}} \\ &= W_{\mathcal{N}_{i-1}}(G'). \end{aligned}$$

Given a cell planning instance, let  $z_n^*, b_{k,n}^*, p_{k,n}^*$  be an optimal solution of Eq.(4). Let  $r_{k,n}^* = b_{k,n}^* \log_2(1 + p_{k,n}^* H_{k,n} / b_{k,n}^*)$  be the corresponding transmission rate. Denote  $\mathcal{N}_m^* = \{n | z_n^* = 1, n \in \mathcal{N}_m\}$ ,  $\mathcal{N}_p^* = \{n | z_n^* = 1, n \in \mathcal{N}_p\}$  and  $\mathcal{N}_r^* = \{n | z_n^* = 1, n \in \mathcal{N}_r\}$  as the set of selected macro BSs, pico BSs and relay nodes in the optimal solution, respectively.

We temporarily treats the macro BS and its associated relay nodes as a unity. Denote  $\mathcal{N}_n^* = \{n\} \cup (\mathcal{N}_r^* \cap \mathcal{M}_n)$  as the set of macro BS  $n$  and its associated relay nodes which selected in the optimal solution,  $\forall n \in \mathcal{N}_m^*$ . Define  $\mathcal{N}^* = \mathcal{N}_m^* \cup \mathcal{N}_p^*$ . For all  $n \in \mathcal{N}^*$ , denote  $R_n$  as the total transmission rates of BS  $n$ , where

$$R_n = \begin{cases} \sum_{k \in \mathcal{K}} r_{k,n}^* + \sum_{n' \in \mathcal{N}_r^* \cap \mathcal{M}_n} \sum_{k \in \mathcal{K}} r_{k,n'}^* & n \in \mathcal{N}_m^*, \\ \sum_{k \in \mathcal{K}} r_{k,n}^* & \text{otherwise.} \end{cases}$$

Each macro BS  $n \in \mathcal{N}_m^*$  satisfies  $R_n$  rate requirements with the total cost of  $c(\mathcal{N}_n^*)$ . Each pico BS  $n' \in \mathcal{N}_p^*$  satisfies  $R_{n'}$  rate requirements with the total cost of  $c_{n'}$ .

**Theorem 1.** *Algorithm 1 achieves an approximation factor of  $O(\log R)$  for the minimum cost cell planning problem, where  $R = \max_{n \in \mathcal{N}} R_n$ .*

*Proof:* Let  $z'_n, b'_{k,n}, p'_{k,n}$  be the solution of Algorithm 1. Denote  $L$  as the number of iterations and  $\mathcal{N}_L = \{n | z'_n = 1, n \in \mathcal{N}\}$  as the corresponding set of selected BSs by Algorithm 1.

We inductively define for each iteration  $l$  and  $n \in \mathcal{N}^* \setminus \mathcal{N}_l$  a value  $a_l(n)$ . This value is a part of the rate requirements  $R_n$  that still not be satisfied after  $l$  iterations. If we add BS  $n$  (if  $n$  is macro BS, also add the associated relay nodes) to  $\mathcal{N}_l$ , the remaining rate requirements  $a_l(n)$  obviously can be satisfied. The following equations always hold,

$$a_l(n) \leq \begin{cases} w_{\mathcal{N}_l}(\mathcal{N}_n^*) - w(\mathcal{N}_l), & \forall n \in \mathcal{N}_m^*, \\ w_{\mathcal{N}_l}(\{n\}) - w(\mathcal{N}_l), & \forall n \in \mathcal{N}_p^*. \end{cases} \quad (8)$$

Now we consider macro BS  $n \in \mathcal{N}_m^* \setminus \mathcal{N}_l$ . According to Eq.(7) and Eq.(8), for each iteration  $l$ , it holds

$$\begin{aligned} W_{\mathcal{N}_{l-1}}(G_l) &\geq \max_{|G| \subseteq \mathcal{M}_n} W_{\mathcal{N}_{l-1}}(\{n\} \cup (G)) \\ &\geq W_{\mathcal{N}_{l-1}}(\mathcal{N}_n^*) \\ &\geq a_{l-1}(n)/c(\mathcal{N}_n^*). \end{aligned}$$

As same as macro BS, for each pico BS  $n' \in \mathcal{N}_p^* \setminus \mathcal{N}_l$ , we also can obtain

$$W_{\mathcal{N}_{l-1}}(G_l) \geq a_{l-1}(n')/c_{n'}.$$

Now, we charge the cost of the BSs that are chosen by Algorithm 1 to the optimal solution. If Algorithm 1 adds  $n \in \mathcal{N}^*$  to  $\mathcal{N}_{l-1}$  at the  $l$ th iteration, we do not charge any selected macro BSs and relay nodes for their cost, since  $\mathcal{N}^*$  also pays for it. Otherwise, we charge each  $\mathcal{N}^* \setminus \mathcal{N}_l$  with

$$c_l(n) = (a_{l-1}(n) - a_l(n)) \cdot W_{\mathcal{N}_{l-1}}(G_l).$$

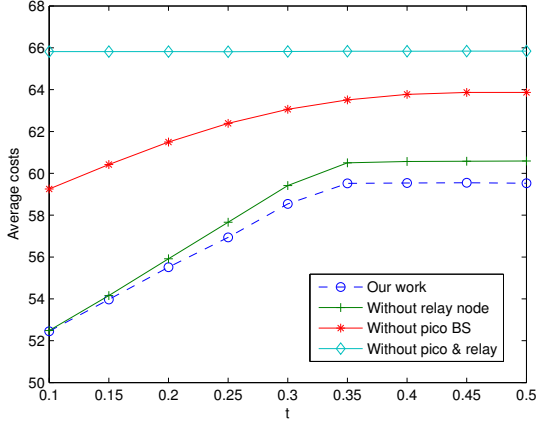


Fig. 2. Average cost as a function of  $t$ .  $N = 450$ ,  $K = 400$ ,  $B = 100\text{MHz}$ ,  $R_{k,\min} = 1\text{Mbps}$  for all  $k$ ,  $P_{n,\max} = 40\text{W}$ ,  $\forall n \in \mathcal{N}_m$ ,  $P_{n,\max} = 1\text{W}$ ,  $\forall n \in \mathcal{N}_p \cup \mathcal{N}_r$ .

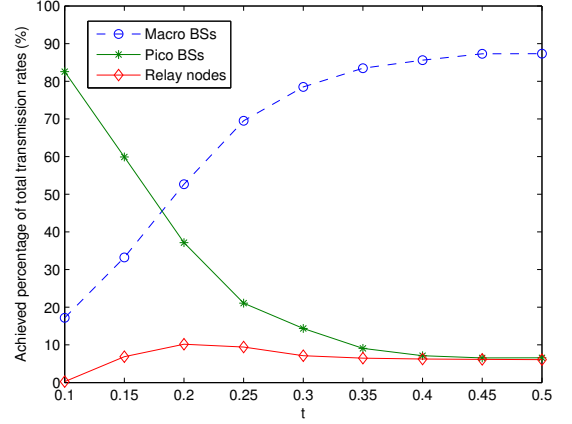


Fig. 3. Average achieved percentage of total transmission rates as a function of  $t$ .  $N = 450$ ,  $K = 400$ ,  $B = 100\text{MHz}$ ,  $R_{k,\min} = 1\text{Mbps}$  for all  $k$ ,  $P_{n,\max} = 40\text{W}$ ,  $\forall n \in \mathcal{N}_m$ ,  $P_{n,\max} = 1\text{W}$ ,  $\forall n \in \mathcal{N}_p \cup \mathcal{N}_r$ .

For each  $n \in \mathcal{N}_m^*$ . The cost charged upon  $\mathcal{N}_n^*$  is

$$\begin{aligned} \sum_{l=1}^L c_l(n) &= \sum_{l=1}^L (a_{l-1}(n) - a_l(n)) W_{\mathcal{N}_{l-1}}(G_l) \\ &\leq c(\mathcal{N}_n^*) \sum_{l=1}^L \frac{a_{l-1}(n) - a_l(n)}{a_{l-1}(n)} \\ &= c(\mathcal{N}_n^*) \cdot H(R_n) \\ &= c(\mathcal{N}_n^*) \cdot O(\log R_n), \end{aligned}$$

where  $H(r)$  is the  $r$ th harmonic number. For each pico BS, we also can obtain  $n' \in \mathcal{N}_p^*$ ,

$$\sum_{l=1}^L c_l(n') \leq c_{n'} \cdot O(\log R_{n'}).$$

Hence, it always holds

$$c(\mathcal{N}_L) \leq O(\log R) \cdot c(\mathcal{N}_m^* \cup \mathcal{N}_p^* \cup \mathcal{N}_r^*).$$

#### IV. SIMULATION RESULTS

In this section, we give numerical results of the proposed algorithm. For comparison, the deployment of BSs without pico BSs, the deployment of BSs without relay nodes and the deployment of macro BSs only are also evaluated. Simulation parameters, such as path-loss models, maximum transmission power, total available bandwidth, etc. are based on specifications proposed in [23].

The deployment area is  $5 \times 5 \text{ km}^2$ . There are 50 possible sites of macro BSs, 100 possible sites of pico BSs and 400 demand nodes in the deployment area randomly. Each possible site of BS is assigned a position with uniform distribution in the deployment area. Each macro BS can connect to 6 relay nodes. Each relay node is distributed at the edge of its donor macro BS with uniform distribution. The total bandwidth is 100 MHz. The maximum transmission power of macro BS is 46dBm and the cost of each is distributed uniformly in the

interval  $(8, 12)$ . For each pico BS, the transmission power is limited by 30dBm and the cost is distributed uniformly in the interval  $(8 \cdot t, 12 \cdot t)$ , where  $t$  is the ratio of average cost of pico to average installation cost of macro BS. The transmission power of relay node is also limited by 30dBm, and the cost is set to  $(6 \cdot t, 8 \cdot t)$ . The resource consumption  $\gamma$  by one relay node is set to 0.05.

Each demand node is assigned a position with uniform distribution in the deployment area and has a rate requirement of 1Mbps. We model path loss (in dB) from macro BS to demand node as  $128.1 + 37.6 \log_{10}(D)$  and path loss from low-power BS to demand node as  $140.7 + 36.7 \log_{10}(D)$  for distance  $D$  in km, lognormal shadowing with a standard deviation 10 dB. The noise power spectral density is -174 dBm/Hz and  $\Gamma$  is set to 7.6288 ( $BER = 10^{-6}$ ).

First, we study average total cost for Eq.(4) over  $t$ , which varies from 0.1 to 0.5. The results are shown in Fig.2, where we can see that the performance of our proposed deployment scheme is better than the case without low-power BSs, even though the cost of low-power BS is not attractive. The average cost of our deployment scheme is 20% and 10% lower than that of the macro BSs only for  $t = 0.1$  and  $t = 0.5$ , respectively. When  $t \geq 0.35$ , the cost of our proposed deployment scheme keeps almost unchanged. In this case, very few low-power BSs can be selected.

In Fig.2, we can observe that the performance of deployment without relays is very close to our proposed one, both of which outperform the case without pico BSs, even though relay is cheaper than pico BS. We investigate the average achievable transmission rate as a function of  $t$  in Fig.3, where the transmission rate of each selected BS is obtained by the optimal solution of Eq.(6).  $t$  varies from 0.1 to 0.5. It can be observed from Fig.3 that the macro BSs serve a small part of total rate requirements when  $t = 0.1$ , which means only a few of macro BSs are selected. Almost no relay is open. Pico BSs serve about 80% total transmission rates. With  $t$

increases, more and more macro BSs are selected for opening since the cost of pico BS becomes unattractive. It is worth noticing that relay nodes have more opportunity to be selected for helping expand coverage and capacity expansion. However, they consume resources of macro BSs. When  $t \geq 0.35$ , the transmission rate becomes invariant, low-power BSs serve about only 17% rate requirements.

## V. CONCLUSION

In this paper we studied the minimum cost cell planning for a heterogeneous cellular network. Different from most of the existing research in cell planning, which is limited to the deployment of macro BSs, we also consider pico BSs and relay nodes in heterogeneous cellular networks. Given the candidate sets of macro BSs, pico BSs, relay nodes and demand nodes, the formulated minimum cost cell planning problem is NP-hard and no efficient algorithm can work out the optimal solutions. We proposed an  $O(\log R)$ -approximation algorithm, which can solve the problem effectively and efficiently, as shown in the numerical results. Our proposal throws some insights on how to deploy heterogenous networks for the next generation cellular system.

## ACKNOWLEDGEMENT

This work was partially supported by the JiangsuSF (BK2011051), Huawei Technologies Co., Ltd (Y-B2012120195, YBWL2012SKL09) and the Fundamental Research Funds for the Central Universities.

## REFERENCES

- [1] C. Lee and H. Kang, "Cell planning with capacity expansion in mobile communications: a tabu search approach," *IEEE Transactions on Vehicular Technology*, vol. 49, no. 5, pp. 1678–1691, Sep. 2000.
- [2] S. Hurley, "Planning effective cellular mobile radio networks," *IEEE Transactions on Vehicular Technology*, vol. 51, no. 2, pp. 243–253, Mar. 2002.
- [3] J. Kalvenes, J. Kennington, and E. Olinick, "Hierarchical cellular network design with channel allocation," *European Journal of Operational Research*, vol. 160, no. 1, pp. 3–18, Jan. 2005.
- [4] N. Weicker, G. Szabo, K. Weicker, and P. Widmayer, "Evolutionary multiobjective optimization for base station transmitter placement with frequency assignment," *IEEE Transactions on Evolutionary Computation*, vol. 7, no. 2, pp. 189–203, Apr. 2003.
- [5] A. Dutta and V. Hsu, "Cellular network design site selection and frequency planning," *Annals of Operations Research*, vol. 106, no. 1, Sep. 2001.
- [6] R. Mathar and T. Niessen, "Optimum positioning of base stations for cellular radio networks," *Wireless Networks*, vol. 6, no. 6, pp. 421–428, Dec. 2000.
- [7] R. Mathar and M. Schmeink, "Integrated optimal cell site selection and frequency allocation for cellular radio networks," *Telecommunication Systems*, vol. 21, no. 2, pp. 339–347, Dec. 2002.
- [8] E. Amaldi, A. Capone, and F. Malucelli, "Planning UMTS base station location: optimization models with power control and algorithms," *IEEE Transactions on Wireless Communications*, vol. 2, no. 5, pp. 939–952, Sep. 2003.
- [9] E. Amaldi and A. Capone, "Radio planning and coverage optimization of 3G cellular networks," *Wireless Networks*, vol. 14, no. 4, pp. 435–447, Aug. 2008.
- [10] A. Kamar, S. Nawaz, M. Patwary, M. Abdel-Maguid, and S.-U.-R. Qureshi, "Optimized algorithm for cellular network planning based on terrain and demand analysis," in *Proc. ICCTD 2010*, Cairo, Egypt, Nov. 2010, pp. 359–364.
- [11] H.-Y. Zhang, Y.-G. Xi, and G. H.-Y., "A rolling window optimization method for large-scale WCDMA base stations planning problem," *European Journal of Operational Research*, vol. 183, no. 1, pp. 370–383, Nov. 2007.
- [12] R. Mathar and M. Schmeink, "Optimal base station positioning and channel assignment for 3G mobile networks by integer programming," *Annals of Operations Research*, vol. 107, no. 1, pp. 225–236, Oct. 2001.
- [13] J. Yang, M. Aydin, J. Zhang, and C. Maple, "UMTS base station location planning: a mathematical model and heuristic optimisation algorithms," *IET Communications*, vol. 1, no. 5, pp. 1007–1014, Oct. 2007.
- [14] J. Sydir and R. Taori, "An evolved cellular system architecture incorporating relay stations," *IEEE Communications Magazine*, vol. 47, no. 6, pp. 115–121, June 2009.
- [15] Y. Yang, H. Hu, J. Xu, and G. Mao, "Relay technologies for WiMax and LTE-advanced mobile systems," *IEEE Communications Magazine*, vol. 47, no. 10, pp. 100–105, Oct. 2009.
- [16] S.-P. Yeh, S. Talwar, G. Wu, N. Himayat, and K. Johansson, "Capacity and coverage enhancement in heterogeneous networks," *IEEE Wireless Communications*, vol. 18, no. 3, pp. 32–38, June 2011.
- [17] S. Parkvall, A. Furuskar, and E. Dahlman, "Evolution of LTE toward IMT-advanced," *IEEE Communications Magazine*, vol. 49, no. 2, pp. 84–91, Feb. 2011.
- [18] I. Guvenc, "Capacity and fairness analysis of heterogeneous networks with range expansion and interference coordination," *IEEE Communications Letters*, vol. 15, no. 10, pp. 1084–1087, Oct. 2011.
- [19] K. Tutschku, "Demand-based radio network planning of cellular mobile communication systems," in *Proc. IEEE INFOCOM '98*, vol. 3, Mar. 1998, pp. 1054–1061.
- [20] A. Goldsmith and C. S.-G., "Variable-rate variable-power MQAM for fading channels," *IEEE Transactions on Communications*, vol. 45, no. 10, pp. 1218–1230, Oct. 1997.
- [21] J. Chuzhoy and J. Naor, "Covering problems with hard capacities," in *Proc. IEEE FOCS*, Vancouver, Nov. 2002, pp. 481–489.
- [22] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press: New York, 2004.
- [23] 3GPP, "Evolved universal terrestrial radio access (E-UTRA); further advancements for E-UTRA physical layer aspects (TR 36.814)," Mar. 2010.